Guidance Concept for a Mars Ascent Vehicle First Stage

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Abstract

This paper presents a guidance concept for use on the first stage of a candidate Mars Ascent Vehicle (MAV). The guidance is based on a calculus of variations approach similar to that used for the final phase of the Apollo Earth return guidance. A three degree-of-freedom (3DOF) Monte Carlo simulation is used to evaluate performance and robustness of the algorithm.

1 Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{E}$</td>
<td>Energy ($m^2/s^2$)</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravitational acceleration ($m/s^2$)</td>
</tr>
<tr>
<td>$H$</td>
<td>Hamiltonian</td>
</tr>
<tr>
<td>$\mathcal{H}$</td>
<td>Angular Momentum ($m^2/s$)</td>
</tr>
<tr>
<td>$h$</td>
<td>Altitude (m)</td>
</tr>
<tr>
<td>$L$</td>
<td>Lift (N)</td>
</tr>
<tr>
<td>$m$</td>
<td>Vehicle mass (kg)</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Planetary radius (m)</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius from planet center (m)</td>
</tr>
<tr>
<td>$T$</td>
<td>Thrust (N)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (s)</td>
</tr>
<tr>
<td>$V$</td>
<td>Velocity ($m/s$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Angle of attack (Thrust Angle) (deg)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Out-of-plane thrust angle (deg)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Flight path angle (deg)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Variation</td>
</tr>
<tr>
<td>$\partial$</td>
<td>Partial Derivative</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Costate</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Gravitational parameter ($m^3/s^2$)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Latitude (deg)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Dummy variable for time</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Cost functional</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Heading angle (deg)</td>
</tr>
<tr>
<td>$BO$</td>
<td>Value at Burnout</td>
</tr>
<tr>
<td>$a$</td>
<td>Value at Apoapsis</td>
</tr>
<tr>
<td>$targ$</td>
<td>Target Value</td>
</tr>
</tbody>
</table>

2 Introduction

Although efforts are underway to continue improvements in reliability and sensitivity of robotic planetary probes, they will not, in the foreseeable future, be able to match the examination and analysis capabilities available here on Earth. One solution to this dilemma is to retrieve planetary samples for analysis here. This has been proposed for samples from Mars starting with the 2003 launch opportunity.

One plan calls for a lander to be sent to Mars to collect soil samples and launch them into orbit around Mars. The samples will remain in orbit until the orbiter and lander launched in 2005 reach Mars. The 2005 lander will collect more samples and launch them into Mars orbit, and the 2005 orbiter will then rendezvous with both sets of samples and return them to Earth.

The Mars Ascent Vehicle (MAV) is subject to severe design constraints. In addition to the usual premiums on weight, volume, and budget, the MAV must oper-
Figure 1. MAV Configuration.

ate somewhat autonomously after being subjected, unattended, to a severe environment for nearly a year. As a result of these and other constraints, The MAV has unique challenges in its design, especially for guidance and control.

Because of the long travel times, the MAV will have solid-fuel motors. Figure 1 shows the general configuration considered in this report.

The ascent trajectory from the Martian surface begins with a high thrust phase that lasts approximately 20 seconds. The MAV then coasts for approximately 200 seconds, at which time it repoints, spins up to 20 rpm, separates the spent first stage and fires the second stage. One half orbit later, the third stage motor (which is mounted backwards to the other stages) is fired to circularize the final orbit. The second and third stages are not guided, though the repointing maneuver may be modified to account for an off-nominal first stage burn/coast. This sequence is illustrated in figure 2.

While the first stage motor is burning, the vehicle is controlled by vanes in the rocket exhaust. After first stage burnout, aerodynamic surfaces are available to reorient the vehicle, but because of the low density of the Martian atmosphere, the ability to adjust the first stage trajectory is limited. Thus, the objective of the first stage guidance is to achieve the highest degree of accuracy in the desired burnout conditions, subject to uncertainties in the winds, atmospheric density, vehicle/payload mass, and total impulse of the motor. The short burn time requires that the first stage guidance be very fast and robust to a rapidly changing plant.

The scheme employed for guidance during the first stage uses an approach similar to that used for the final phase of the Apollo Entry Guidance [1, 2, 3]. Based on a nominal trajectory, the sensitivities of the final state (here the burnout state) to variations in the current state are determined and used to drive those variations to zero at the final time.

The next section gives the derivation of the feedback equations for both the in-plane and out-of-plane components. Section 4 describes the implementation of the algorithm in a numerical simulation and describes some results of that implementation.

3 Theoretical Development

The in-plane differential dynamical equations for a rocket ascent are
as follows. For altitude, 

\[ \dot{h} = V \sin(\gamma) \]  

(1)

where \( h \) is the altitude, \( V \) is the velocity, and \( \gamma \) is the flight path angle. The equation for velocity is 

\[ \dot{V} = \frac{T \cos(\alpha)}{m} - g \sin(\gamma) \]  

(2)

and the equation for flight path angle is 

\[ \dot{\gamma} = \frac{T \sin(\alpha)}{mV} + \left( \frac{V^2}{R_e + h} - g \right) \cos(\gamma) \]  

(3)

The function to be maximized is the energy at burnout, so the Hamiltonian is [4]: 

\[ H = \lambda_h V \sin(\gamma) + \lambda_V \frac{T \cos(\alpha)}{m} - \lambda_v g \sin(\gamma) + \lambda_\gamma \frac{T \sin(\alpha)}{mV} + \lambda_{\gamma'} \frac{V^2 \cos(\gamma)}{(R_e + h)} - \lambda_\gamma g \cos(\gamma) \]  

(4)

and the costate equations are: 

\[ \dot{\lambda}_h = -\frac{\partial H}{\partial h} = \frac{\lambda_v V^2 \cos(\gamma)}{(R_e + h)^2} \]  

(5)

\[ \dot{\lambda}_V = -\frac{\partial H}{\partial V} = -\lambda_h \sin(\gamma) - 2\lambda_\gamma V \cos(\gamma) + \lambda_\gamma T \sin(\alpha) \frac{V^2}{mV^2} \]  

(6)

\[ \dot{\lambda}_\gamma = -\frac{\partial H}{\partial \gamma} = -\lambda_h V \cos(\gamma) + g \lambda_V \cos(\gamma) + \lambda_\gamma V^2 \sin(\gamma) \frac{R_e + h}{R_e + h} - \lambda_\gamma g \sin(\gamma) \]  

(7)

From [1], assuming that the perturbation in the control will be constant, 

\[ \delta \alpha = \frac{-\lambda^T(t) \delta x(t)}{\lambda_\alpha(t)} \]  

(8)

where 

\[ \lambda_\alpha(t) = -\int_t^{t_f} \lambda^T(\tau) \frac{\partial f(\tau)}{\partial \alpha(\tau)} d\tau \]  

(9)

or 

\[ \dot{\lambda}_\alpha(t) = \frac{\lambda_V T \sin(\alpha)}{m} - \frac{\lambda_\gamma T \cos(\alpha)}{mV} \]  

(10)

The above equations (5), (6), (7), (10) are integrated backwards from the final condition using states from the nominal trajectory. It is desired to match apoapsis to put second stage burn at the right position. So, let 

\[ \phi = -(r_a - r_{\text{targ}})^2 \]  

(11)

The boundary conditions for the costates are: 

\[ \lambda_\gamma(t_{BO}) = \left. \frac{\partial \phi}{\partial \gamma} \right|_{t=t_{BO}} = 2(r_a - r_{\text{targ}}) \frac{\partial r_a}{\partial h} \bigg|_{t=t_{BO}} \]  

(12)

\[ \lambda_h(t_{BO}) = \left. \frac{\partial \phi}{\partial h} \right|_{t=t_{BO}} = 2(r_a - r_{\text{targ}}) \frac{\partial r_a}{\partial h} \bigg|_{t=t_{BO}} \]  

(13)

\[ \lambda_V(t_{BO}) = \left. \frac{\partial \phi}{\partial V} \right|_{t=t_{BO}} = 2(r_a - r_{\text{targ}}) \frac{\partial r_a}{\partial V} \bigg|_{t=t_{BO}} \]  

(14)

From orbital mechanics: 

\[ V_{BO} r_{BO} \cos(\gamma_{BO}) = V_a r_a \]  

(15)

and 

\[ \frac{V_{BO}^2}{2} - \frac{\mu}{r_{BO}} = \frac{V_a^2}{2} - \frac{\mu}{r_a} \]  

(16)

where the subscript 0 denotes quantities at burnout and the subscript a denotes quantities at apoapsis. Define the energy and angular momentum 

\[ E = \frac{V_{BO}^2}{2} - \frac{\mu}{r_{BO}} \]  

(17)
\( \mathcal{H} = V_{BO} r_{BO} \cos(\gamma_{BO}) \) \quad (18)

Solving,

\[
 r_a = r_{BO} V_{BO} \cos \gamma_{BO} = \frac{\mathcal{H}}{V_a} \quad (19)
\]

substitute eqn(19) into eqn(16) and solve for \( V_a \):

\[
 V_a = \frac{\mu \pm \sqrt{\mu^2 + 2 \mathcal{E} \mathcal{H}^2}}{\mathcal{H}} \quad (20)
\]

The higher velocity is the periapsis root. The minus root is desired. Note: Energy will be negative for an elliptic orbit. The velocity at apoapsis will still be positive.

\[
 V_a = \frac{\mu - \sqrt{\mu^2 + 2 \mathcal{E} \mathcal{H}^2}}{\mathcal{H}} \quad (21)
\]

Substituting into eqn(19) and simplifying

\[
r_a = \frac{\mathcal{H}^2}{\mu - \sqrt{\mu^2 + 2 \mathcal{E} \mathcal{H}^2}} \quad (22)
\]

The boundary conditions on the costates are then:

\[
 \frac{\partial r_a}{\partial h} = \frac{1}{V_a} \frac{\partial \mathcal{H}}{\partial h} - \frac{\mathcal{H}}{V_a^2} \frac{\partial V_a}{\partial h} \quad (23)
\]

\[
 \frac{\partial \mathcal{H}}{\partial h} = V_{BO} \cos \gamma_{BO} \quad (24)
\]

\[
 \frac{\partial \mathcal{E}}{\partial h} = \frac{\mu}{r_{BO}^2} \quad (25)
\]

The \( V_{BO} \) and \( \gamma_{BO} \) derivatives are exactly analogous, with

\[
 \frac{\partial r_a}{\partial V} = \frac{1}{V_a} \frac{\partial \mathcal{H}}{\partial V} - \frac{\mathcal{H}}{V_a^2} \frac{\partial V_a}{\partial V} \quad (26)
\]

\[
 \frac{\partial \mathcal{H}}{\partial V} = r_{BO} \cos \gamma_{BO} \quad (27)
\]

\[
 \frac{\partial \mathcal{E}}{\partial V} = V_{BO} \quad (28)
\]

\[
 \frac{\partial r_a}{\partial \gamma} = \frac{1}{V_a} \frac{\partial \mathcal{H}}{\partial \gamma} - \frac{\mathcal{H}}{V_a^2} \frac{\partial V_a}{\partial \gamma} \quad (29)
\]

\[
 \frac{\partial \mathcal{H}}{\partial \gamma} = -r_{BO} V_{BO} \sin \gamma_{BO} \quad (30)
\]

\[
 \frac{\partial \mathcal{E}}{\partial \gamma} = 0 \quad (31)
\]

Note that the final state will never depend on the final control so the “control costate”, \( \lambda_a \), will always have a final condition of 0. When this is used in equation (8) it implies that a state perturbation at the final time requires infinite control to be corrected at the final time, i.e. instantaneously.

Also note that, for flight implementation, very little of this process occurs on-board. The costates, \( \lambda_{r}, \lambda_{V}, \) etc, are determined from a nominal trajectory prior to flight. The costates are stored as tables or polynomials and the control is then a simple function of these stored values and the current state.

The out-of-plane equation is

\[
 \dot{\psi} = \frac{L \sin(\phi) + T \sin(\beta)}{m V \cos(\gamma)} - \frac{V}{r} \cos(\gamma) \cos(\phi) \tan(\nu) \quad (32)
\]

where \( \nu \) is the latitude, \( \beta \) is the out-of-plane thrust angle (similar to sideslip angle), and \( L \) is the lift force. It is assumed that the lift force is negligible compared to the \( T \sin(\beta) \) term and that the entire flight takes place near enough to the equator that the last term can be neglected. Writing \( \dot{\psi} \) as a finite difference and solving for \( \sin(\beta) \), results in

\[
 \sin(\beta) = \left( \frac{\psi_{\text{new}} - \psi_{\text{old}}}{\Delta t} \right) \frac{m V \cos(\gamma)}{T} \quad (33)
\]

where \( \psi_{\text{new}} \) is the commanded heading angle and \( \psi_{\text{old}} \) is the current heading angle. For this implementation, the command was chosen as a ramp in time that brings the nominal trajectory to a 45 degree inclination at burnout.
4 Numerical Simulation Results

The guidance algorithm described above was implemented in a numerical 3DOF simulation using the Program to Optimize Simulated Trajectories (POST) program [5]. The simulation was incorporated into a Monte Carlo analysis with dispersions as listed in Table 1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Range</th>
<th>Dist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Launch Altitude</td>
<td>0-2 km</td>
<td>U</td>
</tr>
<tr>
<td>Launch Latitude</td>
<td>±0.1 deg</td>
<td>G</td>
</tr>
<tr>
<td>Launch Longitude</td>
<td>±0.1 deg</td>
<td>G</td>
</tr>
<tr>
<td>Launch Azimuth</td>
<td>41.74 ±1.8 deg</td>
<td>G</td>
</tr>
<tr>
<td>Launch FPA</td>
<td>48.91 ±1.8 deg</td>
<td>G</td>
</tr>
<tr>
<td>E-W Wind</td>
<td>±50 m/s</td>
<td>U</td>
</tr>
<tr>
<td>N-S Wind</td>
<td>5-30 m/s</td>
<td>U</td>
</tr>
<tr>
<td>Propellant Mass</td>
<td>38.418 kg ±0.3%</td>
<td>G</td>
</tr>
<tr>
<td>Payload Mass</td>
<td>2.80 ±0.4 kg</td>
<td>G</td>
</tr>
<tr>
<td>Thrust</td>
<td>5872.0 N ±4.0%</td>
<td>G</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>279s ±1%</td>
<td>G</td>
</tr>
<tr>
<td>$C_A$</td>
<td>±5%</td>
<td>G</td>
</tr>
<tr>
<td>$C_N$</td>
<td>±5%</td>
<td>G</td>
</tr>
</tbody>
</table>

The first column of Table 1 lists the quantities that were dispersed within the limits shown in the second column. The final column denotes the type of random distribution sampled; ‘G’ for Gaussian and ‘U’ for uniform. Random atmosphere variations were also included based on MarsGRAM [6]. The simulation was executed 2000 times with these dispersions.

For this mission, because of the long (uncontrolled) coast phase and the need for eventual rendezvous by the orbiter, the apoapsis and inclination are the most critical parameters. Figure 3 shows the final apoapsis and inclination for 2000 cases. For all cases the apoapsis is between 97 and 103 km, and the inclination is between 44.7 and 45.3 degrees.

Table 2 summarizes some statistics from the Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Max</th>
<th>Min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>6.958</td>
<td>7.363</td>
<td>6.681</td>
</tr>
<tr>
<td>Inclination</td>
<td>45.01</td>
<td>45.27</td>
<td>44.73</td>
</tr>
<tr>
<td>Apoapsis</td>
<td>100.22</td>
<td>102.92</td>
<td>97.43</td>
</tr>
<tr>
<td>Periapsis</td>
<td>-3357</td>
<td>-3355</td>
<td>-3360</td>
</tr>
<tr>
<td>Tot. Impulse</td>
<td>105.1</td>
<td>106.3</td>
<td>104.0</td>
</tr>
</tbody>
</table>

5 Conclusions

A guidance algorithm for the first stage of a proposed Mars Ascent Vehicle has been developed. This algorithm is based on a calculus of variations approach, using influence coefficients to drive the vehicle state to a desired terminal state. The algorithm is designed to provide good performance with very little on-board computation. While the exact configuration is subject to change, this algorithm is potentially useful across a wide range of applications.

The proposed guidance algorithm has
been implemented and tested in a 3DOF Monte Carlo simulation. The results show that the algorithm controls the vehicle to relatively tight tolerances under reasonable environmental dispersions, keeping the final condition within about a quarter degree in inclination and three kilometers apoapsis.

References


This paper presents a guidance concept for use on the first stage of a Mars Ascent Vehicle (MAV). The guidance is based on a calculus of variations approach similar to that used for the final phase of the Apollo Earth return guidance.