A Theoretical Model for Thin Film Ferroelectric Coupled Microstripline Phase Shifters

R.R. Romanofsky
Glenn Research Center, Cleveland, Ohio

A.H. Qureshi
Cleveland State University, Cleveland, Ohio

March 2000
The NASA STI Program Office ... in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the Lead Center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.

- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office’s diverse offerings include creating custom thesauri, building customized data bases, organizing and publishing research results ... even providing videos.

For more information about the NASA STI Program Office, see the following:


- E-mail your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA Access Help Desk at (301) 621-0134

- Telephone the NASA Access Help Desk at (301) 621-0390

- Write to: NASA Access Help Desk NASA Center for AeroSpace Information 7121 Standard Drive Hanover, MD 21076
A Theoretical Model for Thin Film Ferroelectric Coupled Microstripline Phase Shifters

R.R. Romanofsky
Glenn Research Center, Cleveland, Ohio

A.H. Qureshi
Cleveland State University, Cleveland, Ohio

Prepared for the
International Magnetics Conference, Intermag 2000
sponsored by the Magnetics Society of the Institute of Electrical and
Electronics Engineers
Toronto, Canada, April 9–13, 2000

National Aeronautics and
Space Administration

Glenn Research Center

March 2000
A Theoretical Model for Thin Film Ferroelectric Coupled Microstripline Phase Shifters

R.R. Romanofsky*
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135
Phone: 216-433-3507
e-mail: Robert.R.Romanofsky@grc.nasa.gov

and

A.H. Qureshi
Cleveland State University
Cleveland, Ohio 44115
Phone: 440-892-7936
e-mail: qureshi@csx.csuohio.edu

Abstract—Novel microwave phase shifters consisting of coupled microstrip lines on thin ferroelectric films have been demonstrated recently. A theoretical model useful for predicting the propagation characteristics (insertion phase shift, dielectric loss, impedance, and bandwidth) is presented here. The model is based on a variational solution for line capacitance and coupled strip transmission line theory.

Index Terms—Ferroelectricity, Phase shifter circuits

I. INTRODUCTION

Recently, microwave phase shifters have been demonstrated that used thin ferroelectric films and superconducting or normal metal coupled microstrip lines that also served as biasing electrodes [1,2]. The insertion loss of these Ku- and K-band phase shifters was substantially better than their semiconductor microwave integrated circuit counterparts. More than 400 of continuous phase shift with about 4 dB loss at 77 K was obtained using YBa2Cu3O7-δ electrodes and the incipient ferroelectric SrTiO3, sequentially deposited onto 0.25 mm thick LaAIO3 substrates. An insertion loss of about 5 dB was obtained from a similar design using gold electrodes and Ba0.5Sr1.5 TiO3 films on LaAIO3 at room temperature. The films were grown by pulsed laser ablation.

These low loss phase shifters enable a new type of phased array antenna called a ferroelectric reflectarray [3,4]. A reflectarray antenna combines the best features of a gimbaled parabolic dish (i.e., high efficiency and low cost) and a direct radiating array (e.g., vibration-free scanning). It consists of a surface of printed elements illuminated by a radiating feed. The energy from the feed can be re-radiated to form a cophasal beam. In the past, reflectarray antennas have been implemented using gold elements interconnected with diodes to achieve far field phase shift [5]. However, their performance has been limited by the losses of the phase shifting elements and the finite number of phase shifting bits.

Fixed beam printed microstrip patch reflectarrays have also been reported [6]. But a viable technique for including variable phase shift with printed radiators was elusive. The phase shifters analyzed here hold promise for this application because they are compact, low-loss, and can be fabricated lithographically on the same surface as the radiating element.

II. VARIATIONAL FORMULATION OF LINE CAPACITANCE

The designs are based on a series of coupled microstrip lines interconnected with short sections of nominally 50 Ω microstrip. Bias up to 400 V is applied to the sections via printed bias-tees consisting of a quarter-wave radial stub in series with a very high impedance quarter-wave microstrip. A sketch of the cross-section is shown in Fig. 1. By concentrating the fields in the odd mode, the phase shift per unit length is maximized and by using the film in thin film form the effects of high loss tangent are minimized. The amount of phase shift can be increased by cascading coupled line sections. Though methods for calculating the propagation parameters of coupled transmission lines are well known, coupled lines on stratified substrates are difficult to analyze. And the high permittivity of the ferroelectric layer causes unacceptably long computation time by full-wave electromagnetic simulators because the geometry must be fractured into many thousands of cells. The multi-layer structure is analyzed here using a computationally efficient variational method to calculate the complex propagation constant and characteristic impedance. The method is quite general and can be used for multiple layers of various dielectrics or other types of transmission lines. For example, a multi-layer microstrip can be analyzed by allowing the strip spacing (s) to become much greater than the substrate thickness (h) or strip width (w).

In the case of cascaded coupled lines increasing phase shift occurs at the expense of bandwidth since the structure...
resembles a multi-pole filter. Changing the dielectric constant of the ferroelectric film to change insertion phase also modifies the pass band characteristic, resulting in a net bandwidth of \( \approx 10\% \). The bandwidth compression phenomenon is shown in Fig. 2 for an 8-section \( Ba_{0.60}Sr_{0.40}TiO_3 \) on 0.3 mm thick MgO phase shifter [10]. Roll-off at the high frequency end is attributed to the bias tees. The impedance matrix of the cascade network can be derived by traditional coupled line theory using the superposition of even and odd mode excitation [7]. Then an equivalent S-parameter model can be extracted and used to predict the pass-band characteristics of the phase shifter.

Line capacitance (C) can be calculated by adapting the quasi-TEM variational expression from Koul and Bhat [8].

\[
\frac{1}{C} = \frac{1}{q^2} \int \phi(x,y) \frac{d\phi(x,y)}{dl} \tag{1}
\]

where \( q \) = \( \int \rho(x,y)dl \) and \( \phi(x,y) \) is the potential distribution which satisfies Poisson’s equation and can be expressed in terms of the admittances \( (Y_{1,2}) \) at the charge plane \( (y = h_1 + h_2) \) looking in the positive and negative \( y \) directions. According to Crampagne, et al., the admittance can be expressed in terms of the dielectric constant of each layer. The charge distribution \( \rho(x,y) \) for the even and odd mode excitations was assumed to have the form:

\[
\rho(x,y)_{e,o} = \frac{1}{\lambda_g/4} \left[ 1 + A_{e,o} \right] \left( \frac{W}{2} \right) \left( \frac{W-2w}{2} \right) \left( \frac{W-w}{2} \right) \tag{2}
\]

and using the transverse transmission line method of Crampagne, Ahmadpanah and Guiraud [9].

\[
\frac{1}{C} = \frac{1}{q^2} \int \phi(x,y) \frac{d\phi(x,y)}{dl} \tag{1}
\]

where \( A_{e,o} \) are constants derived by maximizing the even (e) and odd (o) mode capacitance. That is, a trial function that maximizes capacitance yields the most accurate result. It is an attribute of the variational method that the trial function for the charge distribution does not have to be known precisely a priori in order to evaluate capacitance. The expression for \( A_{o} \) is given in (3) to (6).

The variational approach is only valid for electrically thin stratified substrates. But since the coupling to surface waves represents an operational limit, it is appropriate for practical microwave applications.

\[
A_{o} = \sum_{n} \left( \frac{4 \cdot M(n) - L(n)}{4 \cdot M(n) + L(n)} \right) \cdot \frac{g(n)}{M(n) \cdot g(n)} \tag{3}
\]

where

\[
L(n) = \sin \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) \cdot \sin \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) \tag{4}
\]

and

\[
g(n) = \frac{4}{n \cdot \pi \cdot Y(n)} \left( \frac{2 \cdot W}{n \cdot \pi \cdot W} \right)^2 \tag{5}
\]

and

\[
M(n) = \left( \frac{2 \cdot W}{n \cdot \pi \cdot W} \right)^3 \cdot \sin \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) \cdot \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right)^2 \cdot \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right)^2 - 2 \cdot \sin \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) + 6 \tag{6}
\]

\[
ge(x) = \cos \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) + \cos \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) \cdot \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right)^2 - 6 \cdot \sin \left( \frac{n \cdot \pi \cdot W}{2 \cdot W} \right) + 6 \tag{6}
\]
Here, \( n \) is summed over all odd integers. A summation from 1 to 1999 was adequate for convergence. \( A_e \) is derived similarly, by summing over all even integers.

In the even mode currents in the strips are equal in amplitude and flowing in the same direction. In the odd mode currents in the strips are equal in amplitude but flow in opposite directions. So, \( Z_{ee} \) is the characteristic impedance of one strip to ground with equal currents in the same direction and \( Z_{oo} \) is the characteristic impedance of one strip to ground with equal currents in opposite directions. The microstrip mode exists when \( s \gg h, w \) and \( Z_{ee} = Z_{oo} \). The thin FE film is most effective when the phase velocity is dominated by the odd mode fields. The propagation constant is given by:

\[
\beta = \omega / v_p = (\pi / \lambda_o) \left( \varepsilon_{EVEN}^{1/2} + \varepsilon_{ODD}^{1/2} \right)
\]

where

\[
\varepsilon_{EVEN} = C_{E}/C_{Eair} \quad \text{and} \quad \varepsilon_{ODD} = C_{O}/C_{Oair}
\]

and \( C_{Eair} \) and \( C_{Oair} \) are obtained by replacing all dielectrics with air (i.e., \( \varepsilon_r = 1 \)).

The admittance at the charge plane, corresponding to Fig. 1, is easily shown to be:

\[
Y(n) = \varepsilon_0 \cdot \left[ \frac{\varepsilon_2 \cdot \coth \left( \frac{n \cdot \pi \cdot h_2}{W} \right) \coth \left( \frac{n \cdot \pi \cdot h_1}{W} \right) + \varepsilon_1}{\varepsilon_1 \cdot \coth \left( \frac{n \cdot \pi \cdot h_1}{W} \right) + \varepsilon_2 \cdot \coth \left( \frac{n \cdot \pi \cdot h_2}{W} \right)} + \coth \left( \frac{n \cdot \pi \cdot h_0}{W} \right) \right]
\]

A more general formulation for the admittance consisting of an additional dielectric layer, say air, with thickness \( h_0 \) between the host substrate and the ground plane can be expressed as:

\[
Y(n) = \varepsilon_0 \cdot \coth \left( \frac{n \cdot \pi \cdot h_0}{W} \right) + \varepsilon_0 \cdot \varepsilon_1 \cdot \frac{\coth \left( \frac{n \cdot \pi \cdot h_3}{W} \right) + \varepsilon_2 \cdot \tanh \left( \frac{n \cdot \pi \cdot h_2}{W} \right)}{\varepsilon_2 + \coth \left( \frac{n \cdot \pi \cdot h_3}{W} \right) \tanh \left( \frac{n \cdot \pi \cdot h_2}{W} \right)} + \varepsilon_0 \cdot \varepsilon_1 \cdot \tanh \left( \frac{n \cdot \pi \cdot h_1}{W} \right)
\]

Finally, the odd mode capacitance becomes:

\[
C_O = \sum_n \frac{(1 + \frac{A_o}{4})^2}{g(n) \cdot (L(n) + A_o \cdot M(n))^2}
\]

And the even mode capacitance is similarly obtained, by summing over even integers with \( A_e \) replacing \( A_o \). For a TEM transmission line, the characteristic impedance is obtained as \( Z_0 = \left[ (CCair)^{1/2} \right] \) where \( c \) is the speed of light in vacuum.

In general, minimum attenuation is obtained when the effect of the ground plane loss is minimized (i.e., in the odd or balanced mode current flows into one strip and returns through the other). Maximum attenuation occurs in the even or unbalanced mode when equal currents flow into both strips and return through ground.

A comparison between the quasi-TEM approximation and a full-wave electromagnetic simulation is given in Table I for microstrip. The spacing \( s \) was allowed to increase just until the even and odd mode capacitance was equivalent. Choosing arbitrarily large values for \( s \) yields anomalous results.

<table>
<thead>
<tr>
<th>Er Ferroelectric Layer</th>
<th>Eeff (Sonnet &quot;%&quot;)</th>
<th>Eeff (Variational)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>18.76</td>
<td>18.43</td>
</tr>
<tr>
<td>600</td>
<td>21.34</td>
<td>21.00</td>
</tr>
<tr>
<td>900</td>
<td>23.49</td>
<td>23.09</td>
</tr>
<tr>
<td>1200</td>
<td>25.41</td>
<td>24.93</td>
</tr>
<tr>
<td>1500</td>
<td>27.18</td>
<td>26.59</td>
</tr>
<tr>
<td>1800</td>
<td>28.84</td>
<td>28.12</td>
</tr>
</tbody>
</table>

The quasi-TEM solution runs on a Pentium II machine in about 10 sec regardless of the value entered for the ferroelectric film dielectric constant. The same calculation on a commercial electromagnetic simulator using finite element techniques can take several hours because the geometry must be fractured into thousands of cells for these very high dielectric constants.
For coupled strip lines using the superposition of the even and odd modes:

\[ Z_{11} = Z_{22} = \frac{-j}{2}(Z_{oe} + Z_{oo}) \cot \theta(V) \]  
(12)

and

\[ Z_{13} = Z_{31} = \frac{-j}{2}(Z_{oe} - Z_{oo}) \csc \theta(V) \]  
(13)

Where \( \theta(V) \) is the voltage dependent electrical length of the coupled lines obtained from (7) and the physical length of the coupled region (i.e., \( \theta(V) = \delta(V)^{0} \)). The characteristic impedance can be expressed as the geometric mean of the even and odd mode impedance such that:

\[ Z_{0} = (Z_{oe}Z_{oo})^{1/2} \]  
(14)

Equation (14) is strictly valid only for pure TEM propagation and ignores frequency dependence. However, for a practical geometry with moderate coupling, like that considered here, the expression is appropriate. To facilitate the calculation, the \( Z \)-parameters are converted into ABCD or chain parameters for the cascaded sections. The conversion is:

\[ A_f = \frac{Z_{11f}}{Z_{22f}} \]  
(15)

\[ B_f = \frac{Z_{11f} \cdot Z_{22f} - Z_{12f} \cdot Z_{21f}}{Z_{22f}} \]  
(16)

\[ C_f = \frac{1}{Z_{22f}} \]  
(17)

\[ D_f = \frac{Z_{22f}}{Z_{22f}} \]  
(18)

where the indices are used to show frequency dependence. If the short intervening sections of microstrip line (between each coupled line section) are assumed to be of zero length the overall two-port chain matrix becomes:

\[
\begin{bmatrix}
    a_i & b_i \\
    c_i & d_i
\end{bmatrix} = 
\begin{bmatrix}
    A_i & B_i \\
    C_i & D_i
\end{bmatrix} \cdot 
\begin{bmatrix}
    A_i & B_i \\
    C_i & D_i
\end{bmatrix} \cdot 
\begin{bmatrix}
    A_i & B_i \\
    C_i & D_i
\end{bmatrix}\]  
(19)

Where the indices \( i \) and \( f \) are intended to be interchangeable. Finally, the two-port S-parameters of, in this case an 8-section phase shifter, can be converted back according to:

\[ s_{11} = \frac{a_i + \frac{b_i}{Z_0} - (c_i \cdot Z_o) - d_i}{a_i + \frac{b_i}{Z_0} + c_i \cdot Z_o + d_i} \]  
(20)

The predicted bandpass characteristic for a Ba,Sr\textsubscript{1-x}TiO\textsubscript{3} film on 0.25 mm thick LaAIO\textsubscript{3} is shown in Fig. 3 and is in good agreement with experiment.

The ferroelectric layer thickness is crucial to performance. In principle the phase shift for a 2 \( \mu \)m thick film is 2.2 times greater than that of a 0.5 \( \mu \)m film. However maintaining the crystalline quality of the pulse laser ablated Ba,Sr\textsubscript{1-x}TiO\textsubscript{3} films past a thickness of 0.5 \( \mu \)m or so has proven to be difficult [10].

![Figure 3.—Theoretical calculation for the bandpass characteristic of an 8-section coupled microstripline phase shifter using a 0.5 \( \mu \)m Ba,Sr\textsubscript{1-x}TiO\textsubscript{3} film on 0.3 mm thick MgO. The coupled length was 350 \( \mu \)m, \( w = 30 \mu \)m, and \( s = 10 \mu \)m. The permittivity of the film was tuned from 3500 (1-j0.05) (no bias) to 500 (1-j0.005) (maximum bias).](image-url)

III. CONCLUSIONS

The phase shifters analyzed here hold promise for reflectarray applications because they are compact, low-loss, and can be lithographed on the same surface as the radiating element. A key advantage of this technology is the relatively
large feature size. Active devices at the frequencies of interest here would necessitate submicron gate length GaAs FETs. The finest feature size associated with the coupled line phase shifters is the electrode separation $s$, typically $\approx 10 \, \mu m$. Whereas the GaAs FET performance is largely dictated by transconductance and hence carrier transit time across the gate region, the coupled line phase shifters are static devices. The electrode gap separation determines the degree of electromagnetic coupling and the dc potential required to tune the film. It is clear from the experimental and modeled data that the inherent dielectric loss of epitaxial ferroelectric films isn’t necessarily devastating insofar as microwave device performance is concerned. Indeed the loss tangent of a thin dielectric film ($h_1 < 2 \, \mu m$) on a good substrate ($\tan \delta < 0.001$) can deteriorate substantially ($\tan \delta < 0.05$) before the insertion loss of the structures presented here is compromised. If the $\tan \delta$ of the ferroelectric film could be maintained at 0.005 or less, the films contribution to total loss would be essentially negligible except for the mismatch it introduces as it is tuned. But even a $\tan \delta$ of $< 0.05$ is manageable and can result in a 3 dB loss phase shifter that would enable a practical scanning reflectarray antenna. A theoretical model for these devices has been developed and has been shown to be in good agreement with experiment and electromagnetic simulators. Both the insertion phase shift and pass-band characteristics can be closely approximated.

IV. REFERENCES

A Theoretical Model for Thin Film Ferroelectric Coupled Microstripline Phase Shifters

R.R. Romanofsky and A.H. Qureshi

National Aeronautics and Space Administration
John H. Glenn Research Center at Lewis Field
Cleveland, Ohio 44135–3191


Novel microwave phase shifters consisting of coupled microstriplines on thin ferroelectric films have been demonstrated recently. A theoretical model useful for predicting the propagation characteristics (insertion phase shift, dielectric loss, impedance, and bandwidth) is presented here. The model is based on a variational solution for line capacitance and coupled strip transmission line theory.