Abstract—Many space-science experiments need an active isolation system to provide them with the requisite microgravity environment. The isolation systems planned for use with the International Space Station (ISS) have been appropriately modeled using relative position, relative velocity, and acceleration states. In theory, frequency-weighting design filters can be applied to these state-space models, in order to develop optimal H₂ or mixed-norm controllers with desired stability and performance characteristics. In practice, however, since there is a kinematic relationship among the various states, any frequency weighting applied to one state will implicitly weight other states. These implicit frequency-weighting effects must be considered, for intelligent frequency-weighting filter assignment. This paper suggests a rational approach to the assignment of frequency-weighting design filters, in the presence of the kinematic coupling among states that exists in the microgravity vibration isolation problem.

Index Terms: Continuous time systems, control systems, feedback systems, linear-quadratic control, linear systems, optimal control
I. INTRODUCTION

The microgravity vibration isolation problem has received considerable attention in recent years; its difficulty and importance are by now well known. It is anticipated that a number of materials processes and fluid physics science experiments, planned for study on the International Space Station (ISS), will experience unacceptably high background acceleration levels if not isolated [1]–[4]. The low-frequency disturbances of greatest concern (below 10 Hz) are a natural accompaniment of space flight with large, flexible, unloaded structures and random, human-induced excitations.

Passive isolation alone is incapable of providing the necessary disturbance attenuation [5], especially in the low frequency range (below about 1 Hz); and even were a sufficiently soft spring physically realizable, it could not isolate against direct disturbances to the experiment. If the experiment is tethered (e.g., for evacuation, power transmission, cooling, or material transport), a passive isolator cannot provide isolation below the corner frequency imposed by the umbilical stiffness.

An active isolator (such as a magnetic suspension system) that merely possesses a low positive stiffness fares no better in the presence of an umbilical, for the same reasons. Furthermore, if the control system seeks to lower the corner frequency by adding negative stiffness (viz., to counteract the umbilical's positive stiffness) the system will at best possess almost no stability robustness. In the face of the usual umbilical nonlinearities and uncertainties, this situation is clearly unacceptable. At very low frequencies, the rattlespace constraints become limiting, so that any isolation system must have unit transmissibility in that region [6], [7]. In short, the isolator must be active; and it must be capable of dealing with the particular frequency-dependent complexities accompanying a tethered payload and a restrictive rattlespace.

Various active isolation systems exist or are under development to address the microgravity vibration isolation problem. The first in space was STABLE ("Suppression of Transient Accelerations By LEvitation"), developed jointly by McDonnell Douglas and Marshall Space Flight Center (MSFC), and flown with USML-2 on STS-73 in October 1995 [8]. The second isolation system was MIM ("Microgravity Vibration Isolation Mount"), developed jointly by the University of British Columbia,
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MPB Technologies, and the Canadian Space Agency, and placed into operation on the Russian Mir Space Station in April 1996. A second version of MIM (MIM 2) was successfully tested on STS-85 in August 1997 [9]. Boeing’s Active Rack Isolation System (ARIS), built under contract with NASA to isolate an entire International Standard Payload Rack (ISPR), was first tested on-orbit aboard STS-79 in September 1996 [10]. MSFC is developing a second-generation experiment-level isolation system (g-LIMIT: “GLovebox Integrated Microgravity Isolation Technology”), building on the technology developed for STABLE [11]. This compact system will isolate microgravity payloads in the Microgravity Science Glovebox (MSG).

Relative-position and absolute-acceleration measurements are typically available for control of these isolation systems. Linearized analytical system models, using these states, exist for MIM [12], and g-LIMIT, and are under development for ARIS. Each model has a state-space form appropriate for centralized controller design by H2 synthesis. Extensive design software has been written in MATLAB to facilitate H2 controller design for MIM and g-LIMIT.

II. PROBLEM

The states of the analytical state-space models for the above six-degree-of-freedom (6DOF) microgravity isolation systems are relative positions and velocities, and absolute translational accelerations. The models assume the experiment platform to be subject to indirect translational acceleration disturbances (i.e., transmitted indirectly through the umbilical) and direct translational and rotational acceleration disturbances (i.e., applied directly). Controller design by H2 synthesis (or as a subproblem of a mixed-norm design approach) uses a quadratic performance index that has the following forms in the frequency domain:

\[
J = \sup_{\omega \geq 0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \begin{bmatrix}
X^T(s) & U^T(s)
\end{bmatrix}
\begin{bmatrix}
H^T_s(\omega)H_s(\omega) & 0 \\
0 & H^T_s(\omega)H_s(\omega)
\end{bmatrix}
\begin{bmatrix}
X(s) \\
U(s)
\end{bmatrix}
d\omega,
\]

(1)

or

\[
J = \frac{1}{2\pi} \int_{-\infty}^{\infty} \begin{bmatrix}
X(s) & 0
\end{bmatrix}
\begin{bmatrix}
H_s(\omega) & 0 \\
0 & H_s(\omega)
\end{bmatrix}
\begin{bmatrix}
X(s) \\
U(s)
\end{bmatrix}
d\omega,
\]

(2)
where

\[ X_f(s) = W(x)X(s) \]  

is the frequency-weighted state vector, and

\[ U_f(s) = W_u(s)U(s) \]  

is the frequency-weighted control vector. In particular,

\[ X_f(s) = \begin{bmatrix} W_1(s)X_1(s) \\ W_2(s)X_2(s) \\ W_3(s)X_3(s) \end{bmatrix} \]  

where

\[ X_1(s) = X(s) - D(s) \]  

is the relative position vector,

\[ X_2(s) = s[X(s) - D(s)] \]  

is the relative velocity vector, and

\[ X_3(s) = \frac{\omega_0^2 s^2 X(s)}{s + \omega_0} \approx s^2 X(s) \]  

represents the absolute acceleration for sufficiently large \( \omega_0 \). The control engineer seeks to shape the closed-loop acceleration transmissibility so as to pass low-frequency acceleration disturbances (to accommodate rattlespace constraints), to reject intermediate-range acceleration disturbances, to dampen resonances, and to "turn off" the controller below frequencies of unmodeled system dynamics.

In principle, this shaping of the closed-loop system can be accomplished by judicious choices of frequency-weighting filters \( W_i(s) \) (\( i = 1, 2, 3 \)). In practice, however, there is an "implicit frequency weighting," due to the kinematic coupling among the states, that clouds the choices of these design weighting filters. For example, one might wish to weight relative-position states with some filter \( W_1(s) \) (perhaps a low-pass filter or an integrator) to induce large effective umbilical stiffnesses at low frequencies. However, due to the kinematic coupling between relative position \( X_1(s) \) and relative velocity \( X_2(s) \), which equals \( sX_1(s) \), a frequency weight of \( W_1(s) \) on relative position is equivalent (in the cost functional) to a weight of \( \frac{1}{s} W_1(s) \) on relative velocity. Alternatively, such a weighting is equivalent to combined weights of \( \frac{1}{s^2} W_1(s) \) on absolute acceleration \( s^2 X(s) \) and of \( \frac{-1}{s^2} W_1(s) \) on indirect acceleration disturbance \( s^2 D(s) \). There are analogous implicit effects due to design weights on each
other state. In short, the effective frequency weighting on \( X_i(s) \) is the sum of the direct frequency weighting \( W_i(s) \) and of any implicit frequency weightings due to the effects of kinematic coupling among states. The net effect of these direct and implicit frequency weightings can be less than fully intuitive; without a rational procedure one can be left essentially with a trial-and-error approach to frequency weighting selections. In order to use the cost functional to choose appropriate frequency weights one must first place it into a more suitable form.

III. DESIGN FILTER SELECTION

A. Cost-Functional Form

The integrand of the cost functional \( J \) can be expressed as

\[
I(s) = I_X(s) + I_U(s),
\]

where

\[
I_X = X_f^* X_f = X_1^* W_1^* W_1 X_1 + X_2^* W_2^* W_2 X_2 + X_3^* W_3^* W_3 X_3
\]

and

\[
I_U = U_f^* U_f.
\]

Since \( H_2 \) or mixed-norm design methods require solving a matrix Riccati equation (M.R.E.), the presence of a control penalty \( I_U \) couples the state- and control vectors (through this M.R.E.) in a manner that can greatly cloud intuition in design filter selection. The relationships among state weightings and closed-loop transmissibilities can be all but obscured in some problems (such as the one at hand). This algebraic "firewall" can be partially removed by assuming that the control is "cheap," an assumption which permits neglecting \( I_U \). Under this assumption,

\[
I(s) \approx I_X(s).
\]

In terms of relative-position-, relative-velocity-, and (for low enough frequencies) acceleration states, and with the substitutions

\[
Q_i := W_i^* W_i, \quad (i = 1, 2, 3),
\]

the cost-functional integrand is

\[
I_X(s) = (X - D)^* Q_1 (X - D) + [s(X - D)]^* Q_2 [s(X - D)] + (s^2 X)^* Q_3 (s^2 X).
\]

Let "\( T_{XD} \)" and "\( I \)" represent, respectively, the closed-loop transfer function from \( D \) to \( X \), and the identity matrix. With the substitution of \( T_{XD} D \) for \( X \), the integrand can be written as follows:
The usefulness of the above two forms for $I_X$ becomes evident when one considers that neither $X$ nor $D$, in general, has a uniform power spectrum. Further, the power spectrum of $X$ varies as a function of the designer's choice of control. These facts make (14) particularly unattractive for use in selecting frequency-weighting filters. If the acceleration disturbance $s^2D$ is assumed to be zero-mean white Gaussian, and independent of the control vector, the above form for $I_X(j\omega)$ is seen to be more useful for frequency-weighting filter selection than a form which contains $X$ explicitly. Since, with the white-noise assumption, the magnitude of $s^2D$ does not vary with frequency $\omega$, only the terms within the square brackets above require consideration, in assigning frequency-related penalties with $I_X(j\omega)$. If the acceleration disturbance is assumed, more realistically, to be filtered white noise, the coloring filters $W_d(s)$ can be incorporated easily into the square-bracketed portion of (16), by replacing $Q_i$ ($i = 1, 2, 3$) with $W_d(s)Q_i(s)W_d(s)$.

**B. Filter Selection Considerations**

One can now, with some degree of insight, attempt to shape the indirect-acceleration transmissibility $T_{s^2X,s^2D}(=T_{XD})$. In doing so, using (16), one must consider (1) the desired approximate shapes of the transfer functions in $T_{XD}$; (2) the alternative possibilities for the design frequency-weights (the $Q_i$'s, or, equivalently, the $W_i$'s); and (3) the effect of the kinematic frequency factors $\omega^{-4}$ and $\omega^{-2}$, along with any disturbance-coloring filters $W_d(s)$.

Considerations will include the following:

(1) An acceptable closed-loop transmissibility $T_{XD}$ will have unit magnitude up to some corner frequency (e.g., 0.01 Hz), to pass low-frequency disturbances that cannot be attenuated without exceeding
rattlespace constraints. It will then drop off in the intermediate frequencies, and finally rejoin the open-loop transmissibility curve when the controller is to be turned off (e.g., at 100 Hz) to avoid exciting high-frequency system modes.

(2) The frequency-weighting filters should be selected so that at low frequencies $X - D$ dominates $J$, to force the flotor to track the stator. At intermediate frequencies, $s(X - D)$ should be significant, to dampen out resonances. And at higher (and also, to the extent possible, intermediate) frequencies $s^2X$ should be dominant, to increase effective system mass for acceleration-disturbance ($s^2D$) attenuation. State costs should roll off so that control action will not be required above some frequency range.

(3) The “kinematic” frequency factors $\omega^{-4}$ and $\omega^{-2}$ (so designated here, because they arise from the kinematic relationships among the states) must be considered in the choice of the design frequency weights; these factors effectively trade off those weights against one another, in a frequency-dependent fashion. In effect, the frequency factors represent additional “implicit frequency weightings” that do not appear directly in the design frequency weights themselves. Observe, for example, from the expression for $I_x(j\omega)$ that a frequency weight of $W_i(s)$ on relative position is equivalent to a weight of $\frac{1}{s}W_i(s)$ on relative velocity (as previously noted). Both choices would have equivalent effects on the cost functional, and would therefore place equivalent demands on the $H_2$ controller design “machinery.” (However, it might be possible only to implement one or the other of the choices exactly, the “equivalent” choice perhaps being unrealizable.)

C. Filter Choices

By building on the above considerations, it can be shown that a rational choice of frequency-weighting filters $W_i(s)$ for the microgravity vibration isolation problem would be band-pass filtering of relative velocity states, and constant weighting of all other states. (Other choices are possible.) To see this consider band-pass filters, with poles located at $\omega_1$ and $\omega_2$ ($\omega_1 < \omega_2$), to be applied to the relative
velocities. Assume that the first and third legs of the filters have 20 dB/decade slopes. This filter choice affects $I_x(j\omega)$ as follows:

1) For low frequencies ($\omega << \omega_2$),

$$I_x(j\omega) = \left[ T_{XD}^{-1} \left[ \frac{1}{\omega_3 Q_1} \right] \right] \left[ \left[ \frac{1}{\omega^2} \right] \right]_{s=j\omega}$$

That is,

$$I_x(j\omega) \approx (X-D)^* Q_1 (X-D),$$

so that relative positions will dominate $I_x(j\omega)$.

2) For intermediate frequencies ($\omega_1 < \omega < \omega_2$), relative velocity gains in significance. In particular, if $Q_2 >> \frac{1}{\omega_2^2} Q_1$ and $Q_3$ in this range,

$$I_x(s) = \left[ \left[ \frac{1}{\omega^2} \right] \right]_{s=j\omega}$$

In this case,

$$I_x(s) = [s(X-D)]^* Q_2 [s(X-D)].$$

Since $s(X-D)$ is significantly weighted in this region, there will be relative velocity feedback, adding damping to the system. If the bandpass filter poles bracket any open-loop system natural frequencies, the controller will tend to dampen out those system resonances.

3) For high frequencies ($\omega >> \omega_2$),

$$I_x(j\omega) = \left[ \left[ \frac{1}{\omega^2} \right] \right]_{s=j\omega}$$

In another form,

$$I_x(j\omega) = \left[ \left[ \frac{1}{\omega^2} \right] \right]_{s=j\omega}$$

so that the contribution of $s^2X$ to $I_x(j\omega)$ will be significant. Since $s^2X$ and $Q_2$ both roll off at 40 dB per decade, and since $Q_1$ and $Q_3$ both have zero slope, all terms of $I_x(j\omega)$ will have an 80-dB-per-decade roll-off. This means that at very high frequencies control action will not be required: the controller will "turn off." Note that if an additional high-frequency pole is added to $Q_1$ and to $Q_2$ the high-frequency integrand reduces to
Acceleration, then, will dominate the high-frequency cost.

In summary, then the use of band-pass frequency-weighting filters on relative velocity states can be expected to produce $H_2$ controllers that address the concerns presented previously, in Section III-B.

**IV. RESULTING CONTROLLERS**

Use of these filters leads to $H_2$ controllers that, in fact, yield desirable transmissibilities. The following figures present typical transmissibility plots for a microgravity vibration isolation system that provides relative position, relative orientation, and absolute acceleration measurements to an $H_2$ controller. (The relative velocity and relative angular velocity states are reconstructed in a Luenberger observer.) Fig. 1 shows the predicted transmissibilities to indirect acceleration disturbances, with both input and output accelerations directed along the same axis of an experiment-platform accelerometer. (The plots are very similar for the other orthogonal axes.)

![Figure 1. Indirect-Disturbance Attenuation](image1)

![Figure 2. Direct-Disturbance Attenuation](image2)

Fig. 2 shows the predicted transmissibilities to direct translational disturbances, for the same controller and in the same direction. (The lower curve in each figure is the closed-loop curve.) Note that both types...
of disturbance experience significant attenuation in the 0.01 Hz to 10 Hz frequency range. As indicated by the first figure, the indirect (umbilical-induced) disturbances are simply transmitted without attenuation in the range below 0.01 Hz, as required by rattlespace constraints.

V. A POSSIBLE DESIGN APPROACH

The foregoing treatment suggests the following general design approach, for designing frequency-weighting filters in the face of kinematically linked states:

1) Develop a state-space model for the open-loop system, complete with all inputs (controls and disturbances) and outputs. (In the microgravity vibration-isolation example above, the outputs were relative positions, relative velocities, and absolute accelerations).

2) Consider the weighted state-energy portion, $I_X(j\omega)$, of the quadratic cost functional $J$. (In effect, this means that the present approach assumes "cheap control.") Express $I_X(j\omega)$ formally [i.e., as in (16)] in terms of the following:
   a) a vector consisting of a single, "base" type of inertial kinematic quantity [e.g., either absolute positions $X(s)$, or absolute velocities $sX(s)$, or absolute accelerations $s^2X(s)$—or even absolute position or acceleration disturbances $D(s)$ or $s^2D(s)$, respectively—but not a combination of the above];
   b) the undetermined design frequency-weighting filters [e.g., the filters $W_f(s)$];
   c) closed-loop transfer functions [e.g., $T_{XD}(s)$];
   d) and kinematic frequency factors (i.e., powers of $\omega$).

The base kinematic type may include, simultaneously in one vector, both translational and rotational elements (such as position coordinates and orientation angles). It is required only that the units of time be consistent throughout this base vector. For example, since positions have no temporal units, whereas velocities have temporal units of inverse time, these two types of kinematic quantities should not occur together in the chosen base vector.
3) Choose a desired approximate shape (e.g., as portrayed by asymptotic Bode plots) for each of the closed-loop transfer functions (CLTF's) appearing in the above expression for $I_X(j\omega)$. Typically this selection will subdivide the problem into various frequency regions of interest. For example, in the microgravity problem addressed above, unit acceleration transmissibility is desired below some corner frequency, maximum acceleration-disturbance attenuation is desired in an intermediate frequency region, and controller "turn-off" is desired at higher frequencies.

[Note: It should be obvious that not all desirable CLTF's are, in fact, achievable. However, the choice of reasonable, desired, CLTF's can provide a starting point for filter design selection.]

4) Use the desired CLTF’s and the kinematic frequency factors to aid in the selection of the frequency-weighting filters. Choose the filters such that $I_X(j\omega)$ weights appropriately the most significant states or kinematic quantities, in each frequency region of interest. By adjusting these weights judiciously, the designer seeks to cause the desired terms of $I_X(j\omega)$ to dominate in the appropriate frequency regions.

VI. CONCLUDING REMARKS

The suggested approach, while not a simple, artless procedure, does permit the designer to incorporate a degree of physical intuition into the frequency-weighting selection task, even when faced with kinematic coupling among the states. Note that this approach retains its utility with states such as relative positions ($x - d$) and relative velocities ($\dot{x} - \dot{d}$), which include internally the plant disturbances ($d$). The approach can help to inform the filter-selection task, to aid in threading the entanglements of intrinsic frequency weightings and penetrating the “firewall” of the M.R.E. This can relieve the designer of resorting to a mere trial-and-error approach, and can lead to a considerable savings of time and effort in the design process.

The method was applied to a 6DOF microgravity vibration isolation problem. Resulting filter selections were shown to produce an effective $H_2$ controller.
As a final note, for some problems (such as the microgravity vibration isolation problem addressed above), it is possible to re-express the integrand $I_x(s)$ as the sum of a quadratic in a sensitivity-function matrix ($S_{XD}$) and a quadratic in a complementary sensitivity-function matrix ($T_{XD}$). This can be seen readily from (16), by using the substitution $S_{XD} = I - T_{XD}$. In such reformulations the weights on $S_{XD}$ and $T_{XD}$ appear as functions of the $i$ state weighting matrices $W_i(s)$. The problem of state weight-selection can then be treated conveniently as a mixed sensitivity problem. This will be the subject of a future paper.

REFERENCES


**Figure Captions:**

Figure 1. Indirect-Disturbance Attenuation

Figure 2. Direct-Disturbance Attenuation

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Figure 1. Indirect-Disturbance Attenuation
Figure 2. Direct-Disturbance Attenuation
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