Generalized Wall Function for Complex Turbulent Flows

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ABSTRACT

A generalized wall function was proposed by Shih et al (1999). It accounts the effect of pressure gradients on the flow near the wall. Theory shows that the effect of pressure gradients on the flow in the inertial sublayer is very significant and the standard wall function should be replaced by a generalized wall function. Since the theory is also valid for boundary layer flows toward separation, the generalized wall function may be applied to complex turbulent flows with acceleration, deceleration, separation and recirculation. This paper is to verify the generalized wall function with numerical simulations for boundary layer flows with various adverse and favorable pressure gradients, including flows about to separate. Furthermore, a general procedure of implementation of the generalized wall function for National Combustion Code (NCC) is described, it can be applied to both structured and unstructured CFD codes.

INTRODUCTION

Standard wall function (Millikan, 1938, Spalding, 1961, Tennekes and Lumley, 1972) has been widely and successfully used in various CFD applications for high Reynolds number turbulent flows. It is realized that the derivation of the standard wall function is valid only for flows with zero or small pressure gradients; however, experienced CFD engineers can always find their ways to fix the problems where the standard wall function is not valid and does cause the troubles in numerical calculations. Of course, the degree of success varies among the individuals and the type of flows. Tennekes and Lumley (1972) described a method to deal with a boundary flow with zero wall stress. Shih et al (1999) extended this method to a general boundary flow and obtained a generalized wall function with the standard wall function as its special case of zero-pressure gradient. The generalized wall function is valid for various pressure gradients and zero-wall stress, hence overcomes the weakness of the standard wall function. This will provide CFD engineers a unified way of using wall function for the CFD applications without ad hoc fixing. We will first introduce the generalized wall function followed by the comparisons between the theory and numerical calculations of boundary layer flows with various pressure gradients. Then we describe the method of implementation of the wall function for general 3-D CFD codes, especially for NCC code which is a 3-D unstructured code. A backward-facing step flow and dumpcombustor flows with and without combustion will be presented as the examples.
GENERALIZED-WALL FUNCTION

The generalized wall function in the inertial sublayer proposed by Shih et al (1999) is listed as follows

\[
\frac{U_{it}}{u_c} = \frac{1}{\kappa} \ln \left( \frac{u_c y}{\nu} \right) + C_1 + \frac{\nu (dP/dx_i)_{tw}}{u_c^3} \left[ \frac{u_p}{u_c} \ln \left( \frac{u_c y}{\nu} \right) + C_2 \right]
\]

where \( U_{it} \) is the mean velocity vector parallel to the wall or called tangential mean velocity, \( \tau_{iw} \) is the wall stress vector, \( (dP/dx_i)_{tw} \) is the streamlined tangential pressure gradient at the wall. \( u_r \) and \( u_p \) are the characteristic velocity defined by the wall stress \( \tau_{iw} \) and pressure gradient \( (dP/dx_i)_{tw} \) as follows

\[
u u_r \left( \frac{u_r y}{\nu} \right) + \frac{\nu (dP/dx_i)_{tw}}{u_c^3} \left[ \frac{u_p}{u_c} \ln \left( \frac{u_c y}{\nu} \right) + C_2 \right]
\]

which may be called the skin friction velocity and the pressure gradient velocity, respectively. \( u_c \) is a hybrid velocity scale consisting of \( u_r \) and \( u_p \):

\[
u u_r + u_p
\]

This velocity scale \( u_c \) was used by Shih et al for scaling a general boundary layer flow. Note that \( u_c \) will never become zero by its definition. \( y \) is the normal distance to the wall. \( \kappa \) is a constant \( \approx 0.41 \). \( C_1 \) and \( C_2 \) are defined below

\[
C_1 = \frac{1}{\kappa} \ln \left( \frac{u_r}{u_c} \right) + C \quad (4)
\]

\[
C_2 = \frac{u_p}{u_c} \left[ \alpha \ln \left( \frac{u_p}{u_c} \right) + \beta \right] \quad (5)
\]

where \( \alpha \approx 5 \), \( \beta \approx 8 \) and \( C \approx 5.0 \). Eq. (1) is valid in the region of inertial sublayer where \( u_c y/\nu \gg 1 \) and \( y/\delta \ll 1 \). For the region very close to the wall, \( u_c y/\nu \leq 5 \) (viscous sublayer), Shih et al suggest

\[
\frac{U_{it}}{u_c} = \tau_{iw} \left( \frac{u_c y}{\nu} \right) + \frac{1}{2} \frac{\nu (dP/dx_i)_{tw}}{\nu^3} \left( \frac{u_c y}{\nu} \right)^2
\]

which shows that the mean velocity is no longer linear versus the normal wall distance due to the effect of pressure gradients. For the region between the viscous sublayer and the inertial sublayer, i.e., the buffer layer, Shih et al derived a formulation following Spalding's approach. This formulation actually covers the whole surface layer from the wall all the way to the inertial sublayer. Therefore, they obtained an unified wall function valid for viscous sublayer, buffer layer and inertial sublayer. See the paper of Shih et al (1999) for the details.

BOUNDARY LAYER FLOWS WITH VARIOUS PRESSURE GRADIENTS

Boundary layer flow with zero pressure gradient

The calculations start with the uniform profiles of mean velocity, turbulent kinetic energy and its dissipation rate. Our calculations with various grid resolutions, from 5 grid points to 100 grid points, all approach the fully developed turbulent boundary layer. Fig. 1 shows the skin friction versus the Reynolds number based on the momentum thickness. The reference Reynolds number is 10000. Five calculations with five different grid resolutions (uniformly distributed across the boundary layer) are shown in Fig. 1 compared with the experimental data. Fig. 2 shows the results of mean velocity compared with the experimental data at \( Re_o = 7700 \). All calculations agree well with the experimental data when the calculated turbulent boundary is fully developed from their initial uniform profiles. Fig. 3 shows the calculations for different reference Reynolds numbers, ranging from \( 10^3 \) to \( 10^7 \). The mean velocity profiles are plotted at \( Re_o = 7700 \) and compared with experimental data. Good agreements are obtained. We conclude that the wall function implemented in this code has produced
the grid independent and reference Reynolds number (based on the arbitrary reference velocity and length scale) independent solutions for fully developed turbulent boundary flows.

Figure 1: Skin friction of boundary layer with zero pressure gradient

Boundary layer flow with adverse pressure gradients

The wall function newly introduced by Shih et al (1999) includes the effect of pressure gradients. This wall function is claimed to be valid for large adverse pressure gradient including flows about to separate from the wall. To test this new wall function, we use a parabolic code and impose a constant adverse pressure gradient on a fully developed zero pressure gradient boundary layer flow to obtain a fully developed boundary layer flow with the given adverse pressure gradient. Fig. 4 shows four cases, from the zero pressure gradient to the maximum adverse pressure gradient. The latter corresponds to the boundary layer about to separate from the wall. The velocity scale in the figure $u_c$ is defined as $u_r + u_p$. $u_r$ is the normal skin friction velocity, and $u_p$ is defined by the pressure gradient at the wall: $u_p = \left((\nu/\rho)dP/dz\right)^{1/3}$. Note that $u_r$ and $u_p$ may individually approach to zero in certain cases, but their sum, i.e., $u_c$ will never be zero in a boundary layer flow. The ratio, $u_p/u_c$, varies from zero to one. For zero pressure gradient, $u_p/u_c = 0$. For the flow about to separate from the wall, $u_r$ must vanish and $u_p/u_c = 1$. Fig. 4 shows the excellent agreement between the law of the wall (new wall function) and the computations. Both the theory and the computations show the changes of the slope of the mean velocity profile as the adverse pressure gradient changes.

Boundary layer flow with favorable pressure gradients

For accelerating boundary layer flows, i.e. the flows with favorable pressure gradients, we use the same code and impose a constant favorable pressure gradient on a fully developed zero
pressure gradient boundary layer to achieve a fully developed boundary layer flow with a given favorable pressure gradient. For an accelerating flow, the skin friction is nonzero and increases with the pressure gradient, so that \( u_p/u_c \) is always less than one. \( u_p/u_c \) becomes small when Reynolds number is large. For example, in a fully developed channel flow (always favorable pressure gradient), it can shown that \( u_p/u_c = 1/(1 + Re^{1/3}) \), where \( Re = U_{\text{max}}h/v \), \( U_{\text{max}} \) and \( h \) are the centerline mean velocity and the half width of the channel. Therefore, if \( Re \geq 1000 \), then \( u_p/u_c \leq 0.1 \). For accelerating boundary layer flows \( u_p/u_c \) may be larger.

Here, we have carried out the calculations from \( u_p/u_c = 0 \) up to \( u_p/u_c = 0.3 \). The latter represents an extremely large favorable pressure gradient under which the flow still remains turbulent. Five cases have been studied. The results are shown in Fig. 5. Again, the agreement between the theory and computations is excellent. The mean velocity profiles will change significantly with favorable pressure gradient if the Reynolds number is not very large.

**METHOD OF WALL FUNCTION IMPLEMENTATION**

National Combustion Code (NCC) developed in NASA Glenn Research Center is a 3-D finite volume unstructured CFD code. All the variables are Cartesian. We will describe the method of implementation of the wall function for the NCC code. The method is general and can be extended to any other 3-D codes. In Cartesian coordinate system, the momentum equation can be written as

\[
\frac{\partial \rho U_i}{\partial t} + \frac{\partial \rho U_i U_j}{\partial x_j} = \frac{\partial}{\partial x_j} \tau_{ij} - \frac{\partial P}{\partial x_i}
\] (7)

The integration of \( \frac{\partial}{\partial x_j} \tau_{ij} \) in an element (see Fig. 6) gives

\[
\sum_{k=1}^{n} \tau_{ix,k} \frac{A_{x,k}}{\Delta v} + \sum_{k=1}^{n} \tau_{iy,k} \frac{A_{y,k}}{\Delta v} + \sum_{k=1}^{n} \tau_{iz,k} \frac{A_{z,k}}{\Delta v}
\] (8)

where \( A_{x,k}, A_{y,k}, A_{z,k} \) are the three components of \( k^{th} \) face of the element. For the elements on the solid wall, the values of nine stress components \( \tau_{ij} \) at the wall (only six of them are independent) are needed, which cannot be accurately calculated using the values of velocity at the surrounding grid points with coarse numerical grids since the rapid change of turbulent mean velocity near the wall requires very fine grids to numerically calculate the wall stress. The wall function, however, can provide these wall stresses without numerical (differential) calculations.
**Stress tensor in curvilinear coordinates**

Let us denote \((x,y,z)\) as the Cartesian coordinates, \((\xi, \eta, \zeta)\) as the curvilinear coordinates. By definition and chain rule, we may write

\[
\tau_{ij} = (\mu + \mu_T) \left( \frac{\partial U_i}{\partial \xi} \frac{\partial U_j}{\partial \eta} + \frac{\partial U_i}{\partial \eta} \frac{\partial U_j}{\partial \xi} \right) \tag{9}
\]

where \(U_i(U, V, W)\) are the velocity components in the Cartesian coordinates. Now, consider a wall surface element in the curvilinear coordinates. Its normal direction lies in \(\eta\), the surface is in the plane of \(\xi\) and \(\zeta\) (see Fig.7). At the wall, the derivatives of mean velocity with respect to the curvilinear coordinates are all zero except the normal derivatives: \(\frac{\partial U}{\partial \eta}, \frac{\partial V}{\partial \eta}\) and \(\frac{\partial W}{\partial \eta}\). Therefore, the six independent \(\tau_{ij}\) at the wall can be written as

\[
\begin{align*}
\tau_{xx} &= 2(\mu + \mu_T) \frac{\partial U}{\partial \eta} \frac{\partial V}{\partial \eta} \\
\tau_{xy} &= (\mu + \mu_T) \left( \frac{\partial U}{\partial \eta} \frac{\partial V}{\partial \eta} + \frac{\partial U}{\partial \xi} \frac{\partial V}{\partial \xi} \right) \\
\tau_{xz} &= (\mu + \mu_T) \left( \frac{\partial U}{\partial \eta} \frac{\partial W}{\partial \eta} + \frac{\partial U}{\partial \xi} \frac{\partial W}{\partial \xi} \right) \\
\tau_{yz} &= 2(\mu + \mu_T) \frac{\partial V}{\partial \eta} \frac{\partial W}{\partial \eta} \\
\tau_{zx} &= 2(\mu + \mu_T) \frac{\partial V}{\partial \xi} \frac{\partial W}{\partial \xi} \\
\tau_{yy} &= 2(\mu + \mu_T) \frac{\partial V}{\partial \eta} \frac{\partial V}{\partial \eta}
\end{align*}
\]

**Wall function**

The wall function gives relationship between the wall stress \(\tau_{w}\) and the tangential velocity \(U_{it}\) in the inertial sublayer, that is Eq. (1). It can be written as

\[
\tau_{iw} = \frac{U_{it}}{u_c} \frac{\nu (dP/dx_i)_{tw}}{u_p^3} F_2 \frac{\rho u_t^2}{F_1} \tag{11}
\]

where

\[
\begin{align*}
F_1 &= \frac{1}{\kappa u_c} \ln \left( \frac{u_c y}{\nu} \right) + C_1 \\
F_2 &= \alpha \frac{u_p}{u_c} \ln \left( \frac{u_c y}{\nu} \right) + C_2 \tag{12}
\end{align*}
\]

Eq. (11) provides us the wall stress or the normal derivative of the mean velocity at the wall.
\[ (\mu \frac{\partial U_i}{\partial \eta}) \] without to carry out numerical differentiation. That is, if we write
\[ \tau_{iw} = \mu \left( \frac{\partial U}{\partial \eta}, \frac{\partial V}{\partial \eta}, \frac{\partial W}{\partial \eta} \right) \]
then we may write
\[ \left( \frac{\partial U}{\partial \eta} \right)_w = \frac{U_i - \frac{\nu}{\rho} (\nabla P)_{tw,z} F_2}{u_c} \rho \frac{u^2}{\mu F_1} \]
\[ \left( \frac{\partial V}{\partial \eta} \right)_w = \frac{V_i - \frac{\nu}{\rho} (\nabla P)_{tw,y} F_2}{u_c} \rho \frac{u^2}{\mu F_1} \]
\[ \left( \frac{\partial W}{\partial \eta} \right)_w = \frac{W_i - \frac{\nu}{\rho} (\nabla P)_{tw,z} F_2}{u_c} \rho \frac{u^2}{\mu F_1} \]

where, \( U_i, V_i \) and \( W_i \) are the three Cartesian components of tangential velocity in the inertial sublayer, and \( (\nabla P)_{tw,z}, (\nabla P)_{tw,y} \) and \( (\nabla P)_{tw,z} \) are the three Cartesian components of streamlined tangential pressure gradient. The first numerical grid point is usually located in the inertial sublayer for CFD with coarse grids. However, if the first grid point is close to the wall such that \( u_c y / \nu \leq 5 \), then the above relation must be replaced by the following relations:
\[ \left( \frac{\partial U}{\partial \eta} \right)_w = \left[ U_i - \frac{1}{2\mu} (\nabla P)_{tw,z} \frac{y^2}{y} \right] \frac{1}{y} \]
\[ \left( \frac{\partial V}{\partial \eta} \right)_w = \left[ V_i - \frac{1}{2\mu} (\nabla P)_{tw,y} \frac{y^2}{y} \right] \frac{1}{y} \]
\[ \left( \frac{\partial W}{\partial \eta} \right)_w = \left[ W_i - \frac{1}{2\mu} (\nabla P)_{tw,z} \frac{y^2}{y} \right] \frac{1}{y} \]

Calculation of tangential velocity and pressure gradient

Let us denote the normal vector of the surface element by \( \delta x_i \), its length is \( n \) which is the normal distance of the first grid point to the wall. We may calculate the three components of tangential mean velocity and streamlined tangential pressure gradient as follows
\[ U_t = U - (U_j \delta x_j) \frac{\delta x}{n^2} \]
\[ V_t = V - (U_j \delta x_j) \frac{\delta y}{n^2} \]
\[ W_t = W - (U_j \delta x_j) \frac{\delta z}{n^2} \]
and
\[ (\nabla P)_{tw,x} = \frac{(\nabla P)_{tw,z}}{\nabla P_{tw,z}} \]
\[ U_{it} V_{it} \]
\[ (\nabla P)_{tw,y} = \frac{(\nabla P)_{tw,y}}{\nabla P_{tw,y}} \]
\[ U_{it} V_{it} \]
\[ (\nabla P)_{tw,z} = \frac{(\nabla P)_{tw,z}}{\nabla P_{tw,z}} \]

From Eq. (23), we obtain
\[ \frac{\delta x}{n} = \frac{(x - x_f) S_x}{S} \]
\[ \frac{\delta y}{n} = \frac{(y - y_f) S_y}{S} \]
\[ \frac{\delta z}{n} = \frac{(z - z_f) S_z}{S} \]

Calculation of \( \frac{\delta y}{dx}, \frac{\delta y}{dy}, \frac{\delta y}{dz} \)

Let \( x_f \) be the location of the first grid point, \((x_f, y_f)\), the location of the wall surface, \( S_i \) the three components of the surface element with the area of \( S (S_x^2 + S_y^2 + S_z^2 = S^2) \). Then the normal direction vector of the surface element will be \( S_i / S \). The three components of the normal distance \( n \) (the first grid point away from the wall) can be calculated as
\[ n^2 = \delta x^2 + \delta y^2 + \delta z^2 \]
\[ n^2 = \frac{(x - x_f)^2 S_x^2 + (y - y_f)^2 S_y^2 + (z - z_f)^2 S_z^2}{S^2} \]

From Eqs. (24, 25, 26), we obtain
\[ \frac{d\eta}{dx} = \frac{(x - x_f) S_x^2 / S^2}{n} \]
\[ \frac{d\eta}{dy} = \frac{(y - y_f) S_y^2 / S^2}{n} \]
\[ \frac{d\eta}{dz} = \frac{(z - z_f) S_z^2 / S^2}{n} \]

With all the above relations, we will be able to calculate the six \( \tau_{ij} \) at the wall using Eq. (10).
COMPLEX-TURBULENT FLOWS

Backward-facing step flow

A backward-facing step flow of KKJ (Kim et al, 1980) was used to verify the implementation of the wall function in the NCC code. The results using the new wall function are compared with the results from the old wall function built into the NCC code in Fig. (8). Apparently, the old wall function implementation method in NCC is not quite correct. Fig. (9) shows the effect of pressure gradient built into the new wall function on a backward-facing step flow. The results show that the new wall function with pressure gradient effects gives better prediction in terms of the reattachment length (7.0 of step height due to the data of the experiment). The calculation gives 6.8 with the effect of pressure gradient, versus 7.9 without the effect of pressure gradient.

Dumpcombustor flow

Another validation case is the dumpcombustor flow. Fig. (10) shows the flow with the combustion. The stream lines, the contours of pressure, NO and OH were plotted. Fig. (11) compares the same flow with and without combustion. Fig. (12) compares the calculations with the results from a low-Reynolds number turbulence model built into NCC, which does not need the wall function. This confirms that the results from the new wall function are quite reasonable.

Conclusion

The theory of generalized wall function has been verified by numerical simulations of fully developed...
New wall function
(4983 iterations, converged)

Pressure

NO

OH

Figure 10: Dumpcombustor flow 1

Low Reynolds model
(4518 iterations, converged)

Pressure

Cold flow (new wall function)
(10000 iterations)

Pressure

Figure 11: Dumpcombustor flow 2

New wall function
(4983 iterations, converged)

Pressure

Low Reynolds model
(4518 iterations, converged)

Pressure

Figure 12: Dumpcombustor flow 3

oped turbulent boundary layer flows with various pressure gradients. It shows that the standard wall function should be replaced by the generalized wall function for general boundary flows with large pressure gradients. A method of implementing wall function for 3-D complex flows has been introduced. The examples of complex 2-D flows show the success of implementation of the wall function in NCC. Further validation against 3-D complex flows will follow.

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Generalized Wall Function for Complex Turbulent Flows

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