ERROR SOURCES IN ASTEROID ASTROMETRY

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Asteroid astrometry, like any other scientific measurement process, is subject to both random and systematic errors, not all of which are under the observer's control. To design an astrometric observing program or to improve an existing one requires knowledge of the various sources of error, how different errors affect one's results, and how various errors may be minimized by careful observation or data reduction techniques.

Introduction

There are three ingredients to astrometry:

$$\text{Observations} + \text{Reference star catalog} + \text{Data reduction} \rightarrow \text{Results}.$$

No scientific measurement is free from error, and astrometric observations are no exception. This applies to the reference star catalog as well as to one's own observations. Furthermore, the process of reducing our observations requires some judgment on the part of the "measurer" (to use the MPC's term), and this in turn means that the data reduction can in principle yield erroneous results even if the observations themselves were free of error. There are many sources of error, ranging from natural causes such as photon statistics or atmospheric effects to ~~stupid~~ mistakes by the observer. SUBJECT MATTER Not all of these error sources are under the observer's control, although in some cases careful observing or data reduction techniques can help to some extent.

The purpose of this paper is not merely to provide an overview of the various kinds of errors that can affect astrometry, but also to give some methods for minimizing those errors. Attention to detail can pay off: if an observer can manage to reduce his postfit residuals from $1''$ to $0''.25$, then each observation can be weighted sixteen times more heavily. In other words, one good observation would be worth 16 bad ones, and if you can achieve this performance for less than 16 times as much work, you're coming out ahead.

Types of errors

Most textbooks divide errors into two classes: *random* and *systematic*. Random errors are those which are inherent to the measurement process, and they result from *noisy data* of one form or another. Since these errors are random, taking additional measurements will gradually reduce the effects of noise, leaving you with increasingly accurate results. Systematic errors, on the other hand, will affect all measurements in the same way, and taking additional measurements will not remove this bias. These errors *may* be removed to some extent in data reduction, but not always, and never completely.

The situation for astrometry is a bit more complicated. Yes, there are random errors, and there are systematic errors, but some errors that start out random wind up having a systematic effect on one's results. Of course, there are also silly mistakes, and since those are easiest to understand that's where I'll start.

Blunders. Some errors are just plain stupid mistakes. Examples from my own experience include: using the wrong coordinates for my observing site; writing down the wrong date or time; using the wrong filter or wrong exposure; pointing the telescope at the wrong place; leaving the telescope cover on; failing to refocus often enough; failing to check the dome; and (worst of all) not realizing that my hard disk was full. I have made each of these errors, and some of them more than once. The only cure I know is to have your procedures written down in checklist form and then to be sure to follow the checklist. After all, we're trying to measure positions to seven or eight significant digits, and there's no room for sloppiness.

Random errors. The most obvious of these arises from the *Poisson* or *shot noise* in the arrival of photons from the target asteroid and from the reference stars. Each image is built up gradually, one photoelectron at a time, and since the exposure time is not infinite, the actual distribution of photoelectrons in the image will not exactly match the expected distribution. This is true regardless of what the atmosphere is doing and regardless of how well you model seeing effects in the point-spread function. The result must show up as a random error in the (x, y) coordinates you determine for the image.

A second important random error is *background noise.* Whether this arises from photons from the sky, thermal electrons in the CCD, or read noise in the amplifier, the result is the same: the background is not flat but noisy. Some of the background noise is bound to interact with your centroiding process, and this will pull your centroids in some random direction. This is true even if you solve for the background height in the fitting process.

A third random error—random in the sense that it is different from one observation to the next—relates to *telescope tracking.* No drive is perfect, and images will therefore not be completely round. Photographic emulsions are nonlinear and notorious for "magnitude terms," since the observed centroid can be influenced by the brightness of the star. CCDs, being linear, are much less prone to this effect. However, asteroids move, and poor tracking can produce trails that are neither straight nor uniformly illuminated, even if the star images are reasonably round. Determining the center (or the endpoints) of a trailed image isn't easy under normal circumstances, but when the trail is wiggly and has bright spots in it, the situation becomes much worse. This too is a random error, since the next exposure will be affected somewhat differently.

Finally, astrometric measurements will be subtly affected by the *atmosphere.* We all know that turbulence in the air causes star images to wind up somewhat larger than what the telescope's optics are capable of producing. If this effect is symmetrical—and of course it isn't—it would have no effect on the centroid. If the effect were the same everywhere in the field of view, it would pull all the centroids by the same amount, and the constant term in your reduction model would account for it very nicely. However, seeing is not quite 100% correlated from point to point in your field of view. The larger the angle, the less the correlation; reference stars on opposite ends of your image may be affected quite differently by the atmosphere. Seeing will thus produce small changes in scale, in orientation, in the zero point, and in any other higher-order term you can think of, and these changes are random from one picture to the next.

Systematic errors. The first systematic error that comes immediately to my mind is a possible *zone error* in the reference star catalog. This means that the positions or proper motions of all of the stars in a particular region of the sky contain a bias, and this bias will be passed through intact into the measured positions of every asteroid in that part of the sky. Using more reference stars will not change things at all. This is not much of a problem any more, thanks to Hipparcos, but the old SAO catalog had some terrible zone errors in the southern hemisphere, exceeding $1''$ in parts of Scorpius and Sagittarius. And since the original Guide Star Catalog (version 1.0) was reduced to the SAO, it is plagued with the same zone errors (plus others that I won't mention here). GSC 1.0 is not at all reliable for astrometry; version 1.2 is much better.

A second systematic measurement error is *quantization error*, which arises because CCDs have pixels and give a set of discrete measurements rather than a continuous sampling of the image. One obviously cannot do subpixel centroiding if a star lights up only one pixel! Four pixels is marginally acceptable; nine pixels (a 3×3 box) is much better; things continue to improve somewhat as you oversample the image more and more. It can sometimes happen that one's centroiding algorithm prefers to give results near the center of a pixel or close to the edge of a pixel. In other words, if you move the true center of light uniformly across a pixel, the measured coordinate may show systematic departures from the truth. These errors, while systematic, usually have the flavor of random errors since one generally cannot reproduce them exactly from one exposure to the next.

Another systematic error is introduced because asteroids are not point sources, and we do not necessarily observe them at zero phase. There will be an offset between the *center of mass* of the asteroid and its *center of light*. For instance, (1) Ceres has a radius of 470 km, or an apparent radius of $0''.37$ at mean opposition. The center of light is offset by an amount roughly proportional to the phase angle, and this offset can come close to $0''.1$ in magnitude, which is becoming measurable with modern techniques. This offset does not concern us observers, since the positions we report are known to be of the center of light. However, this effect should be included by those who perform the orbit fits. We at JPL don't model this effect in our reductions—at least not yet.

Perhaps the most important source of systematic error, however, is the use of an inappropriate model to reduce the observations. Suppose, for instance, that your telescope suffers from third-order distortion. (This is one of the five classical third-order optical aberrations; its effect is to change the distance of an image from the optical axis by an amount proportional to the cube of that distance.) Stars near the edge of the field of view will be displaced systematically from their ideal position. If you use the standard 4-constant or even 6-constant plate model, the linear terms will absorb much of the effect of distortion, but not all of it. Consequently the mapping from the focal plane back to the sky will be almost, but not quite, correct. The part of the true transformation that your adopted model cannot correct will become a systematic error affecting the measured right ascension and declination of the asteroid, and this will persist across multiple frames of the same scene.

One can err just as easily in the other direction, though, and include terms in one's model that are not necessary. It can happen that these unneeded terms will resonate with random errors in the reference star positions—fitting noise, if you will—resulting in a plate model which is improperly stretched from the truth. Here, as above, the incorrect model will produce an erroneous transformation of the asteroid's measured position in the picture into its position on the sky. Determining which terms to include in one's reduction model is still more of an art than a science.

Hybrid errors. There is one important error whose source is random but whose effect is systematic. This error arises from random errors in the reference star catalog. (These are not to be confused with the systematic zone errors mentioned earlier.) Each asteroid is observed relative to a specific set of reference stars. These stars will have their own random errors in the catalog, and these errors will produce errors in the plate constants during reduction. Since the plate constants are wrong, the inferred position of the asteroid will be wrong. This error has its roots in a random phenomenon, but the effect is systematic, since *every* picture that is taken using that particular set of reference stars will be affected in the same way.

What to do about errors

There is no substitute for careful attention to the task at hand! This is just as true for reducing the observations as it is for obtaining them. The sorts of blunders that I mentioned above can all be eliminated if one pays attention to detail.

Random errors arising from photon statistics or from seeing cannot be eliminated in any one picture, but their effect can be attenuated if you take multiple exposures of the same scene. This is statistics at its simplest: the final error is inversely proportional to the square root of the number of measurements. Hence the old saying, "$\sqrt{N}$ is your friend." Four observations will yield one effective measurement with half as much random error as one observation. However, the next factor of two improvement requires 12 more observations, and one rapidly reaches the point of diminishing returns. This is why the folks at the MPC will tell you to take no more than two or three shots of any one object per night.

Random errors from tracking can likewise be improved by taking multiple observations, keeping only the ones whose images appear round, but wouldn't it be better to fix the hardware instead?

The best way to beat down errors induced by the atmosphere is to lengthen one's exposure time. The effects of seeing are generally inversely proportional to the square root of the exposure time, so that doubling the exposure should reduce seeing noise by 40%. Longer exposures also mean brighter images, which helps reduce shot noise as well. Of course, longer exposures run the risks of trailing the asteroid and overexposing the stars.

Errors arising from pixelization effects (and I'm including errors in flat fielding here too) can be brought under control by moving the asteroid around in the field of view. A shift of even 10 or 12 pixels is enough to let you use an entirely different set of pixels from one frame to the next.

Random errors in the positions of individual reference stars—the hybrid error that leads to systematic error in one's results—can be beaten down by using more reference stars from the same catalog. This is easier said than done. If you're using a dense star catalog already, such as the SA 2.0 that Dave Monet made right here in Flagstaff, it may be simply a matter of identifying and measuring more stars in your field of view. Otherwise, you'll need to expand your field of view, by changing your focal length, buying a new camera with a bigger chip, or by taking a mosaic of images and reducing them all together. The last approach is the one we use at Table Mountain: we'll take between 2 and 5 frames of each asteroid, varying the pointing between frames until we have captured a sufficient number of reference stars all around the target asteroid. The trick in the data reduction is to use images of field stars in the overlap regions to constrain the plate constants. We solve for the position of each field star, using all its images to form one position, and this process introduces enough additional equations of constraint to make the whole ensemble of frames hang together. This scheme, first introduced nearly 40 years ago by Heinz Eichhorn, has been used for some time in the development of accurate star catalogs, but I believe I am the first to apply it to asteroid astrometry.

I've just mentioned our desire to image reference stars all around our target. Errors in the plate constants propagate into errors in the measured coordinates of the target in a way that depends on both where the target lies in the field and on where the reference stars appear relative to the target. If you pretend that the reference stars are smeared out uniformly over the field of view (so that you can do integrals instead of discrete sums), you'll find that the uncertainty in the mapping from (x, y) to (α, δ) is a minimum in the center of the field. This is true because the only contribution to the mapping is from the constant terms in the plate model—the scale and orientation don't matter since the target is already at the tangent point. As you move away from the center, the scale and orientation enter linearly into the mapping. Since the uncertainty in the final result is the RSS of the errors that contribute to it, the result is that when you plot the uncertainty as a function of field position, you get a *hyperboloid* with the minimum at the center.

When you replace the smeared-out idealized distribution of reference stars with the real distribution, things change somewhat. Now the point of minimum error lies at the "center of gravity" of the reference stars. The minimum can be fairly broad, and in practice what we require is that the asteroid lie *inside* the smallest convex polygon that encloses the reference stars. Put another way, if you can find a line through the asteroid such that all of the reference stars lie on the same side of the line, you're likely to run into trouble. For in that case you wind up *extrapolating* the asteroid's position in the direction perpendicular to that line. And we all know that extrapolation is a dangerous habit! Of course we can't control where the stars are, but we *can* use the distribution of reference stars to help us decide which targets to observe on a particular night.

Systematic errors in reference star catalogs are much harder to fight, since nothing you do on any particular night can make the problem go away. The process of getting an orbit solution will defeat zone errors to some extent, simply because the asteroid moves around the sky and thus gets measured with respect to many more stars. Nevertheless, there will certainly be some effect (albeit perhaps not a significant one) on the final results. Meanwhile, the best course of action for the observer and measurer is to use the best star catalog available. There's always a trade-off between accuracy per star and the density of the catalog. We have chosen to go with the ACT Catalog, despite its relatively low density, because our field of view is large enough to get the reference stars we need. (Using the overlapping plate method helps immensely here as well.) Other observers with small fields may choose a denser but less accurate catalog. One hopes that this problem will gradually go away as the Hipparcos and Tycho catalogs become more fully used in the development of other catalogs.

This leaves us with errors arising from inappropriate modeling. The only cure for this is to use the right model, and that raises the question, "What *is* the right model, anyway?" This question must be answered empirically, and the answer will be different for each observer. In general, you don't want to use more terms than are necessary—not because you want to avoid additional work, but because those extra terms can wind up taking on unphysically large values if your measurement errors conspire against you, and then your reduced positions will be bad. If you have n reference stars, you're making $2n$ measurements, and you can't possibly determine more than $2n$ plate constants. A rule of thumb is to have at least two or three times as many measurements as plate constants, but that's a very general rule. If your optics or detector have some strange feature that you can't model with 4 or 6 constants, then you simply *must* include those extra terms in *every* reduction you do. Maybe you don't have to solve for them every time; perhaps a term will have a value that stays nearly constant all night, every night, and you can plug in that value and be done with it.

So how does one figure out what to do? Aside from trial and error, one good method is to take several picture of a star cluster in which there are many reference stars. Do a simple reduction and examine the residuals for the reference stars. Plot the residuals in (x, y) as an arrow, and place the arrows at the spots where the stars appeared in the field of view. If you see some kind of pattern when you're done, then you need another term in your model; if the arrows are random, then your model is adequate. Conversely, if you remove terms from the model and the residuals don't begin to show a pattern, you're probably safe in leaving those terms out. Simply looking at the magnitude of the residuals is not sufficient.

Conclusion

In this paper I have given a necessarily brief and qualitative overview of the various sources of error in asteroid astrometry. Careful observing and data reduction techniques can often improve one's error, and careful analysis and calibration are often necessary in order to determine which reduction model is best suited for one's equipment. The payoff, however, is well worth the effort.

References

I have intentionally kept footnotes and references out of the body of the paper since many participants in this conference lack ready access to the academic literature. Nevertheless, this paper would not be complete without some acknowledgment of my sources. The seminal paper on the propagation of errors in photographic astrometry was written by Heinz Eichhorn and Carol Williams (*Astronomical Journal* **68**, 221, 1963). Three years prior, Eichhorn introduced his "overlapping plate method" which we routinely use for getting the most out of our observations (*Astronomisches Nachtrichten* **285**, 233, 1960). Error propagation is a standard topic in any statistics text; I happen to have Bevington's old book, *Data Reduction and Error Analysis for the Physical Sciences* (1969, McGraw-Hill).

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