An Optimization-Based Approach
To
Injector Element Design

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ABSTRACT
An injector optimization methodology, method i, is used to investigate optimal design points for gaseous oxygen/gaseous hydrogen (GO2/GH2) injector elements. A swirl coaxial element and an unlike impinging element (a fuel-oxidizer-fuel triplet) are used to facilitate the study. The elements are optimized in terms of design variables such as fuel pressure drop, $\Delta P_f$, oxidizer pressure drop, $\Delta P_o$, combustor length, $L_{comb}$, and full cone swirl angle, $\Theta$, (for the swirl element) or impingement half-angle, $\alpha$, (for the impinging element) at a given mixture ratio and chamber pressure. Dependent variables such as energy release efficiency, ERE, wall heat flux, $Q_w$, injector heat flux, $Q_{inj}$, relative combustor weight, $W_{rel}$, and relative injector cost, $C_{rel}$, are calculated and then correlated with the design variables. An empirical design methodology is used to generate these responses for both element types. Method i is then used to generate response surfaces for each dependent variable for both types of elements. Desirability functions based on dependent variable constraints are created and used to facilitate development of composite response surfaces representing the five dependent variables in terms of the input variables. Three examples illustrating the utility and flexibility of method i are discussed in detail for each element type. First, joint response surfaces are constructed by sequentially adding dependent variables. Optimum designs are identified after addition of each variable and the effect each variable has on the element design is illustrated. This stepwise demonstration also highlights the importance of including variables such as weight and cost early in the design process. Secondly, using the composite response surface that includes all five dependent variables, unequal weights are assigned to emphasize certain variables relative to others. Here, method i is used to enable objective trade studies on design issues such as component life and thrust-to-weight ratio. Finally, combining results from both elements to simulate a trade study, thrust-to-weight trends are illustrated and examined in detail.

INTRODUCTION
In order to meet future launch program goals, the Spaceliner 100 Technology Roadmap1 specifies very aggressive system goals for safety, life and cost per pound of payload launched into Earth orbit. Spaceliner 100 safety goals would decrease catastrophic events from the current 1 in 200 to 1 in 1,000,000 in 15 years. The life goal would be increased from the current 200 manned missions per year to 2000-5000 per year over the same time period. Concurrently, the cost goal aims to reduce the cost of delivering payloads to Earth orbit from the current $10,000 per pound to $1000 per pound in 10 years and to $100 per...
an empirical design methodology for GOJGH2 used to generate the design data—in this work, discrete entities. The first element is the tool injectors generated by Calhoon et al. 9° The As implied above, or appropriate CFD models. generated with modern experimental techniques methodologies. If required, new data can be from existing databases and empirical design source can be used, design data can be obtained structured so that any pertinent information specified constraints. Since toward an optimum design subject to his guide the designer through the information to generate appropriate design data and then of injector design. Simply put, method i is used to generate appropriate design data and then guide the designer through the information toward an optimum design subject to his specified constraints. Since method i is structured so that any pertinent information source can be used, design data can be obtained from existing databases and empirical design methodologies. If required, new data can be generated with modern experimental techniques or appropriate CFD models. As implied above, method i is comprised of two discrete entities. The first element is the tool used to generate the design data—in this work, an empirical design methodology for GOX/GH2 injectors generated by Calhoon et al.9,10 The second element in method i is a group of optimization techniques. It is the optimization capability that extends method i beyond previous injector design methodologies. The optimization scheme allows large amounts of inter-related information to be managed in such a way that the extent to which variables influence each other can be objectively evaluated and optimal design points can be identified and evaluated with confidence. In this work, the Response Surface Method (RSM)11 is used to facilitate the optimization. The RSM approach is to conduct a series of well-chosen experiments (i.e., numerical, physical, or both) and use the resulting function values to construct a global approximation (i.e., response surface) of the measured quantity (i.e., response) over the design space. A standard constrained optimization algorithm is then used to interrogate the response surface for an optimum design. The approach used to develop and demonstrate this new methodology is divided into three main tasks. Task 1 is a proof of concept where the basic methodology is developed and demonstrated on single element injectors. Work on Task 1 for the shear coaxial element has been reported previously.7,8 The work for the swirl coaxial and impinging elements, which completes the empirical database for Task 1, is presented below. To conclude Task 1, all the design data, along with optimization techniques developed to date, will be demonstrated in an element selection/preliminary design process. Task 2 involves replacing/augmenting the empirical data with data from physical and numerical experiments (i.e., test data and validated CFD analyses). Task 3 involves using CFD analyses and empirical methods to design a multi-element injector consisting of 7-12 elements. Optimization will be done in the context of single element variables plus element pattern, element spacing, film cooling, etc.

SCOPE OF CURRENT EFFORT
This paper presents the design optimization of both a swirl coaxial injector element and a fuel-oxidizer-fuel (F-O-F) impinging injector element. The swirl coaxial element has been used somewhat sparingly in this country, but has been widely used in Russia because of its reported ability to perform well over a large throttle range.12 A schematic of the element is shown in Fig. 1. The empirical design
The methodology of Calhoon et al. uses the oxidizer pressure drop, $\Delta P_o$, fuel pressure drop, $\Delta P_f$, combustor length, $L_{comb}$, and the full cone swirl angle, $\Theta$, as independent variables. Due to stability considerations for this injector design, the $\Delta P_o$ range is set to 10-20% of the chamber pressure, while the $\Delta P_f$ range is set to 2-20% of chamber pressure. The combustor length, defined as the distance from the injector to the end of the barrel portion of the chamber ranges from 2-8 inches. The full cone swirl angle is allowed to vary from 30-90°. The dependent variables modeled are $ERE$ (a measure of element performance), wall heat flux, $Q_w$, injector heat flux, $Q_{ij}$, relative combustor weight, $W_{rel}$, and relative injector cost, $C_{rel}$.

Figure 1. Swirl Coaxial Injector Element Schematic

The F-O-F triplet element type is widely used and is capable of operating at high efficiency levels. A schematic of an F-O-F element is shown in Fig. 2. The empirical design methodology of Calhoon et al. uses the oxidizer pressure drop, $\Delta P_o$, fuel pressure drop, $\Delta P_f$, combustor length, $L_{comb}$, and the impingement half-angle, $\alpha$ as independent variables. For this injector design, the pressure drop range is set to 10-20% of the chamber pressure due to stability considerations. The combustor length again ranges from 2-8 inches. The impingement half angle is allowed to vary from 15-50°. Dependent variables are, again, $ERE$, wall heat flux, $Q_w$, injector heat flux, $Q_{ij}$, relative combustor weight, $W_{rel}$, and relative injector cost, $C_{rel}$.

Figure 2. Schematic of F-O-F Injector Element
In the following sections, the injector models and the generation of design data are briefly discussed. Response surfaces for each of the dependent variables are generated and then combined into a joint surface for each element to facilitate the optimization process. Optimization of each element is demonstrated by applying equal weights for all dependent variables as they are added to the joint response surface one at a time and, then, by applying unequal weights that might reflect specific design priorities and trades. Finally, thrust-to-weight ratio trends are for each element are examined and compared.

**INJECTOR DESIGN MODELS**

This section provides details of the models used to generate the design data for the dependent variables previously noted.

**SWIRL COAXIAL ELEMENT DESIGN MODEL**

The process for generating the design data for the swirl coaxial element is described and sample results are also presented. The chamber pressure, mixture ratio, and propellant flow rates selected for this example are:

\[
P_c = 1000 \text{ psi}
\]

\[
MR = 6
\]

\[
m_{GO_2} = 0.25 \text{ lb/s}
\]

\[
m_{GH_2} = 0.042 \text{ lb/s}
\]

The gaseous propellants are injected at a temperature of 540 R.

**Dependent Variable Models**

Reference to Figure 1 shows that the GO\(_2\), flowing in the center post of the element, exits the element with both radial and axial velocity components. This effect is achieved by introducing the GO\(_2\) tangentially into the center post through small slots. When the GO\(_2\), under hydrostatic head, is forced through the tangential slots, part of the pressure head is converted into a velocity head, causing a rotational velocity in the element. With the operating conditions fixed at the above-noted levels, the work of Doumas and Laster\(^{13}\) is used to define the element geometry required to generate GO\(_2\) swirl angles. Although developed for liquids, this work has been used successfully to design swirl coaxial elements for gaseous propellants\(^{14,15}\). For a specified \(\Delta P_o\) and swirl angle, \(\Theta\), the number and size of tangential slots, the discharge coefficient, the GO\(_2\) center post diameter, \(d_o\), and the radial and axial GO\(_2\) velocity components, \(V_o\) and \(V_a\), are calculated. These quantities are then used to determine the dependent variables for each design condition.

The element \(ERE\), calculated according to the empirical design methodology of Calhoon et al., is a function of all four independent variables noted above. A cold flow mixing efficiency, \(E_{m90}\), for \(\Theta = 90^\circ\), is correlated by:

\[
E_{m90} = 100 - 5 \ln \left( \frac{K_s}{L_{cold}/d_o} \right)
\]

The cold flow mixing length, \(L_{cold}\), is correlated from a known chamber length, \(L_{comb}\). The GO\(_2\) post diameter, \(d_o\), is a function of \(\Delta P_o\) and \(\Theta\). Smaller values of \(d_o\) correspond to large values of \(\Delta P_o\) and smaller swirl angles. The empirical swirl factor, \(K_s\), is a function of the normalized differential injection velocity, \((V_f - V_a)/V_o\). \(K_s\) increases with increasing normalized differential injection velocity for the range of propellant velocities considered in this effort. For fixed propellant mass flow rates, the velocities \(V_o\) and \(V_f\) are functions of their pressure drops across the injector, \(\Delta P_o\) and \(\Delta P_f\), respectively. For a given \(\Delta P_o\), \(V_o\) also depends on the swirl angle. Lower \(V_o\)'s are a product of higher swirl angles. Cold flow mixing is thereby enhanced with higher values of \(V_o\) (i.e. \(\Delta P_o\)) and \(L_{comb}\). Lower values of \(V_f\) (i.e. \(\Delta P_f\)) and \(\Theta\) also tend to enhance cold flow mixing.

A fractional factor, \(f_s\), is applied to \(E_{m90}\) to account for the lower levels of cold flow mixing found with swirl angles less than 90\(^\circ\). The resultant measure of cold flow mixing, \(E_{m,lo}\), is a product of \(E_{m90}\) and \(f_s\). This factor, for a given design, is a function of the normalized differential injection velocity and the ratio of radial to axial GO\(_2\) velocity, \(V_o/V_a\). Increasing values of both quantities increase \(f_s\) with a value of \(f_s = 1\) being found at \(V_o/V_a = 1\) (\(\Theta = 90^\circ\)) for all values of \((V_f - V_a)/V_o\). Larger values of \(f_s\) increase cold flow mixing. These values are found at low \(\Delta P_o\) and high \(\Delta P_f\) and \(\Theta\). There is no dependency of \(f_s\) on chamber length. These trends are opposite those noted above. The effect of the competing influences of the independent
variables on ERE trends will be discussed later. Finally, ERE is proportional to $E_{m,sp}$.

The wall heat flux $Q_w$ is correlated with the propellant momentum ratio as defined by:

$$MR = \frac{m_f u_f}{m_i u_i}$$  \hspace{1cm} (2)

The wall heat flux curve from the Calhoon et al. methodology is fairly flat, varying only about 10% from high to low for the range of pressure drops considered in this effort. $Q_w$ decreases with increasing $V_o$ (high $\Delta P_o$ and low $\Theta$) and decreasing $V_f$ (low $\Delta P_f$). That $Q_w$ would decrease with increasing $V_o$ is counter to intuition. It seems that high values of $V_o$, for any $\Theta$, would result in higher mixture ratios in the wall region as is the case for liquid O$_2$. This effect is not discussed by Calhoon et al. The CFD analysis to be done in Task 2 should clarify this situation.

For this effort, the model for $Q_w$ is used as is. The wall heat flux seen by the injector, $Q_{nj}$, is actually modeled by the distance from the injector at which the propellant streams intersect. This axial distance is measured at the radial position corresponding to the center of the coaxial fuel annulus, or gap. It is here that the streams begin to mix and burn. This measure is qualitative, but captures the trend that higher injector heat fluxes occur the nearer the injector that the combustion begins. The axial distance is affected directly by the swirl angle, and, indirectly, by the propellant pressure drops. $Q_{nj}$ decreases with decreasing swirl angle, increasing GO$_2$ pressure drop and decreasing GH$_2$ pressure drop. Swirl angle has the largest effect, while $\Delta P_f$ is the least significant factor.

The relative combustor weight, $W_{rel}$, is simply a function of the combustor length, $L_{comb}$, the distance from the injector to the end of the barrel portion of the chamber. The longer the combustor, the more it weighs.

The relative injector cost, $C_{rel}$, is a function of the fuel gap width and the width of the tangential slots used to induce the swirl in the GO$_2$ center post. Larger values of both variables result in lower machining costs, and thus lead to lower injector cost. The fuel gap width increases with increasing $\Delta P_o$, and decreasing values of $\Delta P_f$ and $\Theta$. Swirl slot width increases with lower values of $\Delta P_o$ and $\Theta$. Overall, $C_{rel}$ decreases with increasing $\Delta P_o$ and decreasing $\Delta P_f$ and $\Theta$. Fuel pressure drop and swirl angle are the most significant factors.

**Generation of Design Data**

The operating conditions given above and the noted independent variables (constrained to the previously noted ranges) are used to generate the design data for element optimization studies. A matrix of propellant pressure drop combinations was developed and nine combinations were selected for use in populating the design data base. There are 20 combinations of $L_{comb}$ and $\Theta$ for each $\Delta P$ combination, making a total of 180 design points selected.

**F-O-F INJECTOR MODEL**

The process for generating the design data for the F-O-F impinging element is described and sample results are also presented. The conditions selected for this example are the same as for the swirl coaxial element:

- $P_c = 1000$ psi
- $MR = 6$
- $m_{GO_2} = 0.25$ lb$_m$/sec
- $m_{GH_2} = 0.042$ lb$_m$/sec

The gaseous propellants are injected at a temperature of 540 R.

**Dependent Variable Models**

Again, the empirical design methodology of Calhoon et al is used to characterize the ERE and $Q_w$. For ERE, a cold flow mixing efficiency is correlated by

$$E_m = 100 - K_r \ln \left[ \frac{9.5}{L_{cold}/d_o} \right]$$  \hspace{1cm} (3)

The cold flow mixing length, $L_{cold}$, is correlated from a known chamber length, $L_{comb}$. The GO$_2$ orifice diameter, $d_o$, is a function of $\Delta P_o$. For the impinging element, the methodology uses a quantity called the normalized injection momentum ratio, $MR_{ni}$, to correlate the mixing at the different design points. Here

$$K_r = f(MR_{ni})$$  \hspace{1cm} (4)
where
\[ MR_n = \frac{2.3 m_x u_x}{m_f u_f \sin \alpha} \] (5)

The minimum \( K_f \), and thus maximum mixing and ERE, occurs at an \( MR_n \) of 2.0. Since the propellant mass flowrates are fixed, only the propellant velocities and the impingement half-angle influence the normalized injection momentum ratio. The velocities are proportional to the square root of the respective pressure drops across the injector, \( \Delta P_o \) and \( \Delta P_f \). For the flow conditions and variable ranges considered in this problem, \( MR_n \) ranges from 3.2 to 17.8. Accordingly, lowering \( \Delta P_o \), raising \( \Delta P_f \), increasing \( \alpha \), or some combination of these actions tend to increase ERE.

The wall heat flux is again correlated with the propellant momentum ratio as defined by

\[ MR = \frac{m_x u_x}{m_f u_f} \] (6)

For the F-O-F triplet element, the maximum wall heat flux occurs at a momentum ratio of approximately 0.4. High heat flux is the result of over-penetration of the fuel jet which produces a high \( O/F \) in the wall region. For the flow conditions and variable ranges considered in this effort, \( MR \) ranges from 1.06 to 2.11. Hence, increasing the value of this ratio by either increasing \( \Delta P_o \) or decreasing \( \Delta P_f \) lowers the wall heat flux.

The heat flux seen by the injector face, \( Q_{inj} \), is qualitatively modeled by the impingement height, \( H_{impinge} \). The notion being that, as the impingement height decreases, the combustion occurs closer to the injector face, causing a proportional increase in \( Q_{inj} \). Thus, for the purposes of this exercise, \( Q_{inj} \) is modeled as the reciprocal of the \( H_{impinge} \). Impingement height is a function of \( \alpha \) and \( \Delta P_f \). Reference to Fig. 2 shows that as \( \alpha \) is increased, \( H_{impinge} \) is shortened. The dependence of \( H_{impinge} \) on the fuel orifice diameter, \( d_f \), and thus, \( \Delta P_f \), results from making the freestream length of the fuel jet, \( L_f \), a function of \( d_f^{16} \). For each \( \Delta P_f \), \( L_f \) was set to six times \( d_f \) for an impingement half-angle of 30°. So, as \( d_f \) increases (corresponding to decreasing \( \Delta P_f \)), \( L_f \) increases, as does \( H_{impinge} \).

The models for \( W_{rel} \) and \( C_{rel} \) are simple but represent the correct trends. \( W_{rel} \) is a function only of \( L_{comb} \), the combustor length from injector face to the end of the chamber barrel section. The dimensions of the rest of the thrust chamber assembly are assumed to be fixed. So, as \( L_{comb} \) increases, \( W_{rel} \) increases accordingly. The model for \( C_{rel} \) is based on the notion that smaller orifices are more expensive to machine. Therefore, \( C_{rel} \) is a function of both propellant pressure drops. As the \( \Delta P \)'s increase, the propellant velocity through the injector increases and the orifice area decreases. So, as either, or both, \( \Delta P_o \) and \( \Delta P_f \) increase, \( C_{rel} \) increases.

**Generation of Design Data**

The system variables given above and independent variables (constrained to the previously noted ranges) are used to generate the design data for element optimization studies. Since propellant momentum ratio is an important variable in the empirical design methodology, a matrix of momentum ratios was developed over the 100-200 psi propellant pressure drop range. Nine pressure drop combinations were selected for use in populating the design data base. There are 20 combinations of \( L_{comb} \) and \( \alpha \) for each \( \Delta P \) combination, making a total of 180 design points selected.

**RESPONSE SURFACE GENERATION**

In this effort, method i uses the Response Surface Method (RSM) to find optimal values of ERE, \( Q_u \), \( Q_{inj} \), \( W_{rel} \) and \( C_{rel} \) for acceptable values of \( \Delta P_o \), \( \Delta P_f \), \( L_{comb} \) and \( \Theta \) or \( \alpha \). The approach of RSM is to perform a series of experiments, or numerical analyses, for a prescribed set of design points, and to construct a response surface of the measured quantity over the design space. In the present context, the five responses of interest are ERE, \( Q_u \), \( Q_{inj} \), \( W_{rel} \) and \( C_{rel} \). The design space for each element consists of the set of relevant design variables \( \Delta P_o \), \( \Delta P_f \), \( L_{comb} \) and \( \Theta \) or \( \alpha \). The response surfaces are fit by standard least-squares regression with a quadratic polynomial using the JMP statistical analysis software. JMP is an interactive, spreadsheet-based program which provides a variety of statistical analysis functions. A backward elimination procedure based on t-statistics is used to discard terms and improve the prediction accuracy.
When the JMP software is used to analyze the 180 design points, five individual full response surfaces for the variables in the design space are approximated by quadratic polynomials that contain 15 terms each.

In the current study, it is desirable to attempt to maximize ERE and while simultaneously minimizing Qw, Qwy, Wrel, and Cref. One method of optimizing multiple responses simultaneously is to build from the individual responses a composite response known as the desirability function. The method allows for a designer's own priorities for the response values to be built into the optimization procedure. The first step in the method is to develop a desirability, \( d \), for each response. In the case where a response should be maximized, such as ERE, the desirability takes the form:

\[
d_1 = \left( \frac{\text{ERE} - A}{B - A} \right)^s
\]

where \( B \) is the target value and \( A \) is the lowest acceptable value such that \( d = 1 \) for any \( \text{ERE} > B \) and \( d = 0 \) for \( \text{ERE} < A \). The power value \( s \) is set according to one's subjective impression about the role of the response in the total desirability of the product. In the case where a response is to be minimized, such as \( Q_w \), the desirability takes on the form:

\[
d_2 = \left( \frac{Q - E}{C - E} \right)^s
\]

where \( C \) is the target value and \( E \) is the highest acceptable value such that \( d = 1 \) for any \( Q_w < C \) and \( d = 0 \) for \( Q_w > E \). Choices for \( A, B, C, \) and \( E \) are chosen according to the designer's priorities or, as in the present study, simply as the boundary values of the domain of \( \text{ERE} \) and \( Q_w \).

Choices for \( s \) and \( t \) are more difficult, but plots such as Figure 2 can be instructive. Figure 3 shows the appearance of the desirability function for the case of maximizing a response. Desirabilities with \( s << 1 \) imply that a product need not be close to the response target value, \( B \), to be quite acceptable. But \( s = 8 \), say, implies that the product is nearly unacceptable unless the response is close to \( B \).

A single composite response is developed which is the geometric mean of the desirabilities of the individual responses. The composite response is defined as:

\[
D = \left( d_1 \cdot d_2 \cdot d_3 \ldots d_m \right)^{1/m}
\]

The complete joint response surface for the present study is given by:

\[
D = \left( d_{\text{ERE}} \cdot d_{Q_w} \cdot d_{Q_w} \cdot d_{W_{rel}} \cdot d_{C_{ref}} \right)^{1/5}
\]

**OPTIMIZATION RESULTS AND DISCUSSION**

Results are presented for each element in two parts. The first set of results for each element are obtained by building the joint response surface with the addition of one dependent variable at a time. The second set of results for each element illustrate an emphasis on performance and life issues. Finally, results for the two element are combined to compare the thrust-to-weight ratio trends for the different element types.
SWIRL COAXIAL ELEMENT

Two sets of results are presented below to demonstrate the capability and flexibility of method i for the swirl coaxial injector element design. These examples illustrate the effect of each variable on the optimum design and the trade-offs between life and performance issues.

Effect Of Each Variable On Element Design

The results in this section were obtained by building the joint response surface with the addition of one dependent variable at a time. The results are shown in Table 1. Case 1 seeks the maximum performance without regard to the effect on the other dependent variables. ERE is a fairly strong function of L_comb—longer chamber lengths allow more residence time for the propellant to mix and burn. The effect of \( \Theta \) on ERE is strongest at low values of \( \Theta \). ERE increases with increasing \( \Theta \) until about \( \Theta = 80^\circ \) and then fall off slightly due to the competing influences noted earlier. These competing influences also cause the effect of both pressure drops on ERE to be somewhat flat, although since \( \Delta P_c \) affects more variables, its influence is slightly stronger. Maximum performance is found at high values of \( \Delta P_c \), \( \Theta \), and \( L_{\text{comb}} \) and at low values of \( \Delta P_f \). This trend is consistent with other works for similar injector elements.14,15

The value of 98.5 found by the optimizer is indeed the highest predicted by this model. However, since the model developed by Calhoon et al. has been shown to slightly under-predict swirl coaxial element performance, the actual value is likely somewhat higher.

The objective of Case 2 is to simultaneously maximize ERE and the minimize \( Q_w \). Table 2 shows that the exact same design point was chosen as for Case 1. Usually, the design which yields the maximum ERE also produces a high wall heat flux. That is not the case here; this issue has already been noted. The minimum \( Q_w \) is found in the region of high \( \Delta P_c \), and low \( \Delta P_f \).

In this area, \( Q_w \) is almost independent of \( \Theta \). Hence, the minimum \( Q_w \) can still be found at the high value of \( \Theta \) required to maximize ERE. It should be noted that in the low \( \Delta P_c \), high \( \Delta P_f \) region, \( Q_w \) is a function of \( \Theta \). Here, as \( \Theta \) is increased, \( Q_w \) increases since the larger swirl angle forces \( d_s \) to increase and thus decrease \( V_r \). In the Calhoon et al. model, this reduction in GO2 momentum causes an increase in \( Q_w \).

The requirement to minimize \( Q_m \) is added in Case 3. In order to minimize \( Q_m \), the swirl angle is decreased from 81° to 37°, thus reducing the injector face heat flux by approximately a factor of 3. This decrease in \( \Theta \) also lowers ERE which forces use of a longer chamber to offset some of the loss. Still, ERE is reduced by over one percent.

Case 4 considers the desire to minimize the chamber weight, \( W_{\text{rel}} \) in addition to maximizing ERE and minimizing \( Q_w \) and \( Q_m \). Since \( W_{\text{rel}} \) depends only on \( L_{\text{comb}} \), the chamber length is shortened by over half. The weight is reduced, but so is ERE. To mitigate the adverse effect on ERE, \( \Theta \) is increased by almost 10°, simultaneously increasing \( Q_m \). ERE drops again by over a percent, while \( Q_w \) remains constant.

### Table 1

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Constraints</th>
<th>Results Case 1</th>
<th>Results Case 2</th>
<th>Results Case 3</th>
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### Table 2

<table>
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<th>( Q_m )</th>
<th>( W_{\text{rel}} )</th>
<th>( Q_{\text{opt}} )</th>
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Table 1. Effect of Each Variable on the Design-Optimal Designs for Original Constraints and Equal Weights for Swirl Coaxial Element.
Finally, minimizing the injector cost, $C_{rel}$, is added in Case 5. The relative injector cost is lowered by decreasing each pressure drop approximately a factor of 2. Decreasing $\Delta P_f$ results in a larger fuel gap and decreasing $\Delta P_o$ allows for a larger swirl slot. These factors combine to lower the cost by almost 10%.

Although several of the variables included in this exercise are qualitative, an important conclusion can still be drawn. The sequential addition of dependent variables to an existing design results in changes to independent and dependent variables in the existing design. The direction and magnitude of these changes depends on the sensitivity of the variables, but the changes may well be significant. The design in Case 5 is quite different that the one in Case 1. Consideration of a larger design space results in a different design—the sooner the additional variables are considered, the more robust the final design.

**Emphasis on Life and Performance Issues**

The purpose of this section is to illustrate the effect of emphasizing certain aspects of the design during the optimization process. Method i allows this emphasis via the weights applied to the desirability functions in the joint response surface. The set of results shown in Table 2 facilitate the illustration. The baseline results in Table 2 (repeated from Case 5 in Table 1) consider the entire design space using the original constraints and equal weights for the dependent variables. The results in for Case 1 are obtained by emphasizing the minimization of the wall and injector face heat fluxes. Desirability functions for both of these variables are given lower heat fluxes tend to increase component life, weighting these two variables is equivalent to emphasizing a life-type issue in the design. Since $Q_w$ is already at its minimum value, it remains fixed. As expected, $\Theta$ is decreased which decreases the value of $Q_m$ by almost 35%. The lower value of $\Theta$ also produces a lower $ERE$. Both propellant pressure drops and the combustor length are increased to mitigate the drop in $ERE$. The increases in $L_{comb}$ and $\Delta P_f$ cause increases in $W_{rel}$ and $C_{rel}$, respectively. The emphasis on life extracts the expected penalty on performance. Additionally, for the swirl coaxial element model, there are also slight weight and cost penalties.

The results for Case 2 are obtained by emphasizing maximization of $ERE$ and minimization of $W_{rel}$ with desirability weightings of 10 and 5, respectively. Increased weighting for these two variables is equivalent to emphasizing a thrust to weight goal for the injector/chamber. The relative chamber length is shortened to slightly lower $W_{rel}$. $ERE$ is maximized by increasing the GO$_2$ swirl angle by a factor of almost 2.5 and also increasing $\Delta P_f$ by over 35%. The value of $ERE$ rises by over one percent. As noted earlier, increasing $\Theta$ leads to increased injector heat flux. For this case, emphasis on thrust and weight tends to have an adverse affect on $Q_m$. Relative cost for the swirl coaxial element model is also increased significantly.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Constraints</th>
<th>Results Baseline</th>
<th>Constraints</th>
<th>Results Case 1</th>
<th>Constraints</th>
<th>Results Case 2</th>
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<tr>
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<td>104</td>
<td>100-200</td>
<td>200</td>
<td>100-200</td>
<td>200</td>
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<td>20-200</td>
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<td>20-200</td>
<td>3.2</td>
<td>20-200</td>
<td>4.4</td>
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<td>$L_{comb}$</td>
<td>2 - 8</td>
<td>3.4</td>
<td>2 - 8</td>
<td>3.6</td>
<td>2 - 8</td>
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<th>Life Variable Weight</th>
<th>Thrust/Weight Variable Weight</th>
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<td>10</td>
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<tr>
<td>$W_{rel}$</td>
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<tr>
<td>$C_{rel}$</td>
<td>1</td>
<td>0.76</td>
<td>1</td>
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Table 2. Effect of Emphasizing Life and Performance Issues for the Swirl Coaxial Element.
IMPINGING ELEMENT

Two sets of results are presented below to demonstrate the capability of method i for the impinging injector element design. These two examples illustrate the effect of each variable on the optimum design and the trade-offs between life and performance issues.

Effect Of Each Variable On Element Design

The results in this section were obtained by building the joint response surface with the addition of one dependent variable at a time. The results are shown in Table 3. Since current non-optimizer based design methods yield high-performing injector elements, simply maximizing the ERE is not a challenge. Accordingly, the initial results (Case 1) are obtained with a joint ERE and Qw response surface. The results in Case 2 have the impingement height added, Case 3 adds the relative chamber weight and the relative cost is added in Case 4. All results are obtained using the original independent variable constraints and all dependent variables have equal weights of one. The results for Case 1 show that ERE is at its maximum and Qw is very near its minimum desirability limit. Minimizing Qw requires a small ΔP/ relative to ΔP0 as evidenced by the values of 100 psi and 183 psi, respectively. Maximum ERE values are found at the longest chamber length, Lcomb = 8 inches. Even with the relatively high value of 183 psi for ΔP0 and low value of ΔP/ of 100 psi, ERE is maximized to 99.9% with an impingement half-angle of 33.1°. Addition of the impingement height to Case 2 to model the injector face heat flux, Qf, forces α lower to increase Himpinge, and decrease Qf. This decrease in the radial component of the fuel momentum has an adverse affect on ERE. This effect is mitigated to a degree by increasing the ΔP/ by 32 psi to 132 psi. ERE is still reduced by 1.6%. Also, the increase in ΔP/ causes increased penetration of the fuel jet which results in a slightly higher Qw.

Case 3 adds the relative combustor weight to the list of dependent variables modeled. Since Wrel is only a function of Lcomb, minimizing Wrel shortens the combustor length from 8 to 6.6 inches. The shorter Lcomb tends to lower ERE. This effect is offset to a large degree by increases in ΔP/ and α, both of which increase the radial component of the fuel momentum. The increase in ΔP/ also causes a slight increase in Qw. The increase in α causes a significant decrease in Himpinge which increases the injector face heat flux.

Finally, the relative cost of the injector is added in Case 5. Since Crel is only a function of propellant pressure drops, both ΔP0 and ΔP/ are driven to their respective minimum values. This and a slight increase in α allow ERE to be maintained at 98%, even with a slight decrease in Lcomb. The largest effect of this fairly dramatic decrease in propellant pressure drops is on Qw. Even though the values for ΔP0 and ΔP/ fell, ΔP/ increased relative to ΔP0 causing Qw to increase by almost 9%. Impingement height and relative combustor weight are essentially unchanged.

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Constraints</th>
<th>Results Case 1</th>
<th>Results Case 2</th>
<th>Results Case 3</th>
<th>Results Case 4</th>
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<td>ΔP0</td>
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<td>183</td>
<td>179</td>
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<td>ΔPf</td>
<td>100-200</td>
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<td>132</td>
<td>149</td>
<td>100</td>
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<td>Lcomb</td>
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<td>α</td>
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<td>18.9</td>
<td>22.3</td>
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<table>
<thead>
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<th>Dependent Variable</th>
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<th>Results Case 3</th>
<th>Results Case 4</th>
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<td>0.79</td>
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<td>Wrel</td>
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<td>1.15</td>
<td>1.10</td>
<td>1.10</td>
</tr>
<tr>
<td>Crel</td>
<td>0.7-1.1</td>
<td>0.98</td>
<td>1.00</td>
<td>1.01</td>
<td>0.93</td>
</tr>
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</table>

Table 3. Effect of Each Variable on the Design—Optimal Designs for Original Constraints & Equal Weights for the Impinging Element.
The design in Case 4 is quite different that the one in Case 1. Again, consideration of a larger design space results in a different design—the sooner the additional variables are considered, the more robust the final design.

**Emphasis on Life and Performance Issues**

The purpose of this section is to illustrate the effect of emphasizing certain aspects of the design during the optimization process. Method 1 allows this emphasis via the weights applied to the desirability functions in the joint response surface. The set of results shown in Table 4 facilitate the illustration. The Case 1 (baseline) results are repeated from Case 4 in Table 3 where the entire design space is considered with the original constraints and equal weights for the dependent variables. The results in the Case 2 column are obtained by emphasizing the minimization of the wall and injector face heat fluxes. Desirability functions for both of these variables are given a weight of five. Since lower heat fluxes tend to increase component life, weighting these two variables is equivalent to emphasizing a life-type issue in the design. As expected, \( \alpha \) is decreased to increase \( H_{\text{mpen}} \), thus decreasing \( Q_w \). Since the fuel pressure drop is already at the minimum, the oxidizer pressure drop is increased by 58% to decrease \( Q_w \). Both of these changes tend to decrease \( ERE \). While \( ERE \) does decrease, the effect is somewhat mitigated by an increase in \( L_{\text{comb}} \). The increases in \( L_{\text{comb}} \) and \( \Delta P_f \) cause increases in \( W_{rel} \) and \( C_{rel} \), respectively. Again, the emphasis on life imposes a penalty on performance. As with the swirl coaxial element, there are also weight and cost penalties for the impinging element.

The results for Case 3 are obtained by emphasizing maximization of \( ERE \) and minimization of \( W_{rel} \) with desirability weightings of five. Increased weighting for these two variables is equivalent to emphasizing a thrust to weight goal for the injector/chamber. The relative chamber length is shortened to lower \( W_{rel} \). \( ERE \) is maximized by increasing the radial momentum of the fuel jet. Both \( \Delta P_f \) and \( \alpha \) are increased to accomplish \( ERE \) maximization. As noted earlier, increasing \( \Delta P_f \) and \( \alpha \) lead to increased wall and injector heat fluxes, respectively. Reference to Table 4 indicates that to be the case here. For this case, emphasis on thrust and weight tend to have an adverse affect on both \( Q_w \) and \( W_{rel} \). Relative cost, for the impinging element model, is not significantly affected.

**THrust to WeigHT RATIO TRENd COMPARISONS**

Results from both injector elements have been normalized to illustrate an emphasis on high thrust-to-weight designs. The results are shown in Figures 4 and 5. In Figure 4, the results of simultaneously increasing the weighting factors to increase \( ERE \) and decrease \( W_{rel} \) are shown. Although the impinging element has a higher \( ERE \), the \( W_{rel} \) is also higher. With a slightly

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Constraints</th>
<th>Results Case 1</th>
<th>Constraints</th>
<th>Results Case 2</th>
<th>Constraints</th>
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<tr>
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<td>5.2</td>
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<tr>
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<td>15-50</td>
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<td><strong>Life Variable Weight</strong></td>
<td><strong>Thrust/Weight Variable Weight</strong></td>
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</tr>
<tr>
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<td>5</td>
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<tr>
<td>( W_{rel} )</td>
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<td>1</td>
<td>1.14</td>
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<td>1.05</td>
</tr>
<tr>
<td>( C_{rel} )</td>
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<td>1</td>
<td>0.97</td>
<td>1</td>
<td>0.95</td>
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</tbody>
</table>

Table 4. Effect of Emphasizing Life & Performance Issues—Optimal Designs for Original Constraints and Modified Weights for the Impinging Element.
lower ERE but significantly lower $W_{rel}$, the swirl element has the higher thrust-to-weight ratio. However, as weight minimization is emphasized, the impinging element weight continues to decrease after the swirl element weight has become constant. This indicates there is more room for improvement in thrust-to-weight ratio for the impinging element than the swirl element.

Figure 4. Performance and weight trends for swirl and impinging elements.

The impacts on $Q_w$, $Q_{sg}$, and $W_{rel}$ which result from increasing the thrust-to-weight ratio are shown in Figure 5. The $Q_w$ for the impinging element increases rapidly, while $Q_w$ for the swirl element is flat. As expected, increasing ERE imposes a large $Q_{sg}$ penalty on both elements. Also, emphasis on thrust-to-weight results in increased cost for both elements, with the already higher swirl element cost increasing more rapidly than the $C_{rel}$ for the impinging element.

Figure 5. Heat flux and cost trends for swirl and impinging elements.

SUMMARY

Both swirl coaxial and F-O-F impinging GO$_2$/GH$_2$ injector element designs have been employed to facilitate optimization studies. Starting with propellant pressure drops, combustor length, and full cone swirl angle or impingement half-angle, an empirical design methodology was used to calculate the dependent variables for both element types. The dependent variables were energy release efficiency, chamber wall and injector face heat fluxes, relative chamber weight, and relative injector cost. The response surface methodology was used to fit the results for both elements with quadratic polynomials. Desirability functions were used to create joint response surfaces that were used in the optimization studies.

Three sets of results for both elements were generated to illustrate the capability of method i in the context of injector design and optimization. The first set of results started with a design optimized for ERE and then added the other four dependent variables to the design one at a time. Most sequential optimal designs were different than previous designs, with the final design being quite different than the initial design. The result showed the importance of including as many variables as possible early in the design. The optimization techniques embodied in method i facilitate this early inclusion by allowing efficient management of large amounts of data.

The second set of results focused on the inherent design trade-offs between performance and component life. Different weights were applied to emphasize variables related to performance (ERE and $W_{rel}$). While the thrust to weight ratio was improved, the adverse affect on variables related to component life ($Q_w$ and $Q_{sg}$) were clearly shown. Conversely, when $Q_w$ and $Q_{sg}$ were emphasized, the toll on the performance variables was clear. These techniques can be used to identify both qualitative trends and to
examine the quantitative trade-offs present in this and other design processes.

The third illustration combined results from both elements to show the effects on all dependent variables of increasing the thrust-to-weight ratio. Here, objective assessments can be made on the penalties on $Q_w$, $Q_{op}$, and $C_{rl}$ for an individual element. Also, the relative penalties can be compared for different elements. This ability can provide the injector designer to include margins and robustness in the choice of element type for a particular application.

The flexibility and utility of method i have been demonstrated in this effort. Use of method i can allow an injector designer to confidently and efficiently manage large amounts of data to conduct a range of design optimization studies. Constraints on independent variables can be modified to allow optimum designs to be sought in specific portions of the parameter space. Also, individual or specific groups of dependent variables can be emphasized to reflect a designer’s priorities in the design optimization process.

REFERENCES
1. Personal Communication with Garry Lyles.