

**CAPABILITIES AND LIMITATIONS OF RADIO OCCULTATION
MEASUREMENTS FOR IONOSPHERE MONITORING**

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CHUNMING WANG

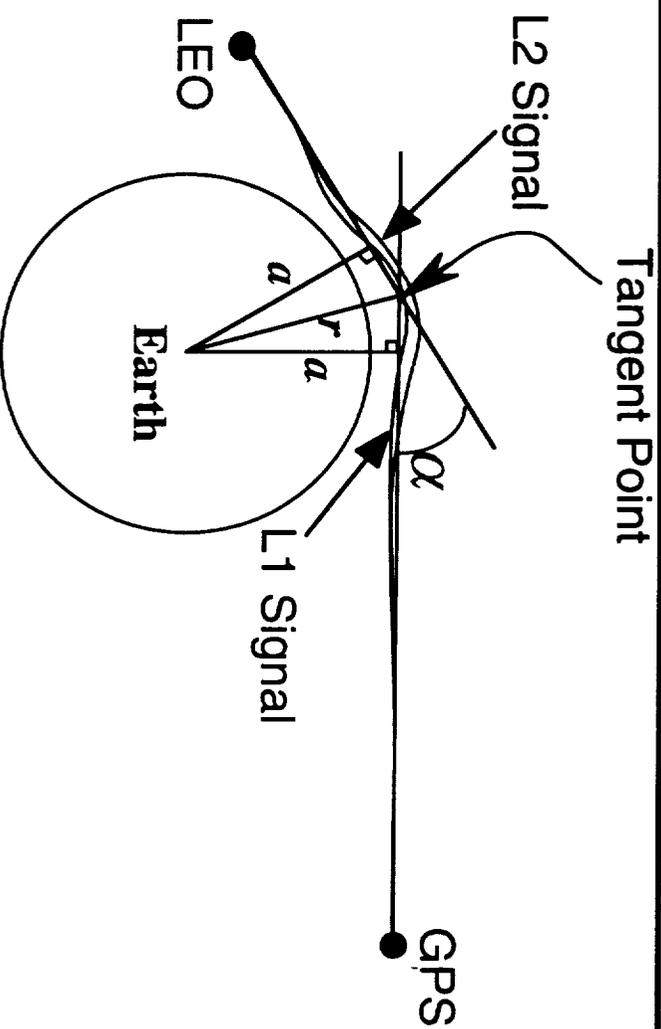
UNIVERSITY OF SOUTHERN CALIFORNIA

**WORKSHOP ON LEO MISSIONS
MARCH 9 - 11, 1999
POTSDAM, GERMANY**

SCOPE OF TALK

- Describe the range of capabilities of GPS radio occultation missions in ionospheric research
 - Ionospheric Profiling
 - Ionospheric Imaging
 - Ionospheric Data Assimilation
 - Measurement of Scintillation
- Identify strengths and weaknesses of measurements
 - Coverage
 - Resolution
 - Uniqueness of Solution

ELECTRON DENSITY PROFILING



- Assuming spherical symmetry:

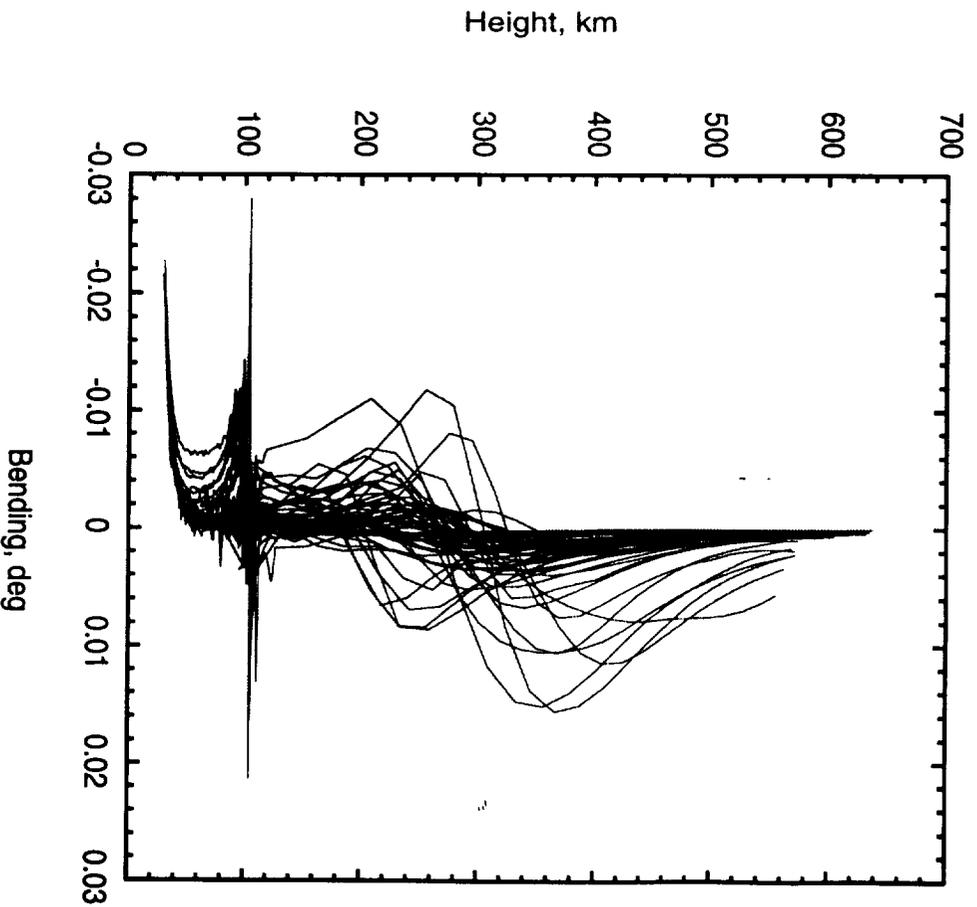
Forward propagation

Abel inversion

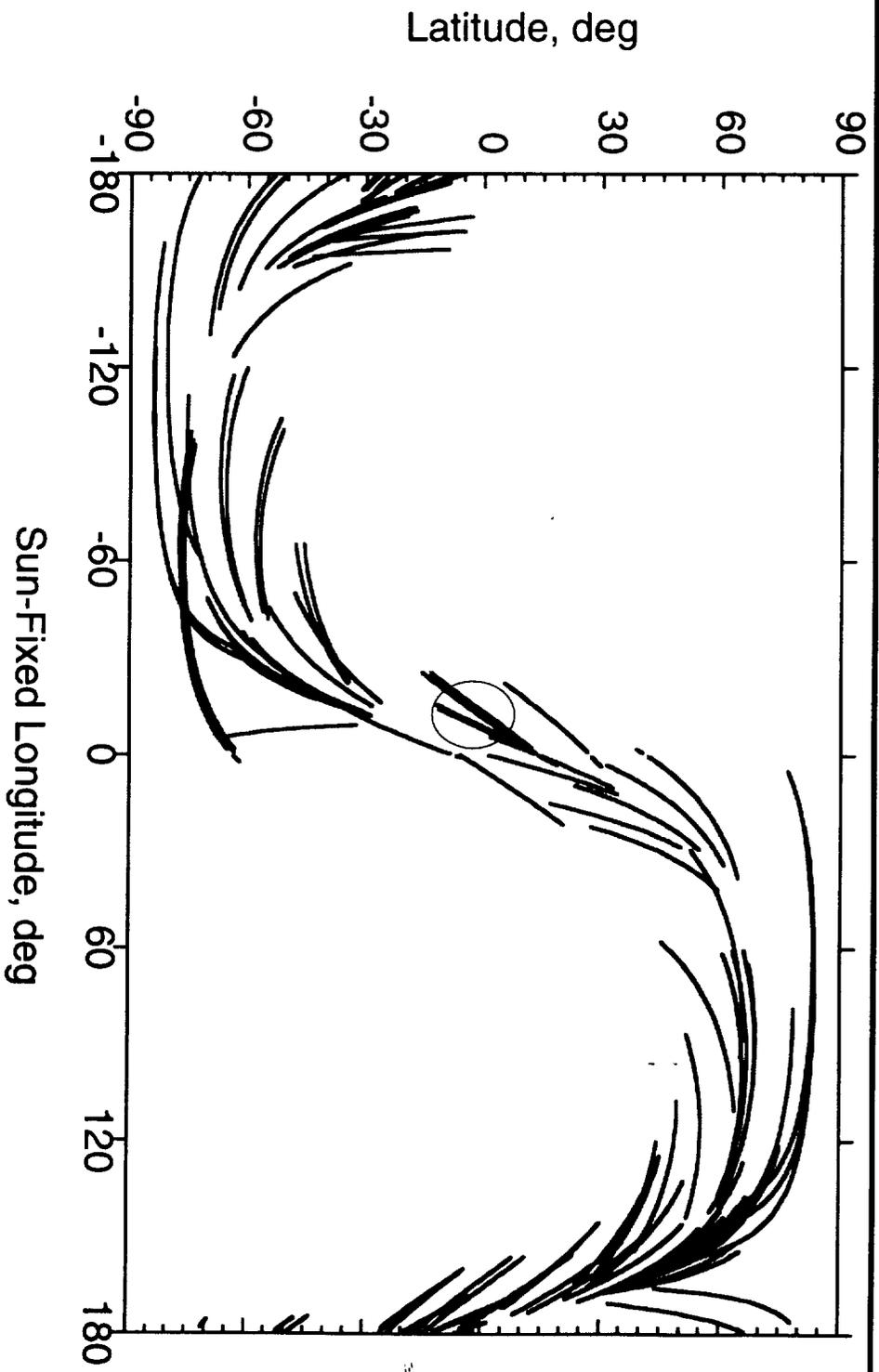
$$\alpha = -2a \int_{r_0}^{\infty} \frac{d \ln(n) / dr}{\sqrt{r^2 n^2 - a^2}} dr \quad \Rightarrow \quad \ln(n(r)) = \frac{1}{\pi} \int_{nr}^{\infty} \frac{\alpha}{\sqrt{a^2 - r^2 n^2}} da$$

BENDING OF OCCULTED SIGNAL

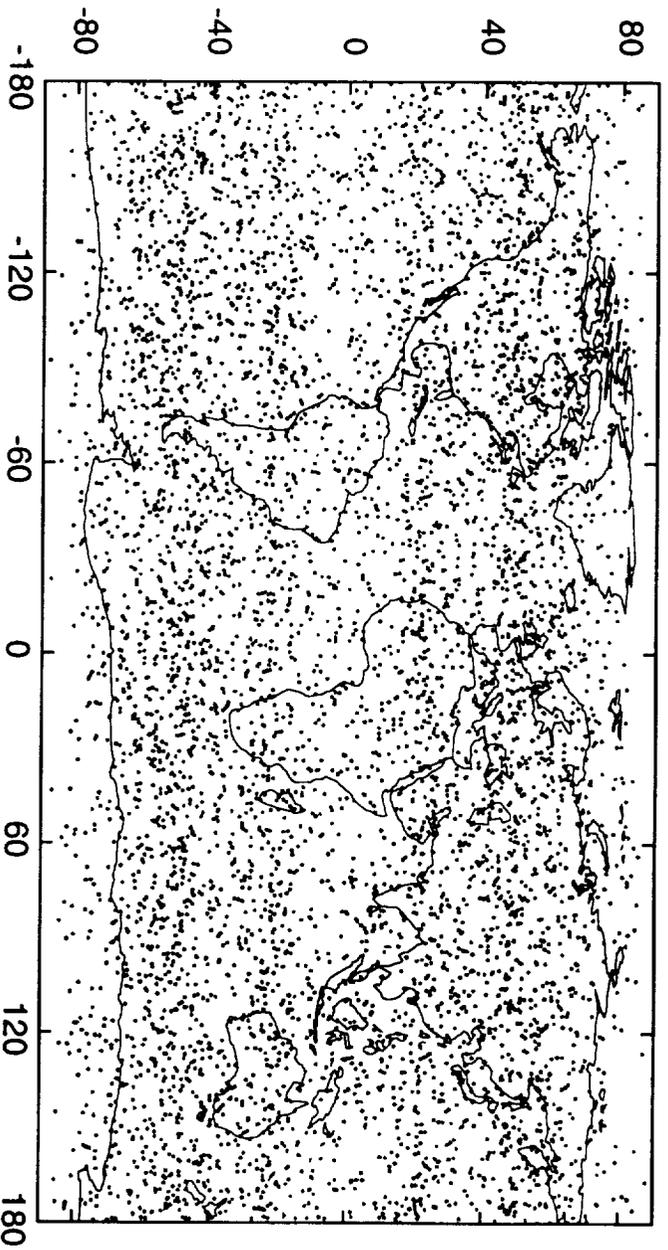
- Shown are bending profiles from 61 occultations on May 4, 1995
- Maximum bending of ~ 0.03 deg corresponds to ~ 1.5 km deviation from a straight line propagation
- For solar maximum conditions we expect a maximum of ~ 0.3 deg of bending (15 km deviation from straight line)



GPS/MET DAILY COVERAGE

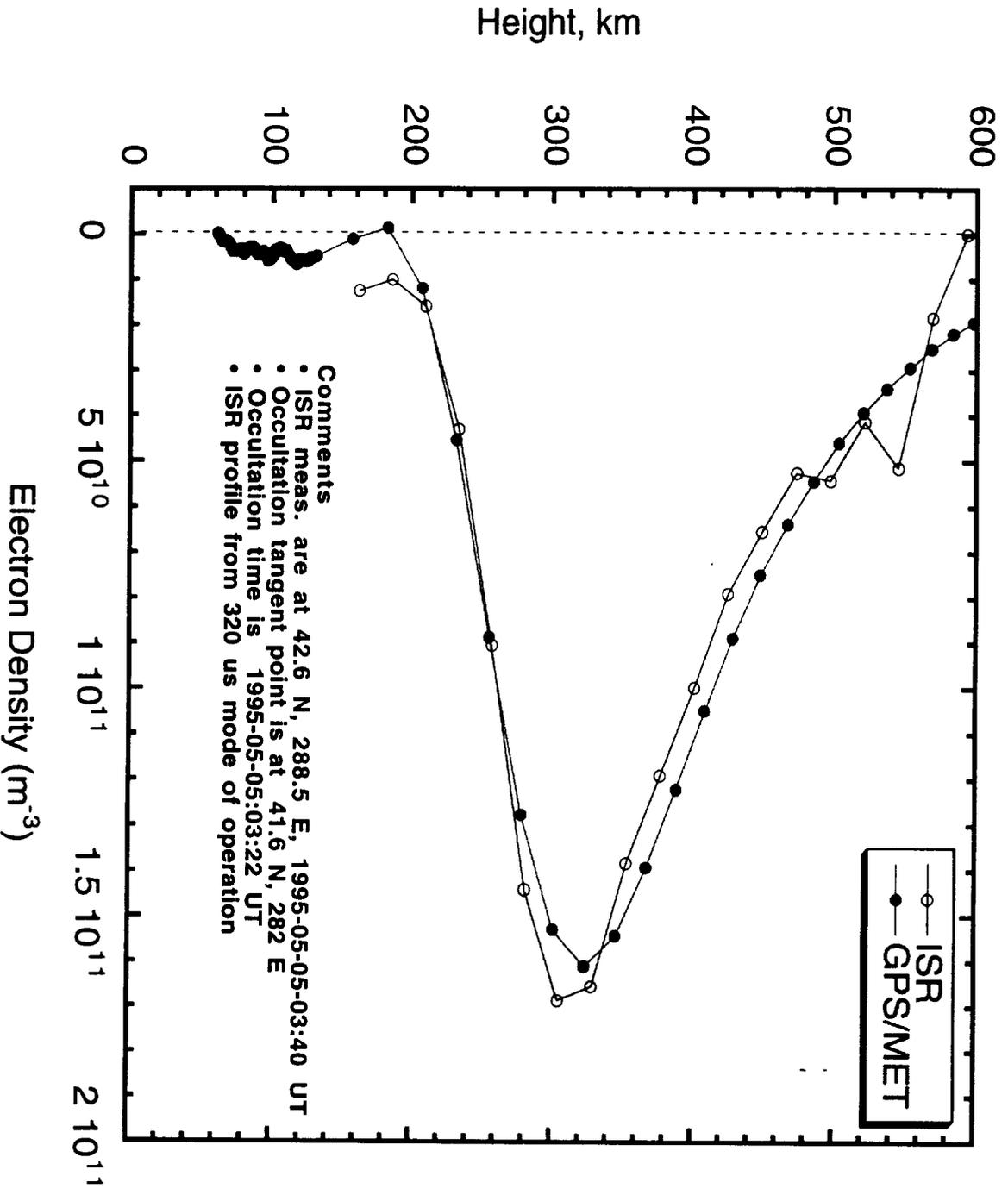


EXPECTED MISSIONS AND DAILY COVERAGE BY 2002



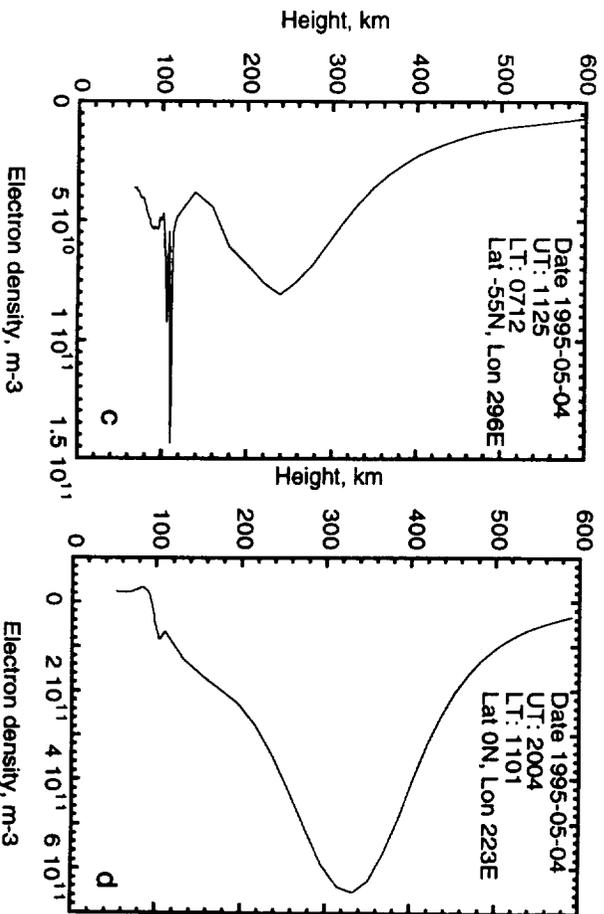
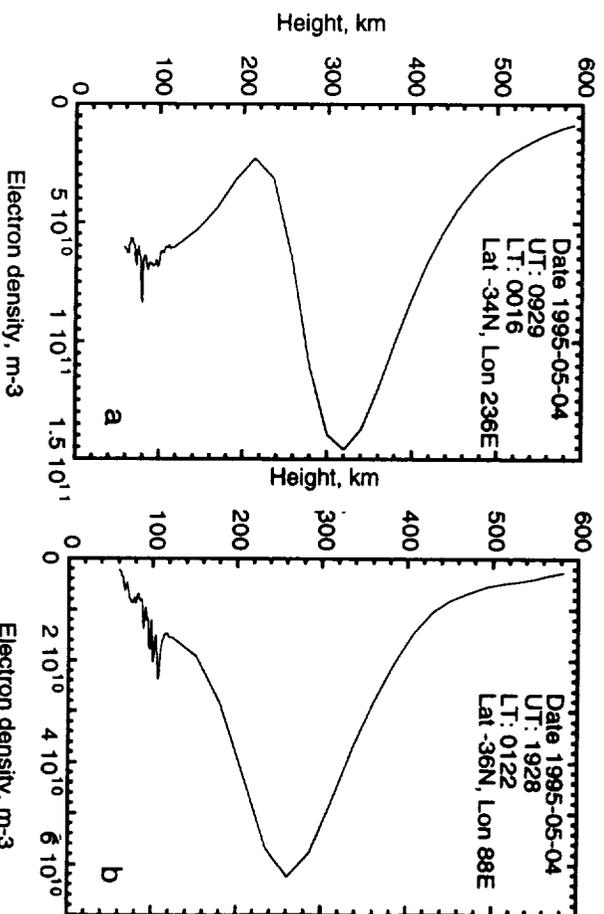
Mission	Occultation Antennas	Altitude (km)	Inclination	Daily Profiles	Launch	Lifetime
Oersted	aft only	400x800	98°	250	Jan 99	3 yrs
Sunsat	aft only	400x800	98°	250	Jan 99	3 yrs
CHAMP	aft only	470	83°	350	June 99	5 yrs
SAC-C	fore/aft	702	97°	700	Sept 99	3 yrs
Jason-1	plasma-sphere	1300	63°	500	Dec 99	5 yrs
GRACE	fore/aft	450	90°	700	Mar 01	5 yrs
COSMIC	fore/aft	700	70°	4000	Jan 02	4 yrs

Electron density profiles measured at the Millstone Hill incoherent scatter radar and derived from a nearby GPS/MET occultation

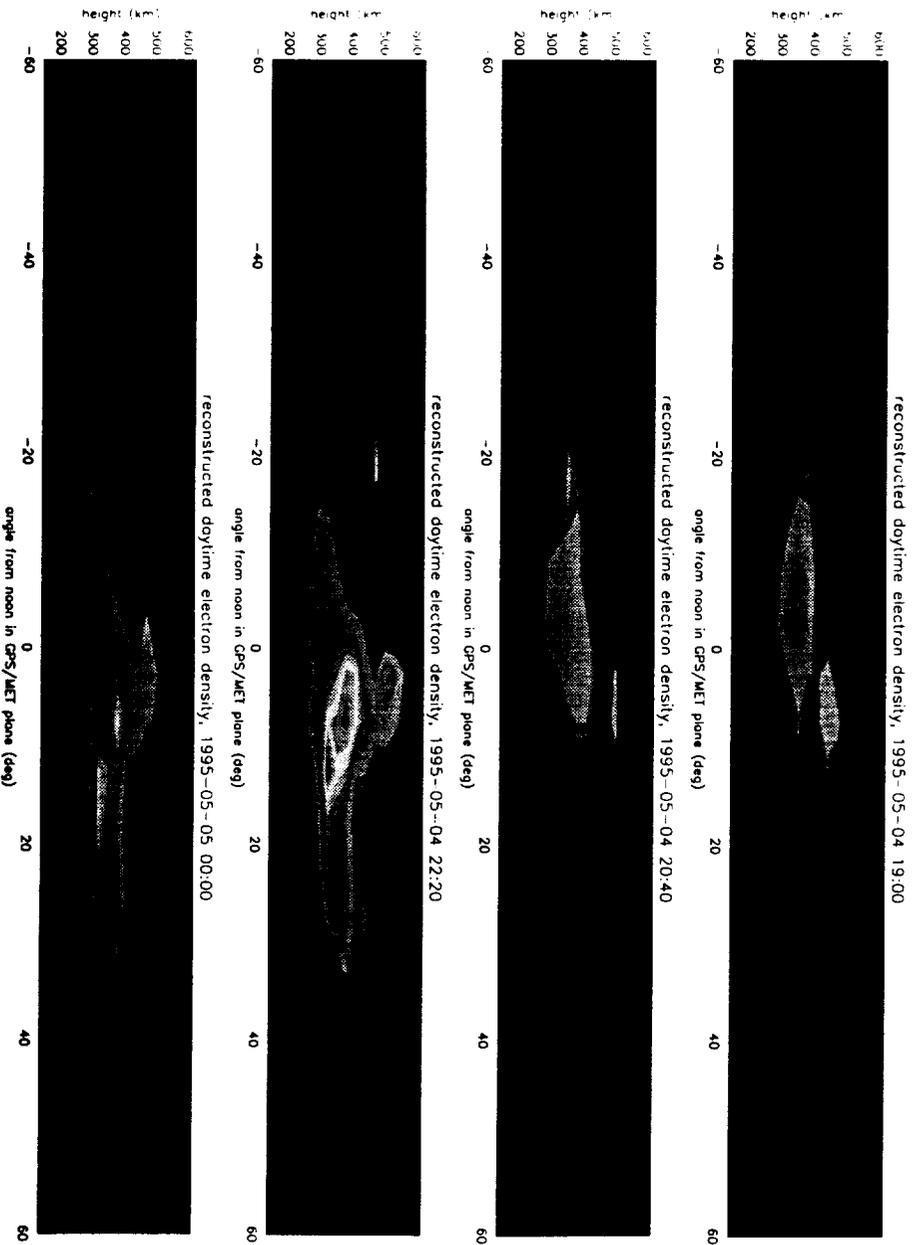


EXAMPLES OF ELECTRON DENSITY PROFILES FROM GPS/MET

- Assumes spherically symmetric medium
- $N_m F_2$ is accurate to 20-30% accurate ($1-\sigma$)
- $H_m F_2$ is accurate to 5-10 km
- Sensitive to structures of short vertical scales, e.g. sporadic E
- Accuracy can be improved by combining nearby occultations in a tomographic fashion

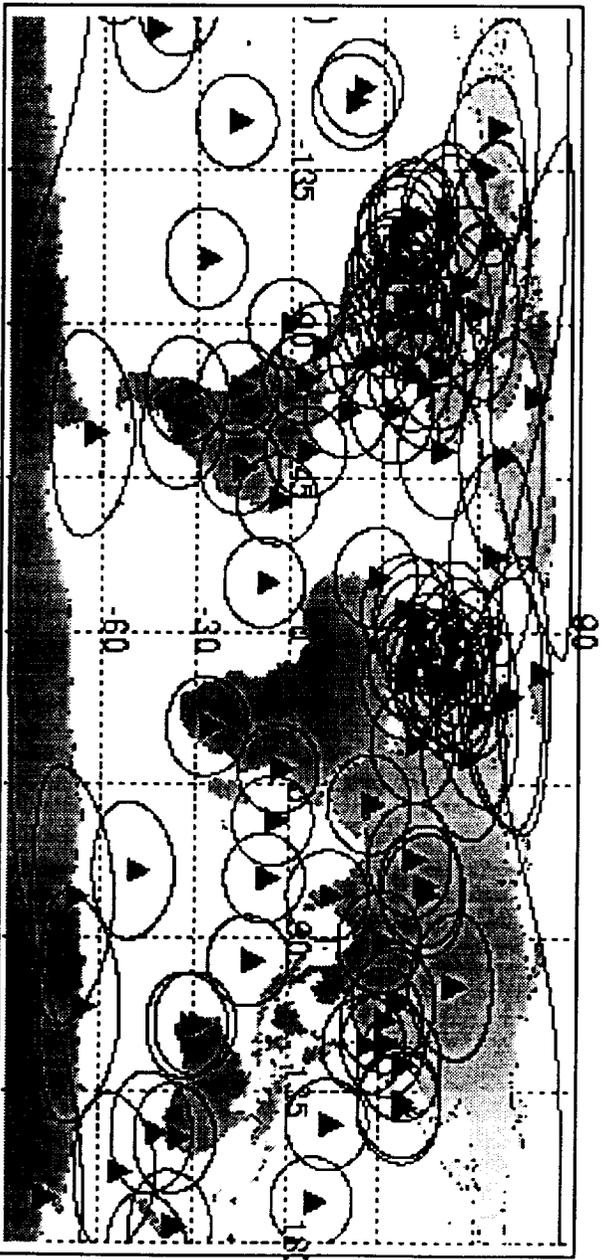


2-D TOMOGRAPHIC RECONSTRUCTIONS OF ELECTRON DENSITY OBTAINED WITH GPS/MET (MAY 4-5, 1995)



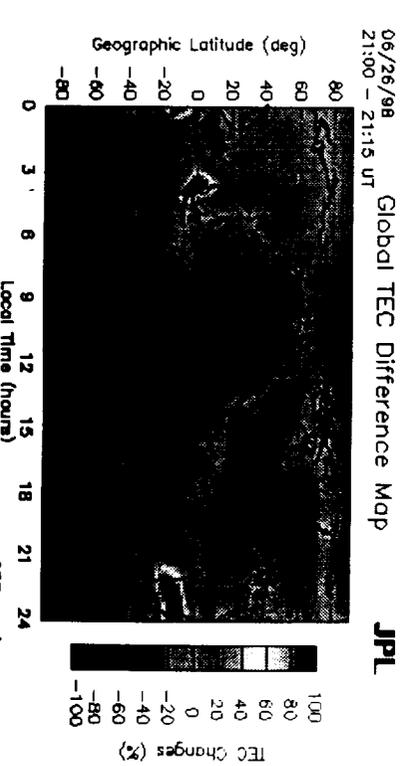
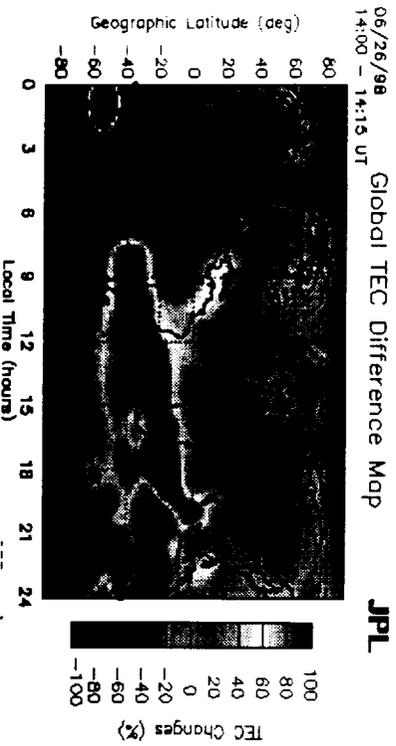
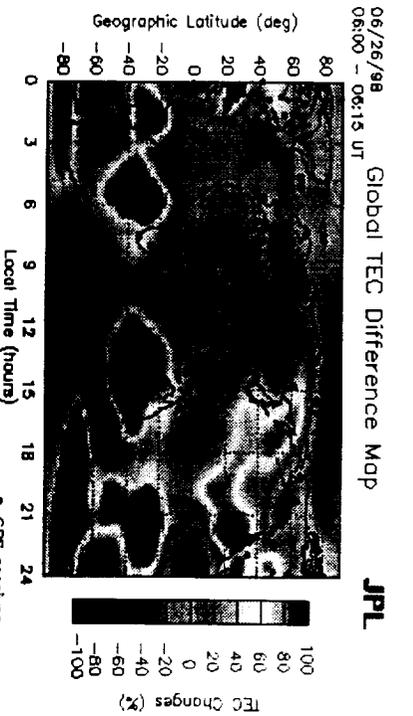
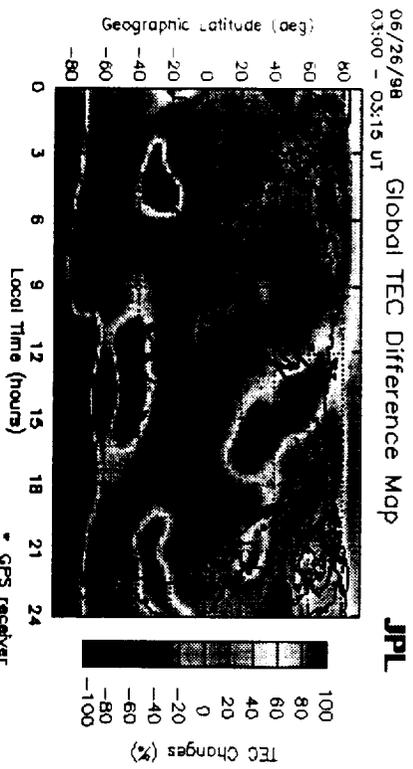
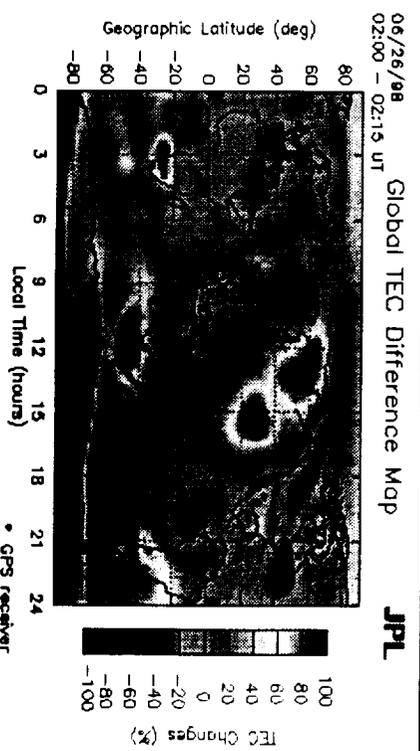
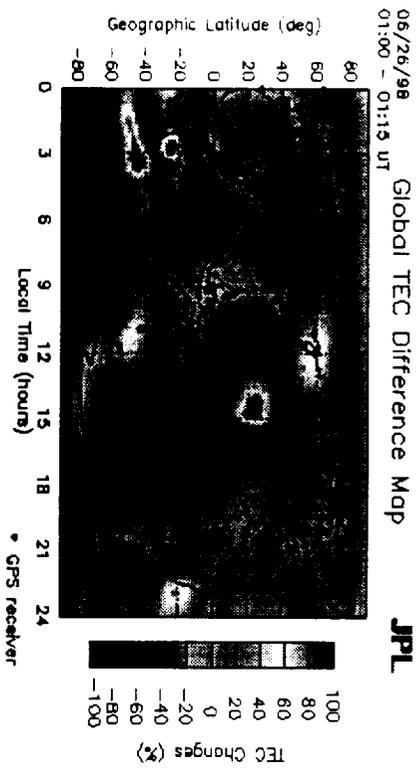
- Images obtained in the orbiter's plane separated by 100 minutes each
- Angle 0 corresponds to local noon

GPS GROUND NETWORK



- Over 200 GPS globally distributed ground receivers, each tracking 8-12 GPS satellites simultaneously
- The coverage is global
- The network operates continuously with minimal intervention
- Many of the receivers are connected to the Internet and are capable of sub-hourly data acquisition (presently 36 such stations exist and 10 more are planned for next year)

GPS DERIVED MAPS OF FRACTIONAL TEC CHANGES DURING A GEOMAGNETIC STORM (JUNE 26, 1998)



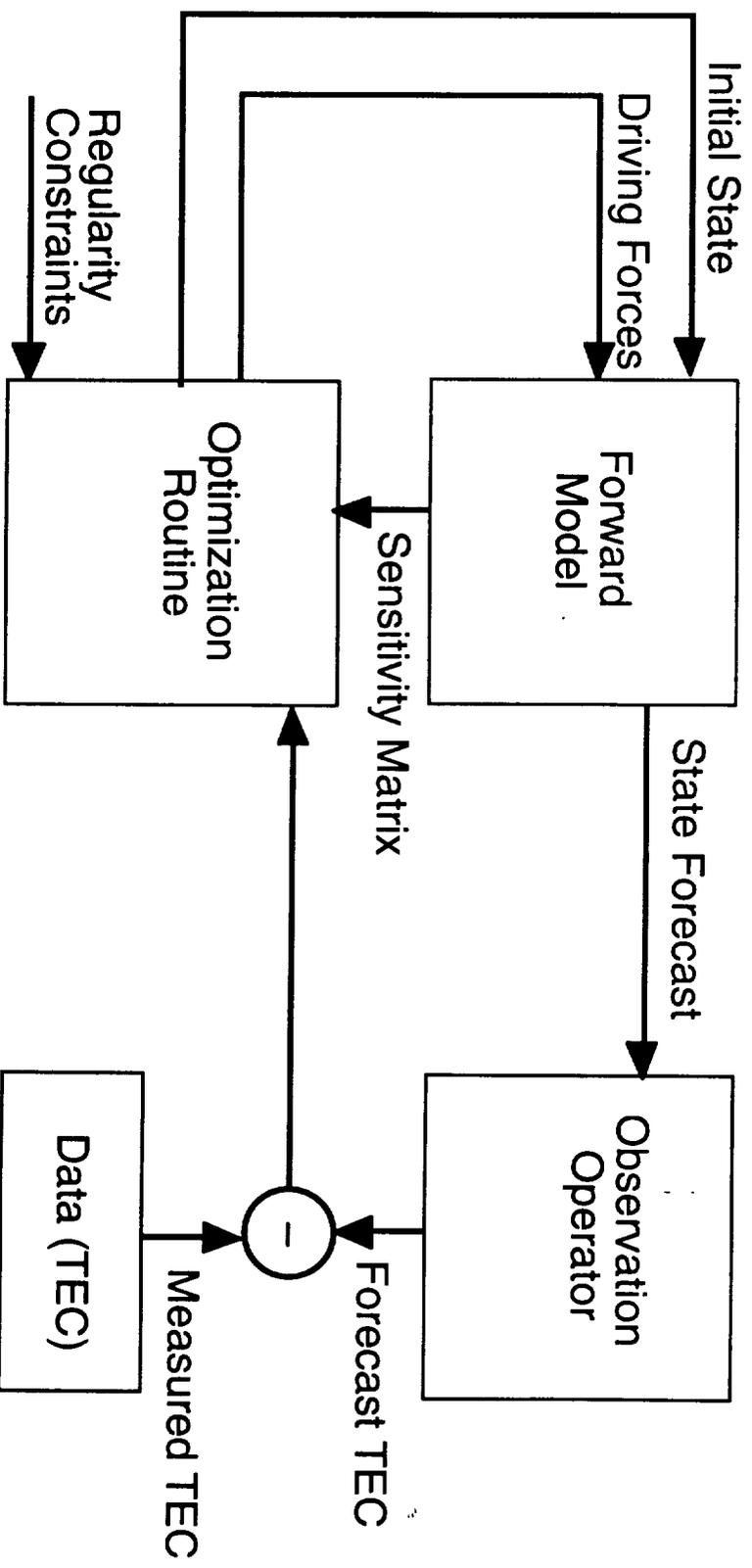
IONOSPHERIC DATA ASSIMILATION

- Start with an ionospheric model (i.e., avoid the complication of using a thermosphere/ionosphere coupled model)
- Recast existing physical models such as USPIM into a numerical framework suitable for data assimilation
- Use a 4-DVar (4-dimensional variational) approach possibly combined with the adjoint method
- Test the effect of assimilating different data types on the stability of the model
- Derive 4D fields of electron density, zonal and meridional components of neutral wind, plasma drift or electric fields, and other driving forces
- Perform Observing System Simulated Experiment (OSSE)
 - Evaluate the accuracy of the models and forecasts by comparing to data-driven models such as the Global Ionospheric Maps (GIM) and 4D tomography, or to incoherent scatter radar measurements
 - Evaluate the predictive power of the model by comparing predicted measurements to actual ones or to direct measurements

THE NEED FOR DATA ASSIMILATION

- The availability of continuous global data on the ionosphere (e.g., GPS TEC data from global network of over 200 ground receivers; the launch of several LEO satellites tracking GPS in an occultation geometry; planned airglow missions)
- The need to combine these data in an optimal solution consistent with the data and the fundamental physical processes of the ionosphere
- Overcome the limitation of ionospheric tomography:
 - Does not make use of the known ionospheric dynamics
 - It lacks the forecasting capability
- Because of the thermosphere-ionosphere-magnetosphere coupling, data assimilation will provide a means to adjust other physical parameters such as neutral wind, plasma drift, density and temperature of neutral constituents, which are of paramount importance as inputs to ionospheric models for successful forecasting

DATA ASSIMILATION APPROACH



FORWARD MODEL

STATE EQUATIONS

$$X_k = F_k(X_{k-1}, Q), \quad X_0 \text{ is the initial state}$$

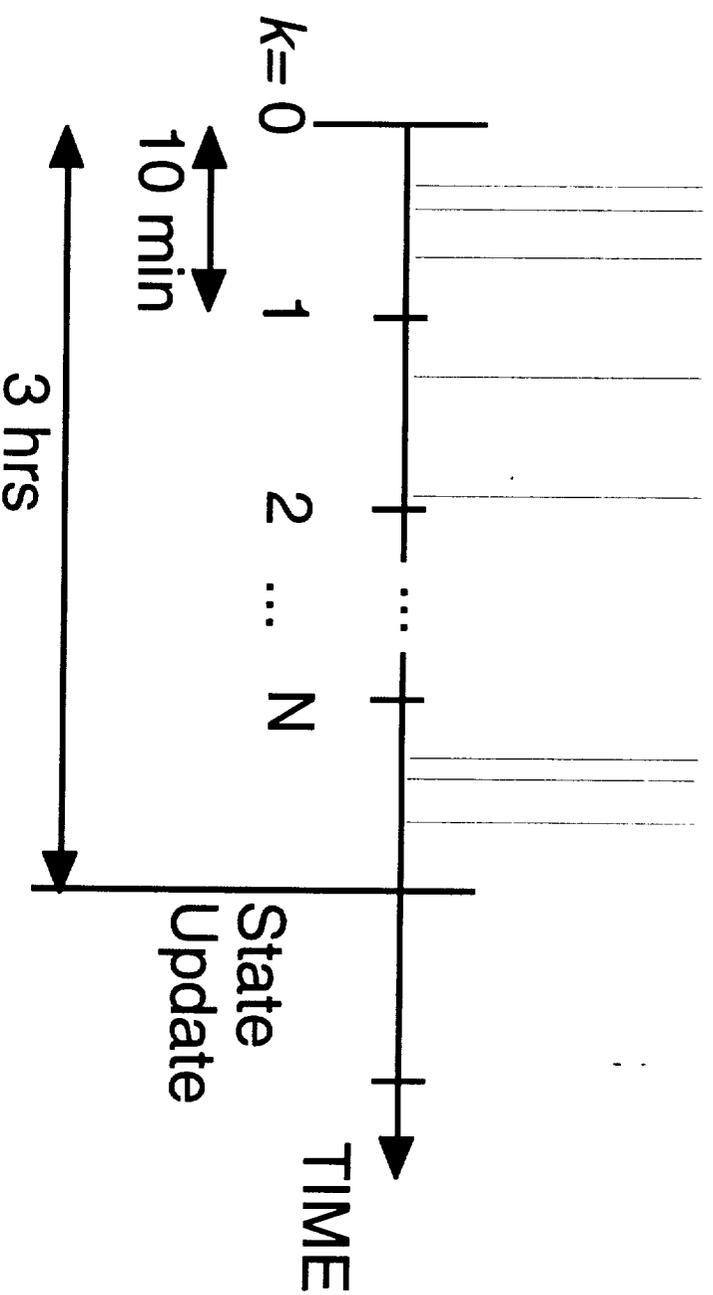
SENSITIVITY EQUATIONS

$$\frac{\partial X_k}{\partial X_0} = \frac{\partial F_k}{\partial X_{k-1}}(X_{k-1}, Q) \frac{\partial X_{k-1}}{\partial X_0}; \quad \frac{\partial X_0}{\partial X_0} = I$$

$$\frac{\partial X_k}{\partial Q} = \frac{\partial F_k}{\partial X_{k-1}}(X_{k-1}, Q) \frac{\partial X_{k-1}}{\partial Q} + \frac{\partial F_k}{\partial Q}(X_{k-1}, Q); \quad \frac{\partial X_0}{\partial Q} = 0$$

OBSERVATION INTERVALS

TEC Measurements



- State is assumed constant over a batch interval (of order 5-15 minutes)
- State is reinitialized at each forecast interval (of order 3-6 hours)
- All data collected at a given forecast interval are used in the optimization

OPTIMIZATION SCHEME

- Given forecasted TEC and measured TEC, we desire to minimize the difference by changing X_0 (initial ion densities) and Q (driving forces)
- We need to define an optimization criterion

$$J(X_0, Q) = \sum_{k=0}^N \left(TEC_k - H_k X_k \right)^T \left(O_k + F_k \right)^{-1} \left(TEC_k - H_k X_k \right) \\ + \left(X_0 - X_b \right)^T M^{-1} \left(X_0 - X_b \right) + \left(Q - Q_b \right)^T W^{-1} \left(Q - Q_b \right)$$

- This can be approximated by a linear least-square problem by expanding the state equation around given backgrounds X_b and Q_b

$$J(X_b + \Delta X_0, Q_b + \Delta Q) \approx \\ \sum_{k=0}^N \left(TEC_k - H_k \left(X_k + \frac{\partial X_k}{\partial X_0} \Delta X_0 + \frac{\partial X_k}{\partial Q} \Delta Q \right) \right)^T \left(O_k + F_k \right)^{-1} \left(TEC_k - H_k \left(X_k + \frac{\partial X_k}{\partial X_0} \Delta X_0 + \frac{\partial X_k}{\partial Q} \Delta Q \right) \right) \\ + \Delta X_0^T M^{-1} \Delta X_0 + \left(Q - Q_b \right)^T W^{-1} \left(Q - Q_b \right)$$

ADJOINT METHOD

State equation

$$X_k = F_k(X_{k-1}, Q)$$

Sensitivity equation

$$\frac{\partial X_k}{\partial \omega} = \frac{\partial F_k}{\partial X_{k-1}}(X_{k-1}, Q) \frac{\partial X_{k-1}}{\partial \omega} + \frac{\partial F_k}{\partial Q} \frac{\partial Q}{\partial \omega}$$

Redefined sensitivity equation

$$\gamma_k = a_k \frac{\partial X_{k-1}}{\partial \omega} + g_k(\omega)$$

Adjoint equation

$$z_k = a_{k+1} z_{k+1} + u_k$$

Partials of cost function

$$\frac{\partial J}{\partial \omega} = 2 \sum_{k=0}^N (TEC_k - H_k X_k)(Q + F)^{-1} H_k \frac{\partial X_k}{\partial \omega} + 2 \sum_{k=0}^N (X_0 - X_b) M^{-1} \frac{\partial X_0}{\partial \omega} + \dots$$

Define

$$u_k = (TEC_k - H_k X_k)(Q + F)^{-1} H_k$$

Partials of cost function

$$\frac{\partial J}{\partial \omega} = 2 \sum_{k=0}^N z_k g_k(\omega) + 2z_0 \gamma_0 - 2a_{N+1} z_{N+1} \gamma_N + \dots$$

OBSERVATIONS

- Solution of sensitivity matrices is computationally very expensive
- Adjoint method allows us to compute the partials derivatives of J directly without using the sensitivity matrices
- Main features of the adjoint method:
 - solves the *adjoint equation* of the sensitivity equation
 - has the same dimension as the state equation
 - always linear
 - the computation of the partial derivatives of J reduces to the computation of a sum

