The use of soft computing techniques in coherent communications phase synchronization provides an alternative to analytical or hard computing methods. This paper discusses a novel use of Adaptive Neuro-Fuzzy Inference Systems (ANFIS) for phase synchronization in coherent communications systems utilizing Multiple Phase Shift Keying (M-PSK) modulation. A brief overview of the M-PSK digital communications bandpass modulation technique is presented and its requisite need for phase synchronization is discussed. We briefly describe the hybrid platform developed by Jang[6] that incorporates fuzzy/neural structures namely the, Adaptive Neuro-Fuzzy Interference Systems (ANFIS). We then discuss application of ANFIS to phase estimation for M-PSK. The modeling of both explicit, and implicit phase estimation schemes for M-PSK symbols with unknown structure are discussed. Performance results from simulation of the above scheme is presented.

1. INTRODUCTION

The functional block diagram shown in Figure 1 illustrates the signal flow through a simplified digital communications system. The upper blocks—format and modulate indicate the signal transformations from the source to the transmitter. The lower blocks indicate the signal transformations from the receiver to the sink; the lower blocks essentially reverse the signal processing steps performed by the upper blocks. The channel in our work is a source of complex additive white Gaussian (AWGN) noise.

![Figure 1. Simplified digital communications system](https://ntrs.nasa.gov/search.jsp?R=20000088620)

In the case of coherent phase modulation also known as Phase Shift Keying (PSK), the receiver is assumed to be able to generate a set of reference signals whose phases are identical to the phases of the signaling alphabet in use at the transmitter. These reference signals are compared to the incoming signals in the process of making maximum likelihood symbol decisions. In order to be able to generate these reference signals, the receiver has to be in synchronization with the received carrier. This means that there has to be phase concurrence between the incoming carrier sinusoid and a replica of it in the receiver. In other words, if there were no information modulated on the incoming carrier, the incoming sinusoid and the replica in the receiver would pass through zero simultaneously. This is what is known as being in phase lock and it is a condition that must be closely approximated if coherently modulated signals are to be accurately demodulated at the receiver. It is in this context that we use the terms synchronization and estimation somewhat interchangeably. We further illustrate these concepts in the remainder of this section before moving on to our
novel application of soft computing to synchronization.

Figure 1 indicates that from the source to the modulator a message such as the ASCII character 'E', is converted in the format block to a baseband signal or bit stream. This bit stream is characterized by a sequence of digital symbols. These digital symbols are uniformly space pulses representing the message. After modulation, the message takes the form of a digitally encoded waveform or digital waveform.

We focus on PSK modulation in this work. PSK modulation is now widely used in both military and commercial communications systems. The general analytic expression for PSK is

\[ x_{sf}(t) = \sqrt{2E/T} \cos(\omega_c t + \theta_i(t)) \quad 0 \leq t \leq T \]  

(1)

where the phase term (baseband signaling alphabet), \( \theta_i(t) \), will have M discrete values, typically given by

\[ \theta_i(t) = \frac{2\pi i}{M} \quad i = 1, \ldots, M \]

The bandpass signaling alphabet (simply the phasors generated by the PSK modulator) are then

\[ a_i = e^{j\theta_i(t)} \quad i = 1, \ldots, M \]

For the binary PSK (BPSK) example in Figure 2, \( M = 2 \). The parameter \( E \) is the symbol energy, \( T \) is the symbol time duration, and \( 0 \leq t \leq T \). In BPSK modulation, the modulating data shifts the phase of the waveform, \( x_{sf}(t) \), to one of two states, either zero or \( \pi \) (180°). The concepts of information, baseband representation, baseband signaling alphabet, bandpass signaling alphabet and bandpass digital waveforms are illustrated in Figure 2 which shows the ASCII binary representation of the letter "E" and the resulting BPSK waveform. We note the rapid phase changes at the symbol transitions. We see that the information is carried in the phase of the sinusoidal carrier wave. For \( M = 4 \) or quadrature phase shift keying (QPSK) the modulator maps 2 baseband bits of the bit stream to one of 4 possible phases (phasors).

The demodulator in Figure 1 has the task making the best estimate \( \hat{\theta}_i \) of the transmitted bandpass signaling alphabet \( a_i \). The optimum receiver in this case is known as a correlation receiver [18]. The demodulation process requires multiplying the received waveform \( x_{sf}(t) \) by a reference waveforms \( r_i(t) \) which have frequency and phase identical to that of the unmodulated sinusoid used to transmit the waveform (set \( M = 1 \) and \( i = 1 \) in (1)) and sampling a matched filter at the optimum sample time \( T \). Figure 3 illustrates the correlation receiver where we have separated the functions of frequency, phase and symbol timing synchronization and we use complex signal notation

\[ e^{j\alpha} = \cos(\alpha) + j \sin(\alpha) \]  

(2)

Figure 2. Bit stream and corresponding digital waveform for Binary-PSK (BPSK)

Recapping, synchronization (estimation) is a critical function in any modern coherent digital communication system. In synchronous digital transmissions the information is conveyed by uniformly spaced pulses, and the received signal is completely known except for the data symbols and a group of variables referred to as reference parameters. Reference parameters in this context include \( \omega, \phi, \) and \( \tau \) the carrier angular frequency, carrier phase and symbol timing, respectively. Though it is the ultimate task of the receiver to generate an accurate replica of the symbol sequence with no regard to the reference parame-
ters, this is only possible by exploiting knowledge of these parameters. Coherent demodulation is used with passband digital communications. In coherent communications the baseband data signal is derived making use of a local reference with the same frequency and phase as the incoming carrier. Carrier or phase synchronization is the function of aligning the phase and frequency of the receiver oscillator with that of the transmitter oscillator when the information is modulated onto the carrier.

The coherent receiver structure that forms the basis for our work is shown in Figure 4. We make the assumption that the frequency and symbol timing are known exactly. This assumption is valid because generally these reference parameters can be estimated independently of each other. Referring to Figure 4, for carrier phase estimation we represent the received signal by the sufficient statistic, namely,

\[ x(k) = a_k e^{j\phi} + n_k \]  

(3)

Here, \( a_k \) is the possibly complex valued bandpass signaling alphabet symbol, \( n_k \) is the unknown carrier phase, and \( n_k \) is complex additive white Gaussian noise (AWGN). It is shown by Van Trees in [18] that \( \{x(k)\} \) forms a set of sufficient statistics for estimating the phase \( \phi \). In simple terms a sufficient statistic indicates that no other information about the signal is needed to obtain the "best" estimate of the parameter(s).

Figure 4. Coherent receiver structure

Historically the approaches to synchronization structure can be divided into two categories, which we denote as ad-hoc structures and derived or analytical structures. Arguably, the most commonly used analytical method for phase estimation is that of maximum likelihood (ML). The ML estimator has several important theoretical advantages which make it very desirable to obtain. These include its' being the best w.r.t the chosen criterion, the that it has the possibility of achieving the lower bound on performance know as the Cramer-Rao lower bound or CRB. An estimator that achieves the CRB is known as an efficient estimator and if an estimator is efficient it is a ML estimator. The ML estimate is given by the likelihood equation

\[ \arg \max p(x|\phi) \]  

(4)

or by the equivalent expression

\[ \frac{\partial \ln p(x|\phi)}{\partial \phi} \bigg|_{\phi = \hat{\phi}_{\text{ML}}} = 0 \]  

(5)

Where \( p(x|\phi) \) the probability density function of the received vector \( x \), given \( \phi \). The ML estimate \( \phi \) can be thought of as the phase \( \phi \) that most likely gave rise to the received signal \( x \).

Unfortunately, in most practical cases where digital modulation is present, derived structure criterion leads to highly non-linear systems, which in general cannot be solved for the optimum solution and only implicit solutions are arrived at. To find explicit solutions approximations must be made, which leads to results that are valid only for ranges of the parameters and are in essence sub-optimal to the true ML estimate.

Equation (6) gives the exact ML estimator of carrier phase for M-PSK modulation obtained in terms of the received signal over the immediate past N symbols. As pointed out by Kam [7] the ML phase estimator is nonimplementable. Implementable approximations by which \( \phi \) can be obtained have been made [7, 16, 12]. An important and fair question is: "Do better estimation procedures than ML exist?" The answer is that if an efficient estimator \( \phi \) does not exist then it is certainly possible. The difficulty is that no procedure exists for finding such an estimator other than trial and error. The highly nonlinear result for the exact ML estimate of \( \phi \) provides the necessary motivation to examine the application of soft computing methods as an alternative for phase estimator design. SC offers a model free approach to estimator design and provides a viable alternative estimator "design" tool.

\[ \cos \hat{\phi}(k) = \sum_{l=k-N}^{k-1} \sum_{l=1}^{M} e^{-S_L} \sinh q_L(l, \hat{\phi}(k)) \Re \{x(l)a^*_l\} \]  

(6)

\[ \sin \hat{\phi}(k) = \sum_{l=k-N}^{k-1} \sum_{l=1}^{M} e^{-S_L} \sinh q_L(l, \hat{\phi}(k)) \Im \{x(l)a^*_l\} \]  

(7)

Where \( S_L = \frac{\sigma^2}{2N}, q_L(l, \phi) = \frac{\sigma^2}{N} \Re \{x(l)a^*_l e^{-j\phi}\}, \) and \( c \) a constant independent of \( \phi \). N is the block size in symbols.

2. Soft Computing and Estimation

"As the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a
threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics.”

Loth Zadeh
Professor, Systems Engineering, 1973

Zadeh’s “Principle of Incompatibility”, quoted above [22], accurately describes the situation we encounter with ML estimation of the phase in M-PSK. Thus we look for alternative estimation algorithm design methodologies. Soft computing is one such alternative. SC techniques enable construction of estimation models using only target system samples or stated another way, SC offers a model-free design approach.

Our aim here is not to necessarily derive a carrier phase recovery scheme for M-PSK that offers improvements in performance or simplification in implementation compared to existing methods, although these goal are believed attainable. Rather, the main contribution lies in developing new model-free tools for the design of estimation schemes and greater insights into carrier phase estimation [3].

Many problems in estimation and identification can be formulated as function-approximation problems [9]. For instance, in conventional system identification, input-output data is gathered from a physical system and a least-squares approach can be used to provide the best approximation for the linear function that maps the system inputs to its outputs. In a similar fashion, in parameter estimation if the given data is such that it associates measurable system variables with an internal system parameter, a functional mapping may be constructed that approximates the process of estimation of the internal system parameter. A system which exhibits universal approximation is capable of approximating any real continuous function on a compact set to any degree of accuracy [6]. Thus in parameter estimation where the given data is such that it associates measurable system variables with an internal system parameter, a functional mapping may be constructed by ANFIS that approximates the process of estimation of the internal system parameter.

ANFIS refers to a class of adaptive network-based fuzzy inference systems which are functionally equivalent to fuzzy inference systems [6]. Specifically the ANFIS system of interest here is functionally equivalent to the Sugeno first-order fuzzy model. We briefly review Jang’s [6] Hybrid Learning Algorithm, which combines gradient descent and the least-squares method, and discuss how the equivalent fuzzy inference system can be rapidly trained and adapted with this algorithm.

As a simple example we assume a fuzzy inference system with two inputs $x$ and $y$ and one output $z$. The first-order Sugeno fuzzy model with two fuzzy IF-Then rules is the following:

Rule 1: If $x$ is $A_1$ and $y$ is $B_1$, Then $f_1 = p_1x + q_1y + r_1$.

Rule 2: If $x$ is $A_2$ and $y$ is $B_2$, Then $f_2 = p_2x + q_2y + r_2$.

The resulting Sugeno fuzzy reasoning system is shown in Figure 5(a). Here the output $z$ is the weighted average of the individual rules outputs and is itself a crisp value. The corresponding equivalent ANFIS architecture is shown in Figure 5(b). Nodes at the same layer have similar functions. We describe the node functions next. The output of the $i$th node in layer $i$ is denoted as $O_{i,i}$.

Layer 1 Every node $i$ in this layer is an adaptive node. Parameters in this layer are referred to as premise parameters.

Layer 2 Every node $i$ in this layer is a fixed node labeled $[\cdot]$ whose output is the product of all the incoming signals. Here each node represents the firing strength of a rule. Other T-norm operators that perform the fuzzy AND can be used as the node function at this layer.

Layer 3 All nodes in this layer are fixed nodes labeled N. The outputs of this layer are called normalized firing strengths.

Layer 4 All nodes $i$ in this layer are adaptive nodes. The parameters of this mode are called consequent parameters.

3. An Brief Review of Neuro-Fuzzy Integration: ANFIS

In this section we provide a brief description of Jang’s [6] hybrid platform that incorporates fuzzy/neural structures namely the, Adaptive Network Fuzzy Inference System (ANFIS). We limit the review of ANFIS to that required to discuss the material presented within this paper. ANFIS is a universal approximator and as such is capable of approximating any real continuous function on a compact set to any degree of accuracy [6]. Thus in parameter estimation where the given data is such that it associates measurable system variables with an internal system parameter, a functional mapping may be constructed by ANFIS that approximates the process of estimation of the internal system parameter.

We then engage ANFIS in identification of a novel explicit (open-loop) phase estimation for blocks of N, M-PSK symbols. We find that this most general estimation approach can be used as the node function at this layer.

Layer 3 All nodes in this layer are fixed nodes labeled N. The outputs of this layer are called normalized firing strengths.

Layer 4 All nodes $i$ in this layer are adaptive nodes. The parameters of this mode are called consequent parameters.

Layer 4
Two-input first-order Sugeno fuzzy model with two rules.

Layer 4 This node is a fixed node labeled $\sum$, that computes the overall output as the summation of all incoming signals.

Through the definitions of layers 1-5 of the ANFIS structure we have an adaptive network-based fuzzy inference system which is functionally equivalent to Sugeno first-order fuzzy inference systems.

As an adaptive system the outputs of the adaptive nodes depend on the modifiable parameters of the adaptive nodes. The learning rule specifies how these parameters should be updated to minimize a prescribed error measure $E$. The error measure is a mathematical expression that measures the difference between the networks actual output and the desired output, such as the squared error. The basic learning rule of the adaptive network is the steepest descent method. In this method the gradient vector is derived by repeated application of the chain rule. Having obtained the gradient, if we use it in a steepest descent method, the resulting learning algorithm is called the backpropagation learning rule[2, 21, 15, 17, 6].

While the backpropagation learning rule can be used to identify the parameters in an adaptive network, this method is slow to converge. The Hybrid Learning Algorithm [6], which combines backpropagation and the least-squares method (LSE) can be used to rapidly trained and adapt the equivalent fuzzy inference system. For hybrid learning applied in batch mode (off-line learning), each epoch is composed of a forward pass and a backward pass as summarized in Table 3.

<table>
<thead>
<tr>
<th></th>
<th>Forward Pass</th>
<th>Backward Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Premise parameters</td>
<td>Fixed</td>
<td>Gradient descent</td>
</tr>
<tr>
<td>Consequent parameters</td>
<td>Least-squares est.</td>
<td>Fixed</td>
</tr>
<tr>
<td>Signals</td>
<td>Node outputs</td>
<td>Error signals</td>
</tr>
</tbody>
</table>

In the forward pass of the hybrid learning algorithm, node outputs go forward until the final layer (layer 4 in Figure 5(b)) and the consequent parameters are identified by the least-squares method. In the backward pass, the error signals propagate backward and the premise parameters are updated by gradient descent.

4. General Structure Identification for ANFIS

We introduced Jang’s [6] ANFIS architecture along with its rules for learning in the previous section. The rules for learning only deal with parameter identification. Methods for structure identification to determine an initial ANFIS architecture are required before parameters can be tuned using the rules for learning. This process is equally important to the successful application of ANFIS.

Structure identification involves the following issues:

- Selection of relevant input variables
- Initial ANFIS architecture
  1. Input space partitioning
  2. Number and type of MFs for each input
  3. Number of fuzzy if-then rules
  4. Premise and consequence parts of rules
- Select initial MF parameters

In a conventional fuzzy inference system, the number of rules is determined by an expert who is familiar with the target system to be modeled. If one has insight into the system at hand and the system is not too complicated then structure identification can easily be done by a human expert. This is typically the Mamdani method of developing fuzzy If-Then rules. The result is a Fuzzy Associative Memory (FAM) that provides the knowledge base for the system. With more complex systems or those where expert rules are not readily available other means are needed to identify the initial ANFIS structure. In such cases where no expert is
available, the number of MFs assigned to each input variable is often chosen by visually inspecting the data sets or simply by trial and error. This situation is also common to neural networks where there is no simple way to determine in advance the minimal number of hidden units required to achieve a desired performance level [6]. In our applications we use simple grid partitioning. In grid partitioning the number of MFs on each input variable uniquely determines the number of rules [2]. The initial values of the premise parameters are set such that the centers of the MFs are equally spaced along the range of each input variable.

Efficient partitioning of the input space can decrease the number of rules and thus increase the speed of learning and adaptation. There are several techniques that have been used successfully in neural network structure identification [10]. One of the most effective structure identification methods for ANFIS is a binary tree partition scheme based on the CART (classification and regression tree) algorithm [1]. The CART algorithm was first applied to structure determination in fuzzy modeling by Jang in [5]. In the CART tree partition, each region can be uniquely specified along a binary decision tree. Tree partitioning eliminates the problem of exponential growth in rules that we see with grid partitioning. But often more MFs are needed per input to define the fuzzy regions, and these MFs do not generally bear clear linguistic meanings.

5. Explicit (open-loop) ANFIS Phase Estimation

Referring to Figure 4 and (3) we see that the observed (received) signal consists of signal plus noise. The signal is composed of transmitted bandpass alphabet symbols $a_k$ that are multiplied (phase rotated) by an unknown phasor, $e^{j\phi}$ plus complex AWGN. Each packet or block is composed of $N$ such observed signals. The target signal is the phasor, $e^{j\omega}$, which is constant over all $N$ observations.

Due to the curse of dimensionality previously described, such an ideal ANFIS structure is not readily implementable. With improved structure or system identification methods such as CART (Classification and Regression Trees) [1] the ideal system may become a reality in the future. That is it may be possible to estimate the phase accurately if all $N$ symbols could be presented to the ANFIS in parallel, an "adequate" number of MFs could be used on each input and the ANFIS trained. But at present the explosion of rules severely limits the number of inputs if many MFs are required.

6. Implicit (closed-loop) ANFIS Phase Detector

In this section we employ ANFIS in identification of a novel, implementable, implicit (closed-loop or error tracking) estimator of phase for continuous transmission or large blocks of M-PSK modulation using no a-priori knowledge of the signal or estimator structure. We discuss the model, results and performance of this new estimation scheme.

A phase error tracking system (Figure 7) is characterized by the following principles of operation. A phase error signal as a function of the phase alignment error is computed in a functional block called a phase error detector. This error signal is then used in a feedback loop to adjust the voltage controlled oscillator (VCO). If properly designed, the feedback circuit forces the error signal to zero. The VCO is then aligned with the received signal and may serve as the reference phase in the receiver. For unmodulated carrier (CW) the phase error detector simply becomes a multiplier [4]. For modulated signals the structure of the phase error detector is much more complex. Many different phase error detectors have been proposed. They include the M-PSK Costas loop [4], generalized Costas loop [12], the tanlock loop [11], the Mth power loop [16] and the approximate

\[ \Phi \text{ Target} \hspace{1cm} \Phi \text{ ANFIS} \]

\[ f(x_1, \ldots, x_N) \]

\[ \text{Operate mode} \hspace{1cm} \text{Training mode} \]

\[ x_1 \hspace{1cm} \cdots \hspace{1cm} x_N \]

\[ \Phi \text{ Target} \hspace{1cm} \Phi \text{ ANFIS} \]

\[ f(x_1, \ldots, x_N) \]

\[ \text{Operate mode} \hspace{1cm} \text{Training mode} \]

\[ x_1 \hspace{1cm} \cdots \hspace{1cm} x_N \]

\[ \Phi \text{ Target} \hspace{1cm} \Phi \text{ ANFIS} \]

\[ f(x_1, \ldots, x_N) \]

\[ \text{Operate mode} \hspace{1cm} \text{Training mode} \]

\[ x_1 \hspace{1cm} \cdots \hspace{1cm} x_N \]
The ANFIS system is trained with the phase of the observed signal and the target signal previously described for the explicit estimation scheme. Figure 8 shows the block diagram of the configuration used in training the closed-loop ANFIS. Having trained the ANFIS to model the phase error detectors for M-PSK we use the ANFIS phase error detectors in a second order, type II PLL [4] to estimate phase for M-PSK as shown in Figure 9 with loop parameters $\zeta = 0.85$, $\frac{\mu}{SR} = 0.19\%$.

The purpose of the phase detector is to generate an output to function as a measure of the phase error [13]. A key tool to investigating phase acquisition and tracking is the average phase detector output commonly called the PD characteristic \(^3\). This is the expected value of the error signal $e(k)$ conditioned on a fixed value of the phase error $\phi_e \equiv \phi - \phi_0$.

$$S(\phi_e) \equiv E[e(k) | \phi_e]$$

From simulation $S(\phi_e)$ is obtained by opening the loop and measuring the average of the phase error signal. In Figure 10, we present the PD characteristics for 2-, 4-, 8-, and 16-PSK obtained from the ANFIS based PD when the SNR=20dB. One detector cycle is plotted for each M, with the average detector output taken as the average of 1000 observations.

\(^3\)The PD characteristic [13, 4] is also known as the S-curve [12]
Having presented the resulting phase detector characteristics of the ANFIS M-PSK phase error detector we evaluate the variance of the error of ANFIS based, closed loop phase estimator. Recall that the variance is a measure power defined as

\[ \sigma_\phi^2 = E[\hat{\phi}^2] - E[\hat{\phi}]^2 \]  

(9)

The variance of the estimation error provides an important measure of the estimator performance. We simulate the implicit (closed loop) ANFIS estimation scheme with Matlab® using the ANFIS derived phase error detector.

To determine the “optimal” number of membership functions required to achieve the minimum variance simulations where performed varying the number of MFs from 2 to 80. We plot results of these studies in Figure 11, where we see that “optimal” performance is achieved with about 20 MFs. Little or no gain is achieved by using greater than 20 MFs, as can be seen from Figure 11.

We see that using only angle information to designing phase error detectors for M-PSK using the ANFIS model free approach, results in performance equivalent to that of the approximate ML solution. It is believed improved performance can be achieved by the ANFIS solution when the complete complex signal can be processed. This leads back to the explosion-of-rules that we have discussed previously as well as methods for restricting or limiting the range of noisy observations, while not reducing the information about phase contained in those observations.

7. Conclusions

We first described application of ANFIS to the general case of explicit (open-loop or feed-forward) phase estimation for blocks of N, M-PSK symbols. We saw that the “ideal” ANFIS phase estimation system is currently not implementable due to the explosion of rules. It may be possible to estimate the phase of a lock of N symbols accurately if all N symbols could be presented to the ANFIS in parallel, an “adequate” number of MFs could be used on each input and the ANFIS trained. But at present the explosion of rules severely limits the number of inputs if many MFs are required. Use of efficient input partitioning schemes such as CART [1] show promise in this area.

We then engaged ANFIS in identification of a novel implicit (closed-loop or error tracking) estimator of phase for continuous transmission or large blocks of M-PSK modulation using no a-priori knowledge of the signal or estimator structure. We discussed the results and performance of this new estimation scheme. We found that using only angle information in training phase error detectors for M-PSK using the ANFIS model free approach, results in performance equivalent to that of the approximate ML solution. The ANFIS techniques successfully applied to the design of M-PSK PDs is directly applicable to more general and complex Quadrature Amplitude Modulation PD design.
In the application of ANFIS to phase estimation the problem of structure identification remains a critical issue. It is believed improved performance can be achieved by the ANFIS solution when the complete complex signal can be processed. This leads back to the explosion-of-rules that we have discussed previously as well as methods for restricting or limiting the range of noisy observations, while not reducing the information about phase contained in those observations. As improved methods for structure identification are developed and applied to ANFIS, the explosion-of-rules problem can be overcome and improved ANFIS estimator performance and expanded applications are expected.

While the focus of this research was been on the application of soft computing to improve the performance of coherent communications phase estimation, soft computing techniques offer an alternative to current hard computing techniques in all areas of communications parameter estimation and synchronization.

References
