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# COSIM—A Finite-Difference Computer Model to Predict Ternary Concentration Profiles Associated With Oxidation and Interdiffusion of Overlay-Coated Substrates

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## Summary

A finite-difference computer program (COSIM) has been written which models the one-dimensional, diffusional transport associated with high-temperature oxidation and interdiffusion of overlay-coated substrates. The program predicts concentration profiles for up to three elements in the coating and substrate after various oxidation exposures. Surface recession due to solute loss is also predicted. Ternary cross terms and concentration-dependent diffusion coefficients are taken into account. The program also incorporates a previously-developed oxide growth and spalling model to simulate either isothermal or cyclic oxidation exposures. In addition to predicting concentration profiles after various oxidation exposures, the program can also be used to predict coating life based on a concentration dependent failure criterion (e.g., surface solute content drops to 2 percent). The computer code, written in an extension of FORTRAN 77, employs numerous subroutines to make the program flexible and easily modifiable to other coating oxidation problems.

## Introduction

Many blades and vanes in gas turbine engines are coated with an aluminide or overlay coating to impart additional oxidation and corrosion protection to the component. These coatings provide protection by the selective oxidation of Al to form a compact and adherent  $\text{Al}_2\text{O}_3$  scale. Under isothermal conditions, the  $\text{Al}_2\text{O}_3$  scale thickens at a parabolic rate (scale thickness proportional to the square root of time). Diffusional transport within the coating supplies Al to the growing oxide scale at a rate consistent with the parabolic growth of the scale. Although Al is continually consumed from the coating, the rate is acceptably low such that the coating typically contains sufficient Al to easily provide protection during isothermal oxidation conditions. Although land-based turbines often operate in a nearly isothermal mode for long periods, aero gas turbines undergo thermal cycling with each flight. This thermal cycling, primarily to ambient temperatures when the engine is shut down, can cause cracking and partial spalling of the protective  $\text{Al}_2\text{O}_3$  scale. This spallation generally occurs randomly across flat component surfaces but is higher at edges and corners. The spalling may occur to the metal surface but more commonly occurs only in the outer layers of the oxide scale. However, loss of the oxide is not catastrophic since selective oxidation of Al continues when the component again reaches high temperatures such that the damaged scale will "heal" and continue to grow. Whenever scale spallation occurs due to thermal cycling during oxidation (i.e., cyclic oxidation), the oxide scale on the surface will, on average, be thinner than a scale grown isothermally for equivalent hot exposure times. Since the rate of scale growth is inversely proportional to the scale thickness, a consequence of this thinner oxide scale is that the average rate of Al consumption is greater during cyclic oxidation accompanied by scale spallation than during isothermal oxidation. Hence, Al is depleted from the coating at a higher rate during cyclic oxidation, the exact rate depending on the growth rate of the alumina scale and the amount of oxide spalling. This paper will focus specifically on degradation of MCrAlY overlay coatings, where M stands for either Ni, Co or Fe. These coatings are often used on components in marine environments where hot corrosion protection is required, and in military applications where higher temperatures are often encountered.

In addition to oxidative attack, overlay coatings are further degraded by interdiffusion with the substrate. Since the coating is, by nature, higher in Al content than the substrate, Al diffuses from the coating into the substrate and generally becomes unavailable to support the growth of the protective alumina scale. Likewise, the Cr concentration

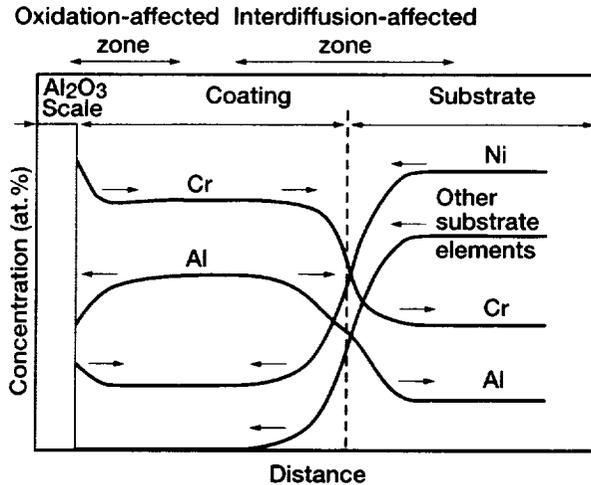


Figure 1.—Schematic concentration profiles after oxidation exposure. Arrows indicate direction of atomic transport.

in the coating is also typically greater than that in the substrate resulting in the diffusion of Cr from the coating into the substrate. In contrast, Ni and other elements in the substrate diffuse into the coating. Although the Y in the alloy strongly affects adherence of the  $\text{Al}_2\text{O}_3$  scale during cyclic oxidation, it is at such low concentration in the coating (e.g., <0.2 at%) that dilution by interdiffusion with the substrate is generally not considered. Schematic concentration profiles in the coating and substrate after a short exposure in an oxidizing environment are shown in figure 1. When a coating is substantially depleted of Al during cyclic oxidation, it can no longer supply sufficient Al to reheat the protective scale. Less protective oxides, such as NiO, FeO or CoO, can form signaling the end of the protective life of the coating. A critical Al content in the coating can be defined to indicate the useful life of the coating. This critical Al content could indicate the time at which less-protective oxides form on the surface, or an earlier time at which the coating could be stripped from the component and a new coating applied. Modeling the Al transport in the coating and substrate during cyclic oxidation allows the coating life to be predicted. The purpose of the present work was to develop a one-dimensional ternary diffusion model to predict the concentration profiles associated with the oxidation and interdiffusion of coated superalloys undergoing cyclic oxidation. This model, given the name COSIM for Coating Oxidation and Substrate Interdiffusion Model, employs finite-difference techniques embodied in a FORTRAN computer program to provide numerical solutions to the appropriate diffusion equations. The computer code employs numerous subroutines to make the program flexible and easily modifiable to other coating oxidation problems. Although the computer program and discussion below are primarily in terms of a NiCrAl coating on a Ni-based substrate, other elements could be substituted for the Al and Cr.

## Ternary Diffusion Equations

Because of the low Y content, MCrAlY overlay coatings can be approximated as ternary alloys with Al and Cr in a matrix of either Ni, Co or Fe. Ternary diffusion equations can be employed to simulate the Al and Cr transport associated with coating oxidation and coating/substrate interdiffusion. The choice of simulating Cr rather than Ni diffusion is inconsequential. In the ternary system, the concentration of the third component, being a dependent variable, is always determined by difference (i.e.,  $C_{\text{Ni}} = 100 - C_{\text{Al}} - C_{\text{Cr}}$ ). Since most superalloys are complex multi-component, multiphase alloys, fully accounting for diffusional interactions of the various superalloy components, as well as that of Y in the coating, on the Al and Cr transport is beyond current capabilities. Hence, these potential interactions, other than those encountered in the ternary system, are ignored. In addition, NiCrAlY and CoCrAlY overlay coatings generally consist of two phases, a high-Al NiAl or CoAl  $\beta$  phase embedded in a Ni or Co solid solution  $\gamma$  phase. This  $\beta$  phase is depleted as Al is consumed as oxide and as Al diffuses into the substrate. It has previously been shown that this  $\beta$  phase is often completely dissolved well before the useful life of the coating has been reached (ref. 1). It was also shown in this same reference that good agreement was achieved between measured

and predicted concentration profiles in the coating at extended times after the  $\beta$  phase had been dissolved by assuming a single-phase coating for the entire oxidation exposure. Hence, in the current model development, the two-phase coating will be represented as a single phase.

Fick's laws describing diffusion in a ternary alloy are:

$$J_j = -D_{j,j} \frac{\partial C_j}{\partial X} - D_{j,k} \frac{\partial C_k}{\partial X} \quad j, k = \text{Al, Cr}^* \quad \text{1st Law} \quad (1)$$

and

$$\frac{\partial C_j}{\partial t} = \frac{\partial \left( D_{j,j} \left( \frac{\partial C_j}{\partial X} \right) \right)}{\partial X} + \frac{\partial \left( D_{j,k} \left( \frac{\partial C_k}{\partial X} \right) \right)}{\partial X} \quad j, k = \text{Al, Cr} \quad \text{2nd Law} \quad (2)$$

where  $J_j$  and  $C_j$  refer to the flux and concentration of component  $j$ ,  $D_{j,j}$  and  $D_{j,k}$  refer to the four ternary interdiffusion coefficients, or diffusivities, and  $X$  and  $t$  refer to distance and time, respectively. The first term on the right hand side (RHS) of equation (2) can be rewritten as:

$$\frac{\partial \left( D_{j,j} \left( \frac{\partial C_j}{\partial X} \right) \right)}{\partial X} = D_{j,j} \frac{\partial^2 C_j}{\partial X^2} + \frac{\partial D_{j,j}}{\partial X} \frac{\partial C_j}{\partial X} \quad j, k = \text{Al, Cr} \quad (3a)$$

Equation (3a) can be further expanded as:

$$\frac{\partial \left( D_{j,j} \left( \frac{\partial C_j}{\partial X} \right) \right)}{\partial X} = D_{j,j} \frac{\partial^2 C_j}{\partial X^2} + \left( \frac{\partial D_{j,j}}{\partial C_j} \frac{\partial C_j}{\partial X} + \frac{\partial D_{j,j}}{\partial C_k} \frac{\partial C_k}{\partial X} \right) \frac{\partial C_j}{\partial X} \quad j, k = \text{Al, Cr} \quad (3b)$$

A similar equation exists for the second term on the RHS of equation (2), namely:

$$\frac{\partial \left( D_{j,k} \left( \frac{\partial C_k}{\partial X} \right) \right)}{\partial X} = D_{j,k} \frac{\partial^2 C_k}{\partial X^2} + \left( \frac{\partial D_{j,k}}{\partial C_j} \frac{\partial C_j}{\partial X} + \frac{\partial D_{j,k}}{\partial C_k} \frac{\partial C_k}{\partial X} \right) \frac{\partial C_k}{\partial X} \quad j, k = \text{Al, Cr} \quad (3c)$$

Substituting equation (3b) and (3c) into the two terms on the right hand side of equation (2) yields:

$$\frac{\partial C_j}{\partial t} = D_{j,j} \frac{\partial^2 C_j}{\partial X^2} + \left( \frac{\partial D_{j,j}}{\partial C_j} \frac{\partial C_j}{\partial X} + \frac{\partial D_{j,j}}{\partial C_k} \frac{\partial C_k}{\partial X} \right) \frac{\partial C_j}{\partial X} + D_{j,k} \frac{\partial^2 C_k}{\partial X^2} + \left( \frac{\partial D_{j,k}}{\partial C_j} \frac{\partial C_j}{\partial X} + \frac{\partial D_{j,k}}{\partial C_k} \frac{\partial C_k}{\partial X} \right) \frac{\partial C_k}{\partial X} \quad j, k = \text{Al, Cr} \quad (4)$$

Initially, transport associated with oxidation and the interdiffusion of the coating and substrate may be viewed as independent problems. Hence, at short times, coating/substrate interdiffusion can be treated as interdiffusion between two semi-infinite materials at a location centered around the coating/substrate interface. Similarly, oxidation of the coating can be treated as oxidation of a semi-infinite material affecting only a thin region below the surface of the coating. These two regions, the "inner" diffusion zone resulting from coating/substrate interdiffusion and the

\*The notation  $j, k = \text{Al, Cr}$  refers to the two conditions where  $j = \text{Al}, k = \text{Cr}$  or  $j = \text{Cr}, k = \text{Al}$ .

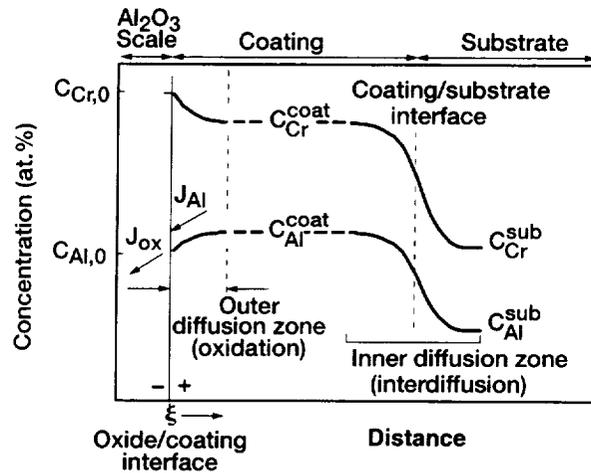


Figure 2.—Schematic concentration profiles after a short oxidation exposure.

“outer” diffusion zone resulting from transport associated with oxidation, are shown schematically in figure 2. At longer times, the two diffusion zones will overlap and diffusion associated with the two regions must be considered together. A solution to Fick’s second law (eq. (2)), whether analytical or numerical, requires initial and boundary conditions. The boundary condition at the oxide/coating interface is discussed in the following paragraphs whereas the initial conditions and other boundary conditions used by the COSIM model are discussed in a later section.

For isothermal oxidation, the boundary condition at the oxide/coating interface, given as the rate of Al consumption, is well defined and decreases uniformly with time (the time dependence of the rate is inversely proportionate to the square root of time). However, for cyclic oxidation, the rate of Al consumption can vary in a nonuniform manner as the thickness of the oxide scale increases during the high temperature exposure but decreases on cooling as spallation occurs (ref. 2). In the present diffusion model, a separate model simulating oxide growth and spallation during cyclic oxidation has been adopted to provide the oxide/coating boundary condition. This oxide growth and spalling model (ref. 3), designated COSP by the authors, predicts the rate of Al consumption during each cycle by continuously tracking the thickness of the oxide scale, accounting for growth during high-temperature exposures, and partial oxide loss on cooling. This rate of Al consumption ( $J_{ox}$ ) predicted by COSP is used as the boundary condition for the diffusion model. Hence, the supply of Al within the coating to the oxide/coating interface must equal  $J_{ox}$ . However, since Al is consumed from the coating, the coating surface recedes due to the loss of matter. This recession ( $\xi$ ) is given as (ref. 4):

$$\xi = -V_{Al}J_{ox}t \quad (5)$$

where  $V_{Al}$  is the partial molar volume of Al in the coating and  $J_{ox}$  is the rate of Al consumption discussed above (which is also the flux of Al entering the oxide).  $J_{ox}$  can also be considered as the Al flux to the left of the moving oxide/metal interface in figure 2, or the Al flux away from the interface. Similarly,  $J_{Al}$  is the flux in the metallic coating towards the interface. Because of the interface motion,  $J_{ox}$  is greater than  $J_{Al}$  according to the relationship:

$$J_{ox}\Big|_{X=\xi^-} = a J_{Al}\Big|_{X=\xi^+} \quad (6)$$

where  $\xi^-$  and  $\xi^+$  refer to the left hand side and right hand side of the oxide/coating interface, respectively, as shown in figure 2. The parameter  $a$  in equation (6) is given as:

$$a = \frac{1}{1 - V_{Al} * C_{Al,0}} \quad (7)$$

where  $C_{Al,0}$  is the Al concentration in the coating at the oxide/coating interface (fig. 2). Obviously, for the hypothetical case for  $V_{Al} = 0$  (the Al atoms in the coating have no volume),  $a = 1$ , the Al flux to the interface,  $J_{Al}$ , becomes equal to the Al flux away from the interface,  $J_{o,x}$ , and the interface is stationary. The partial molar volume for Al was assumed independent of concentration due to a lack of available data for most alloy systems of interest (e.g., the  $\gamma$ , Ni solid solution phase in NiCrAl alloys).

Hence, the boundary condition for Al at the oxide/coating interface is given by equation (6) whereby the rate of Al consumption due to oxide formation, predicted by the COSP oxide model,  $J_{o,x}$ , is equated to the supply, or flux, of Al to the interface within the coating,  $J_{Al}$ , while taking into account the motion of the interface through the parameter  $a$ . The rate of Al transport within the coating to the oxide/coating interface (i.e., the Al flux,  $J_{Al}$ ) is given by equation (1), stated as:

$$J_{Al}|_{X=\xi^+} = -D_{Al,Al} \frac{\partial C_{Al}}{\partial X} - D_{Al,Cr} \frac{\partial C_{Cr}}{\partial X} \quad (8)$$

Although no Cr is assumed to enter the  $Al_2O_3$  scale, the surface recession requires the diffusion of Cr and Ni away from the oxide/coating interface into the coating. The boundary condition for Cr is:

$$J_{Cr}|_{X=\xi^+} = -D_{Cr,Al} \frac{\partial C_{Al}}{\partial X} - D_{Cr,Cr} \frac{\partial C_{Cr}}{\partial X} = C_{Cr,0} \frac{d\xi}{dt} \quad (9)$$

where  $J_{Cr}$  is the flux of Cr in the coating away from the interface and  $C_{Cr,0}$  is the Cr concentration in the coating at this interface (fig. 2). In the computer program, the calculations for COSP have been contained in a subroutine to facilitate the incorporation of other oxide growth and spalling models (e.g., ref. 5).

## Finite-Difference (F-D) Method

The initial step in the F-D technique is to establish a grid of equispaced nodes across the region of the material over which diffusion will occur (i.e., the diffusion zone). Each node has a specific concentration associated with it. Fick's laws (eqs. (1) and (4)) are replaced with F-D equivalents based on small differences in concentration,  $\Delta C$ , distance,  $\Delta X$ , and time,  $\Delta t$ . A solution yielding the concentration profile at some time  $t$  is derived by solving the appropriate F-D equivalents for small time increments ( $\Delta t$ ) in an iterative manner. These iterations, or time steps, are continued until the  $\Delta t$  increments sum to the desired time  $t$ . A portion of a diffusion zone at time  $t$  and the new concentrations at time  $t + \Delta t$  is shown schematically in figure 3 (ref. 6). Because of the typically large number of repetitive and tedious calculations made each iteration, F-D solutions are ideally handled by a computer.

The F-D equivalents to Fick's laws are based on Taylor series expansions (refs. 7 and 8). The F-D equivalent of Fick's 2nd law (eq. (4)) can be given by either an explicit or implicit representation. The explicit form was used throughout this work. F-D expressions for both first and second order partial derivatives for concentration with respect to distance, given on the right hand side of equation (4), were given by first central difference equations, stated as:

$$\frac{\partial C_j}{\partial X} = \frac{-C_{j,n+1}^i - C_{j,n-1}^i}{2\Delta X} \quad (10)$$

$$\frac{\partial^2 C_j}{\partial X^2} = \frac{-C_{j,n+1}^i - 2C_{j,n}^i + C_{j,n-1}^i}{(\Delta X)^2} \quad (11)$$

where  $j$  refers to either Al or Cr,  $n$  refers to the node number and the superscript  $i$  refers to the current iteration at time  $t$ . These equations apply to all nodes  $n$  where nodes  $n - 1$  and  $n + 1$  exist. A first forward difference expression was used for  $\partial C/\partial t$  in the left hand side of equation (4), given as:

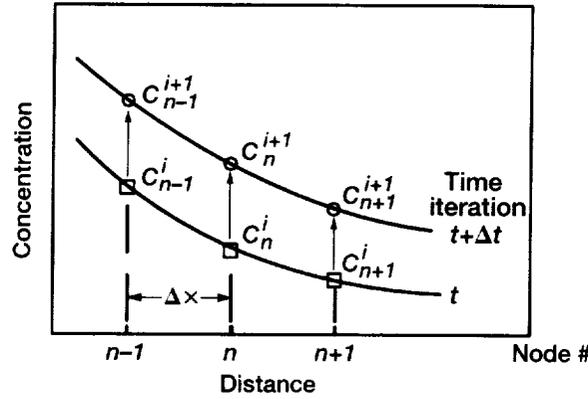


Figure 3.—Schematic diffusion zone for time  $t$  (squares) and time  $t+\Delta t$  (circles). The concentration subscript refers to the node number and the superscript refers to the time iteration.

$$\frac{\partial C_j}{\partial t} = \frac{C_{j,n}^{i+1} - C_{j,n}^i}{\Delta t} \quad j = \text{Al, Cr} \quad (12)$$

where the superscript  $i + 1$  refers to the next iteration at time  $t + \Delta t$ . The time increment  $\Delta t$  for the explicit F-D method is limited by a stability criterion typically given as:

$$D_{\max} \frac{\Delta t}{(\Delta X)^2} \leq 0.25 \quad (13)$$

where  $D_{\max}$  is the appropriate diffusion coefficient which is discussed in a later section. Although  $\Delta t$  is initially very small, the grid expansion scheme described below allows the time increment each iteration to increase, generally allowing long term simulation of oxidation with a reasonable number of program iterations.

The last terms in equation (4) to be discussed are the eight derivatives of the diffusion coefficients ( $\partial D/\partial C$ ). Obviously, the concentration dependence of the four ternary diffusivities is required to evaluate this expression. The concentration dependence of the diffusivities is input to the computer program in polynomial form. The computer program determines an abbreviated polynomial expression for each derivative using the polynomial coefficients. If some, or all of the ternary diffusivities are concentration independent, or if the concentration dependence is not known and cannot be input to the program, the value of the appropriate derivatives is zero. Hence, each of the terms on the right hand side of equation (4) have known values at time  $t$  and can be evaluated. Since  $\Delta t$  is determined from the stability criterion in equation (13) and values for  $C_{j,n}^i$  are known, values for  $C_{j,n}^{i+1}$  from the left hand side of equation 4 for the new time ( $t + \Delta t$ ) can be calculated.

For the boundary condition at the oxide/coating interface (eqs. (8) and (9)), no node exists in the oxide (i.e., no  $n - 1$  node exists, where  $n$  is located at the interface) so that the central difference formulas in equations (10) and (11) cannot be used. Consequently, concentration gradients ( $\partial C_j/\partial X$ ) at the interface were determined using second order forward difference equations. Hence, the F-D equivalent for the Al concentration gradient in both equations (8) and (9) at the interface was given as:

$$\left. \frac{\partial C_{\text{Al}}}{\partial X} \right|_{X=\xi^+} = \frac{-C_{\text{Al},2}^i + 4C_{\text{Al},1}^i - 3C_{\text{Al},0}^i}{2\Delta X} \quad (14)$$

where the second subscript number refers to the node which is numbered sequentially from zero at the interface and the superscript refers to the current iteration at time  $t$  (i.e.,  $C_{\text{Al},0}^i$  is the Al concentration in the coating at the oxide/coating interface,  $X = \xi^+$  shown in fig 2). A similar expression was used for the Cr gradient. The diffusivities for

equations (8) and (9) were evaluated for the Al and Cr concentrations in the coating at the oxide/coating interface (i.e.,  $C_{Al,0}, C_{Cr,0}$ ). Substituting equation (14) into equations (8) and (9) together with equations (5) to (7) are sufficient to yield the interface concentrations  $C_{Al,0}^i, C_{Cr,0}^i$ .

## COSIM Computer Program

The COSIM program initially establishes separate diffusion zones for interdiffusion and oxidation. Typically, these diffusion zones are set to be very narrow (i.e., 0.1  $\mu\text{m}$ ) but are allowed to expand with increasing interdiffusion. When the diffusion zones eventually overlap within the coating, the two separate zones are combined and the simulation continues with a single zone across the coating and into the substrate. This approach allows the use of a reasonable number of nodes in a zone yet with a fine node spacing during the early times when the concentration gradients are steep. For greater accuracy, all variables used in the iterative calculations were defined as double precision. All input to the COSIM program is through the READ11 subroutine, discussed in a following section.

The starting width of the outer and inner zones,  $DXCOAT$  and  $DXSUB$ , and the number of nodes in each zone,  $NCOAT$  and  $NSUB$ , respectively, are parameters input to the program. The spacing between nodes,  $DELX1$  for the outer zone and  $DELX2$  for the inner zone (i.e.,  $\Delta X$  in the F-D equations), is equal to the width of the zone divided by the number of nodes minus one (i.e.,  $NCOAT-1$  and  $NSUB-1$ , the minus one since both zones are bounded by nodes). The concentrations associated with each node for both diffusion zones are stored in arrays labeled 'Al' and 'Cr'. In the outer diffusion zone, the nodes are numbered sequentially starting with zero at the oxide/coating interface through  $NCOATH$  (equal to  $NCOAT-1$ ). The  $NSUB$  nodes in the inner diffusion zone are numbered from  $NSUBL$  (low) to  $NSUBH$  (high) with  $NSUBL$  starting at a value of  $NCOAT+2$ . Central difference equations, such as equations (10) and (11), cannot operate on the endpoints of a zone since concentrations at nodes  $n+1$  and  $n-1$  are required. Although forward and backward difference expressions can be used at these locations, a common technique is to add ancillary nodes to the zone to allow the continued use of central difference equations. Hence, an additional ancillary node was added to the inner end of the outer diffusion zone and an ancillary node was added to each end of the inner diffusion zone ( $NSUBL-1$  and  $NSUBH+1$ ). The ancillary nodes maintain assigned constant concentration values. The node numbering scheme is schematically shown in figure 4 for  $NCOAT=7$  (e.g., 7 nodes numbered 0 to 6) and  $NSUB=11$  (e.g., 11 nodes numbered 9 to 19). The ancillary node for the outer diffusion zone is shown as node 7 and the ancillary nodes for the inner diffusion zone are shown as nodes 8 and 20. As shown, different values for  $DXCOAT$  or  $DXSUB$ , or for  $NCOAT$  or  $NSUB$ , may result in different values for  $DELX1$  or  $DELX2$ .

### Initial Conditions

Initially, each of the nodes in the outer zone are assigned the concentration of the coating. The ancillary node (node 7 in fig. 4) maintains this coating concentration until the inner and outer diffusion zones overlap. These initial concentrations for the outer zone are shown in figure 4. An error function solution was used to assign the coating and substrate compositions to the nodes in order to provide a smooth transition between the coating and substrate compositions. The  $NSUBL$  to  $NSUBH$  nodes in the inner zone are assigned concentrations as:

$$C_j = C_j^{coat} + (C_j^{sub} - C_j^{coat}) * \frac{1}{2} (1 + erf(X_{mod})) \quad j = \text{Al, Cr} \quad (15)$$

where  $C_j^{coat}$ ,  $C_j^{sub}$  refer to the initial concentration of the coating and substrate, respectively and  $X_{mod}$  is a modified distance parameter ranging from  $\pm 2$  across the diffusion zone of width  $DXCOAT$ . This range in  $X_{mod}$  produces a smooth concentration gradient with end compositions within 0.5 percent of  $C_j^{coat}$ ,  $C_j^{sub}$ . The coating composition was assigned to the ancillary node at  $NSUBL-1$  (node 8 in fig. 4) and the substrate composition was assigned to the ancillary node at  $NSUBH+1$  (node 20 in fig 4). Again, these ancillary nodes maintain these assigned concentrations until the diffusion zones overlap (i.e., the ancillary nodes are not operated upon by Fick's 2<sup>nd</sup> law, as discussed below). The initial concentrations for the inner diffusion zone are also shown in figure 4.

Use of the error function solution to assign initial concentration values does not have a significant effect on later concentration values because of the small initial diffusion zone width and the small initial time increments. A value

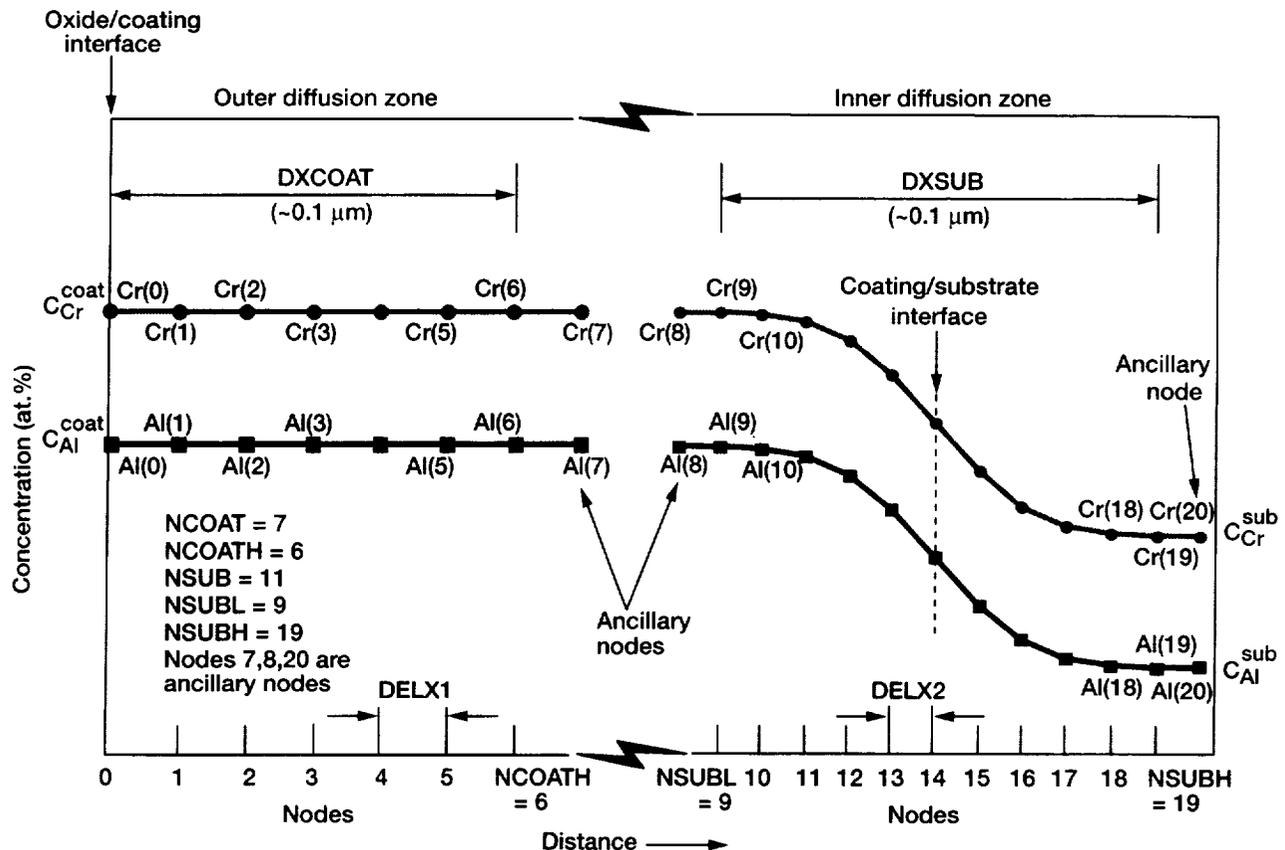


Figure 4.—Schematic of initial concentration profiles before oxidation exposure for NCOAT = 7 and NSUB = 11.

on the order of  $0.1 \mu\text{m}$  was typically used for the initial diffusion zone width,  $DXSUB$ . Typically, 30 to 40 nodes are assigned to the zone so that the node spacing ( $\Delta X$ ) is  $0.0025$  to  $0.003 \mu\text{m}$ . Substituting this latter value into equation 13 and using a typical value of  $10^{-10} \text{ cm}^2/\text{s}$  for  $D_{max}$  yields a time increment  $\Delta t$  of only  $0.000225$  sec. Hence, hundreds to thousands of iterations will typically be performed before the first minute of simulated exposure time allowing ample iterations for the starting concentrations to adjust to satisfy Fick's 2<sup>nd</sup> law (eqs. (2) to (4)).

Fortunately, when diffusion begins to affect the concentration at the ends of the zones, either at node  $NCOATH$ ,  $NSUBL$  or  $NSUBH$ , the node spacings are allowed to expand (as shown in the following section) so that larger time increments can be used each iteration. Since the outer and inner zones may have different node spacings, the program determines the maximum diffusion coefficient ( $D_{max}$  in eq. (13)) for either the coating or substrate composition and uses the smaller of the two node spacings to determine the smallest time increment,  $DELX$  (i.e.,  $\Delta t$  in the F-D equations), according to equation (13).

### Surface Recession, Flexible Zones and Semi-Infinite Boundary Conditions

The COSIM model utilizes a flexible grid technique to account for surface recession and to simulate the semi-infinite boundary conditions. As the outer surface recedes due to Al loss, the entire outer zone is shifted and the concentrations at each node are adjusted. The technique used to accomplish this shift and adjustment is referred to as a "Murray-Landis" (M-L) transformation (ref. 9). This transformation shifts the nodes an amount proportional to their position from the moving boundary ( $X = \xi$ ) to maintain a uniform node spacing. The semi-infinite boundary condition for the outer diffusion zone can be stated as:

$$C_i \Big|_{x=\infty} = C_i^{coat} \quad i = Al, Cr.$$

This boundary condition is approximated by increasing the zone width whenever diffusion “significantly changes” the concentration at the node *NCOATH*. Each iteration, the concentration at this node is changed slightly in accordance with Fick’s 2<sup>nd</sup> law (eq. (4)). A “significant change” used in the COSIM program was taken as 0.005 at%. Varying this value from 0.002 to 0.008 at% changed the number of iterations required to reach a fixed time but had no significant effect on the predicted concentration profiles at short or long times (i.e., 1 and 1000 hr using the sample input data given in Appendix A). Hence, whenever the concentration of node *NCOATH* (either Al or Cr) varied from the coating composition by 0.005 at%, the zone width was expanded by *DELX1* and all node positions and concentrations were adjusted according to the M-L transformation. To further illustrate this operation, Al loss at the surface (node 0) and operation of Fick’s 2<sup>nd</sup> law (eq. (4)) will eventually cause the initial Al profile in the outer diffusion zone (fig. 4) to appear as shown schematically by the solid squares in figure 5. Eventually, the Al concentration at node 6 will decrease to a value 0.005 at% below the concentration of the coating,  $C_{Al}^{coat}$ . During this iteration, the total zone width, *DXCOAT*, will be increased by an amount equal to the node spacing such that:

$$DXCOAT' = DXCOAT + DELX1 \quad (16)$$

The number of nodes remains constant but the new node spacing is given as:

$$DELX1' = DXCOAT' / (NCOAT - 1) \quad (17)$$

The node positions are shifted proportionately such that node 6 is shifted inward by *DELX1* while node 0 at the surface ( $X = \xi$ ) undergoes no shift (fig. 5). The concentrations at each node are shifted in a similar manner with node 6' taking the old position of the ancillary node 7 and being reassigned the concentration of the coating,  $C_i^{coat}$  (the concentration previously held by the ancillary node 7 at the same position). The ancillary node maintains the coating composition at the new position.

The semi-infinite boundary conditions at either end of the inner diffusion zone are simulated in a like fashion. Hence, as diffusion changes the concentrations at nodes *NSUBL* or *NSUBH* in the inner diffusion zone by 0.005 at%, the zone width is expanded by the node spacing, *DELX2* and the node positions and concentrations are adjusted according to the M-L transformation. The expansion occurs at the end of the zone where the concentration change occurred such that the inner diffusion zone expands either into the coating or into the substrate for changes at node *NSUBL* or *NSUBH*, respectively. The ancillary nodes (*NSUBL*-1 and *NSUBH*+1) maintain the substrate composition and are repositioned at the ends of the zones with the new node spacing. Whenever the zones are expanded or contracted, a new time increment *DELTA*, is calculated according to equation (13). As discussed above, the smaller of the

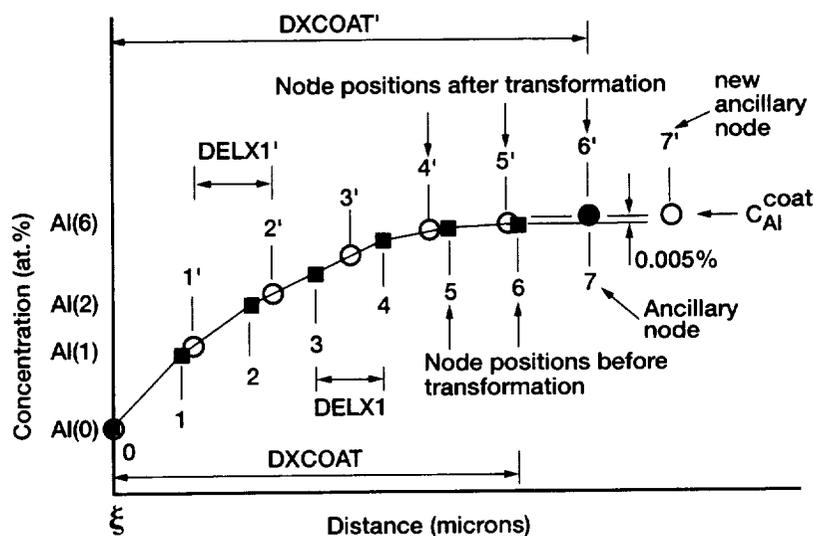


Figure 5. Schematic Al concentration profile in the outer diffusion zone before and after the M-L transformation.

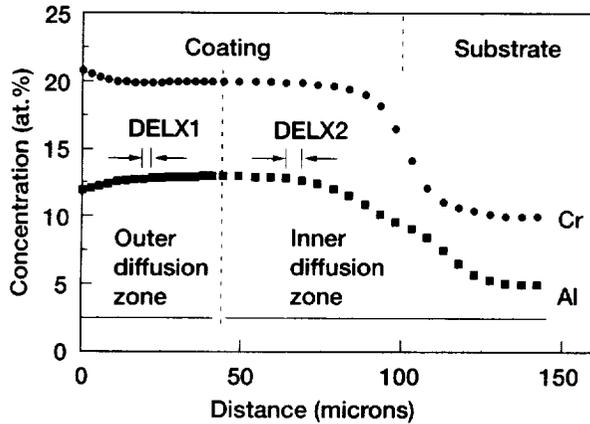


Figure 6.—(a) Al and Cr concentration profiles at the time (time = 1.34 hrs) when the inner and outer diffusion zones overlap. Input data for run given in Appendix A. (Alternate nodes hidden for clarity.)

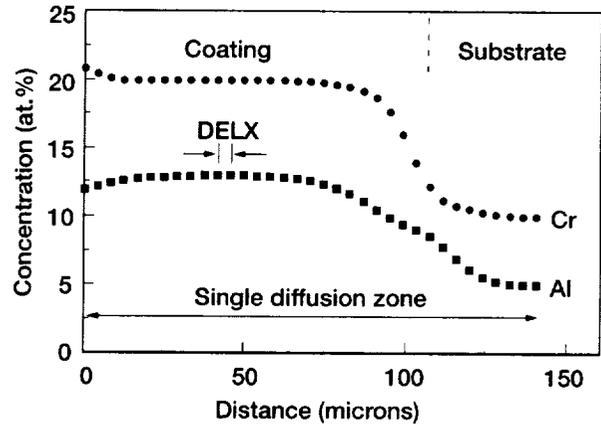


Figure 6.—(b) Al and Cr concentration profiles at the time = 1.34 hrs following the spline operation. Input data for run given in Appendix A. (Alternate nodes hidden for clarity.)

time increments calculated for the outer and inner diffusion zones is always used. Because of the concentration dependence of the diffusivities, one end of the inner diffusion zone may expand more than the other over the course of several thousand iterations. Although the inner and outer zones may appear very different with different widths, different numbers of nodes and different node spacings, both zones always operate with the same time increment such that the total exposure time for both the inner and outer diffusion zones is always identical.

The expansion of the inner and outer zones will eventually result in their impingement, or overlap, within the coating. From this time onward, the two diffusion problems, oxidation and coating/substrate interdiffusion, become coupled and can no longer be operated independently. At the time that the diffusion zones overlap, the COSIM model sums the current width of the inner and outer zones and redefines a single zone of equal width. Equidistant node spacings are also calculated using the combined number of nodes from the two zones (i.e.,  $N_{COAT} + N_{SUB}$ ). The COSIM model then fits a natural cubic spline curve (refs. 10 and 11) through the concentration profiles from both zones and generates a single profile through the coating and into the substrate for both Al and Cr at each of the new node positions. Figure 6(a) shows predicted Al and Cr concentration profiles in the coating and substrate at the time (time = 1.34 hr) when the two diffusion zones overlap. The width of the inner diffusion zone is significantly larger with a larger number of nodes and a larger node spacing than that for the outer diffusion zone. Figure 6(b) shows the same data after the spline interpolation. Note that the nodes are equally spaced across the entire diffusion zone. The spline subroutines are based on equations and code given in reference 11. Following the combination of the two diffusion zones, the COSIM model continues to simulate Al transport to the surface and interdiffusion of the coating and substrate. Both boundary conditions, at the oxide/coating interface and in the substrate remain as before the combination. Loss of Al continues to cause surface recession and a shrinking of the zone while diffusion in the substrate continues to result in zone expansion. Within the program, the transition from two zones to one is reflected in the value of the parameter *ZONE*. The value of *ZONE* changes from two to one after the diffusion zones are combined and the program operates on the single diffusion zone thereafter. Parameters are redefined so that the new, single diffusion zone utilizes parameters associated with the outer diffusion zone (e.g., *DELX1*, *DELTI*, etc.) The model continues to simulate increasing oxidation exposure with each iteration of time. The program can print out concentration data at intermediate times (e.g., 50, 100, 500, 1000, 5000 hr) or at some predetermined failure condition (e.g.,  $C_{Al,0} = 0$  or 5 at%). Figure 7 shows predicted concentration profiles after 1, 100 and 1000 hr. Diffusivities for the current examples were taken from reference 12. The value for the partial molar volume of Al was taken from reference 13.

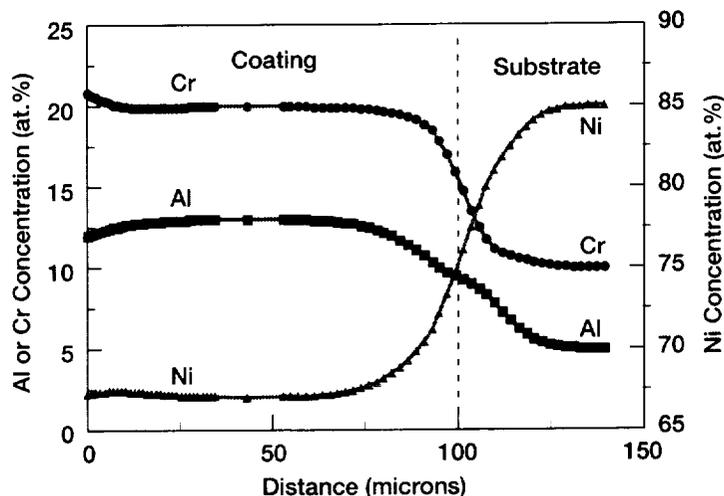


Figure 7.—(a) Al, Cr and Ni concentration profiles at time = 1.0 hr using the input data given in Appendix A.

Occasionally, oscillations in the values of the Al concentration at the oxide/coating interface,  $AL(0)$ , will occur the first few cycles. These oscillations can cause the computed value for the  $AL(0)$  to become negative or to exceed 100 at%. It has also been observed that a very high initial flux from the COSP oxide growth and spalling model can also cause the computed value for  $AL(0)$  to be negative for hundreds of cycles. To protect against this physical impossibility, the program limits the values of  $AL(0)$  to be between 0 and 100 at% (i.e., negative values are set equal to zero and values >100 are set equal to 100). The consequence of these limits is that equation (6) is not satisfied during these iterations. However, monitoring the computed values for  $AL(0)$  typically show convergence to acceptable values at relatively short times (i.e., less than one hour when unreasonably high values for  $K_p$  were used with the example input data given in Appendix A). Since these early oscillations could falsely trigger the test for  $AL(0)$  less than a critical Al concentration (i.e.,  $AL(0) \leq CRITAL$ ) this test is not initiated in the program during the first two hours of oxidation exposure. No problems have been encountered using test cases using various values for  $CRITAL$ , including zero.

Certain constraints on the time increment used each iteration ( $DEL T$ ) are desirable when simulating certain oxidation conditions. For instance, a value for  $DEL T$  much <1 hr might be desirable during cyclic oxidation with 1 hr cycles. In this case, a value of  $DEL T$  of one or 5 min might be desired so that several iterations during oxide growth can be performed between periods of oxide spallation each cycle. In contrast, for isothermal oxidation for long periods, it might be preferable to have no constraint on the values for  $DEL T$  in order to minimize the number of iterations to reach a solution. To provide this flexibility, a maximum value for  $DEL T$  ( $MAXDT$ ) may be input to the program. The default condition within the program is that no constraint be made on  $DEL T$  beyond that given in equation (13).

#### Fortran version

The COSIM program was initially compiled and executed on mainframe computers and later revised for execution on a desktop PC. The current version of the program has been compiled with ANSI FORTRAN95. Very few extensions from the ANSI FORTRAN77 standard were used. These extensions consist of the use of the NAMELIST command, used for ease in reading and writing to files, and extended length variable names exceeding 6 characters and containing special characters which were used for better clarity in the description of the variables.

# Description of Program Elements

## Main Program

The main program defines all common blocks, initializes parameters, calls subroutines to define and initialize both zones and directs program flow. The basic iterations are performed in the main program calling the FSL (Fick's Second Law) subroutine to calculate new concentrations, and JEXT (COSP oxide spalling model) subroutine to provide the boundary condition for the oxide/coating interface. In addition, the main program also checks the end point nodes (*NCOATH*, *NSUBL* and *NSUBH*) to determine when the diffusion zones need to be expanded and calls the M\_L (Murray-Landis) subroutine to shift nodes and concentrations as needed. Checks are also made within the main program for overlap of diffusion zones, whether the maximum number of iterations has been exceeded, and whether a time has been reached at which output is desired. As an option, the program also checks whether the surface Al concentration (*AL(0)*) has decreased below a critical value (*CRITAL*). Output may consist of writing concentration profiles at a specific time (e.g., 100 and 500 hrs) with appropriate headers and documentation to unit 6 (see Appendix B). This output is appropriate for printing. At the same time that concentration profiles are written to unit 6, the concentration data is written to unit 14 with minimal headers and without further documentation (see Appendix C). The data written to unit 14 is useful for reading into a graphing program to plot concentration/distance profiles. As a further option, select data (time (hrs), surface concentrations *AL(0)*, *CR(0)*, rate of Al consumption and total weight of Al consumed) are written to unit 15 on a more frequent time interval (see Appendix D). The data written to unit 15 is useful for plotting and viewing the time dependence of certain important variables. Examples of the data written to unit 15 are shown in figures 8(a) to (c). Subroutines and parameters are discussed below. An alphabetical list of all program variables is given in Appendix E. Flow charts for the main program and subroutines are given in Appendix F.

## Subroutine READ11

The READ11 subroutine reads three descriptive character strings of 80 characters each. These character strings may contain any alphanumeric data and can be used to describe the conditions of the particular run. The names of the character string variables are: *SAMPLE*, *TITLE1* and *TITLE2*. All other data required by the program are entered via three NAMELIST statements, *INPUT*, *OX\_INP*, and *INP15*. All data are read from unit 11. The file assigned in the subroutine to unit 11 is "COSIM.INP". Descriptions of the named input variables are given below. Parameters described as optional may be removed from the COSIM.INP file (i.e., *CRITAL = X* may be removed) or the parameter may remain in the file with the default value given below (i.e., *CRITAL = -1.0* indicates to the program no test for a critical Al concentration). An example set of input data are given in Appendix A.

Parameters and definitions read in through the INPUT namelist are:

<i>ALCOAT</i>	Al concentration in the coating (at%)
<i>CRCOAT</i>	Cr concentration in the coating (at%)
<i>ALSUB</i>	Al concentration in the substrate (at%)
<i>CRSUB</i>	Cr concentration in the substrate (at%)
<i>MW_AL</i>	Molecular weight of Al (gm/mole)
<i>MW_CR</i>	Molecular weight of Cr (gm/mole)
<i>MW_NI</i>	Molecular weight of Ni (gm/mole)
<i>DENSITY</i>	Density of the coating (gm/cm <sup>3</sup> )
<i>VBAR</i>	Partial molar volume of Al in the coating (cm <sup>3</sup> /mole)
<i>THICK</i>	Coating thickness (μm)
<i>TEMP</i>	Oxidation temperature (numeric)
<i>TMAX</i>	Time(s) at which output is desired (hr) (e.g., 100, 500, 2000)
<i>NCOAT</i>	Number of nodes in the "outer" coating diffusion zone
<i>NSUB</i>	Number of nodes in the "inner" coating/substrate diffusion zone
<i>DXCOAT</i>	Initial width of the outer diffusion zone (μm)
<i>DXSUB</i>	Initial width of the inner diffusion zone (μm)

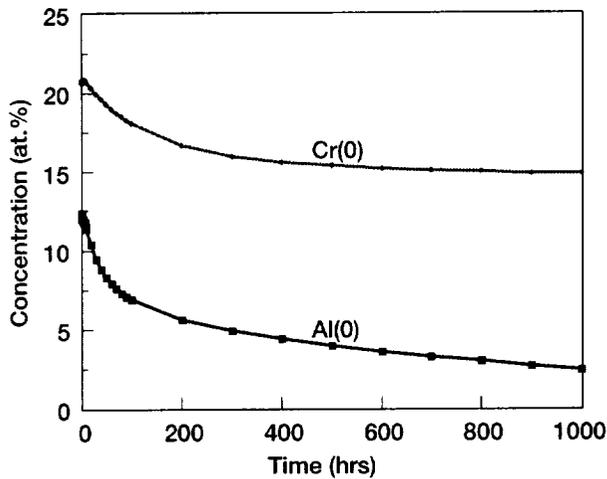


Figure 8.—(a) Time dependence of Al and Cr concentrations at the surface ( $Al(0)$  and  $Cr(0)$ ) from data written to unit 15 (File OUT15) for the input data given in Appendix A.

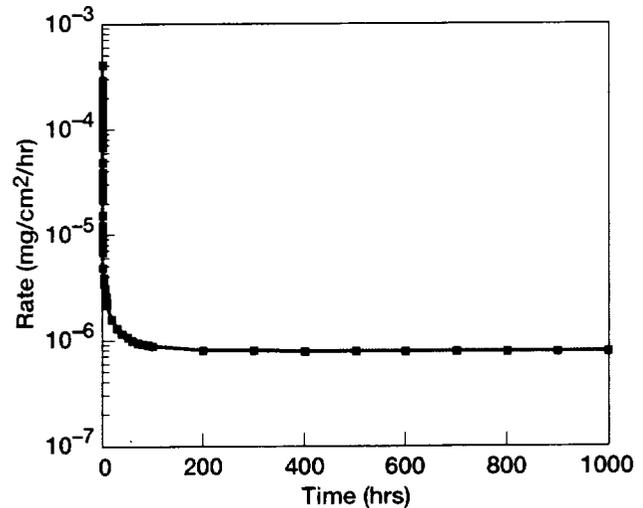


Figure 8.—(b) Time dependence of WMDOT (rate of Al consumption) from data written to unit 15 (File OUT15) for the input data given in Appendix A.

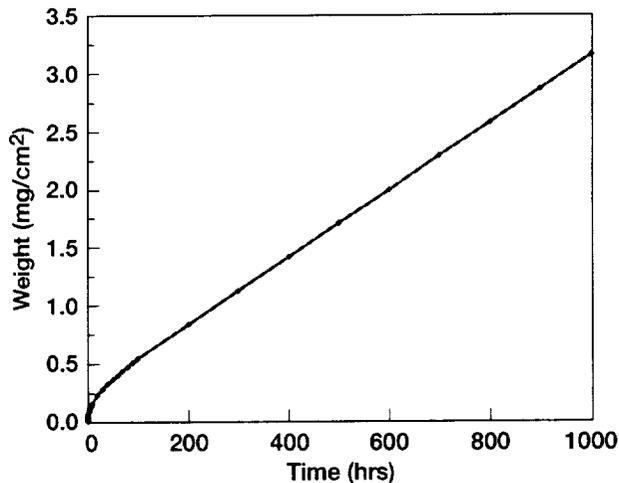


Figure 8.—(c) Time dependence of WMINT (weight of Al consumed) from data written to unit 15 (File OUT15) for the input data given in Appendix A.

- MAXLOOP** Maximum number of iterations (*ILOOP*) allowed. Can be used to halt and print data after a specific number of iterations, or to protect against infinite looping. Parameter is optional. (Default is 1,000,000)
- MAXDT** Maximum desired time increment per iteration (*DELTA*), (min). Parameter is optional. (Default is no limit on *DELTA* which is set in the program as  $MAXDT = -1.0$ .)
- CRITAL** Critical surface Al concentration. When  $AL(0)$  decreases to *CRITAL*, program calls subroutine WRT6F, then stops. Parameter is optional. (Default is no test for a critical surface Al concentration which is set in the program as  $CRITAL = -1.0$ .)

Parameters and definitions read in through the OX\_INP namelist are:

- MODEL** Designates parabolic ( $MODEL = 1$ ) or power law oxide growth kinetics ( $MODEL = 2$ ) (see *AGROW* below)

- KP* Parabolic oxide growth parameter (used with *MODEL* = 1) due to weight of oxygen in the oxide (i.e., not weight of oxide). Units of  $\text{mg}^2/\text{cm}^4/\text{hr}$ .
- AGROW* Power law oxide growth parameter (used with *MODEL* =2) with units of  $\text{mg}/\text{cm}^2$ , defined as:

$$\Delta W = AGROW * time^{BGROW}$$

where time is given in hours.

- BGROW* Power law oxide growth parameter (unitless) as defined above.
- A0* The ratio of the weight of Al in the oxide to the weight of oxygen in the oxide (Unitless) (e.g.,  $A0 = 1.1242$  for  $\text{Al}_2\text{O}_3$ ).
- TCYCLE* The hot cycle duration for each thermal cycle, in hours.
- QO* The spall parameter for the COSP spalling model (ref. 3)
- ETA* Additional spall parameter for the COSP spalling model (ref. 3)

Five parameters are used to define each of the four ternary diffusion coefficients. These parameters are read in through the *D\_COEF* namelist. The parameters are:

A110, A111, A112, A113, A114, A115  
 A120, A121, A122, A123, A124, A125  
 A210, A211, A212, A213, A214, A215  
 A220, A221, A222, A223, A224, A225

The parameters are defined in the equations:

$$D_{AlAl}(C_{Al}, C_{Cr}) = [A110 + (A111 * C_{Al}) + (A112 * C_{Cr}) + (A113 * C_{Al}^2) + (A114 * C_{Cr}^2)] * A115$$

$$D_{AlCr}(C_{Al}, C_{Cr}) = [A120 + (A121 * C_{Al}) + (A122 * C_{Cr}) + (A123 * C_{Al}^2) + (A124 * C_{Cr}^2)] * A125$$

$$D_{CrAl}(C_{Al}, C_{Cr}) = [A210 + (A211 * C_{Al}) + (A212 * C_{Cr}) + (A213 * C_{Al}^2) + (A214 * C_{Cr}^2)] * A215$$

$$D_{CrCr}(C_{Al}, C_{Cr}) = [A220 + (A221 * C_{Al}) + (A222 * C_{Cr}) + (A223 * C_{Al}^2) + (A224 * C_{Cr}^2)] * A225$$

Parameters and definitions read in through the *INP15* namelist are:

- IO15* I/O switch for printing specific output to unit 15. When set to 1, the program prints specific data at predetermined intervals every 0.01, 0.1, 1.0, 10.0, hrs. Parameter is optional. (Default option is no printing to unit 15 which is set in the program as *IO15* = 0.

#### Subroutine WRT6

Subroutine to write out all input data to unit 6 with descriptive format statements. The file assigned in the subroutine to unit 6 is "OUT6". Sample output printed by WRT6 is given in Appendix B.

#### Subroutine WRT6F

Subroutine to write out current status. Simulated exposure time, surface recession, current zone widths and node spacings, weight of Al consumed and concentration profile data is written to unit 6 (file "OUT6"). Sample output printed by WRT6F after 1, 100 and 1000 cycles is given in Appendix B. For convenience in plotting the concentration profiles, node positions from the original surface in microns (i.e., *DIST*), Al and Cr, and Ni concentrations in at% and Time (hrs) are also printed to unit 14 (file "OUT14"). Example output from file OUT14 is shown in Appendix C.

#### Subroutine WRT12

Subroutine which writes intermediate concentration/distance data for both diffusion zones to unit 12 at the time when the two diffusion profiles first overlap. The data is written sequentially as "Distance,  $C_{Al}$ ,  $C_{Cr}$ ." Unit 12 is assigned in the subroutine to file "OUT12." After the data for both diffusion zones are written to unit 12, the file is repositioned to the first record (rewind unit = 12) to prepare the file to be read by the SPLINE subroutine. The SPLINE subroutine is called, reads the data in OUT12 and fits a single smooth curve to the concentration/distance

data. The SPLINE subroutine rewrites the new concentration/distance data over the old data in file OUT12. The WRT12 subroutine then reads the new concentration/distance data from file OUT12. The new data for the single diffusion zone is reassigned in terms of the outer diffusion zone (i.e., the new zone width is assigned to *DXCOAT*, the new node spacing to *delx1*, and the innermost concentrations (*ALCOAT*, *CRCOAT*) are assigned the values *ALSUB*, *CRSUB*.

#### **Subroutine WRT15**

Subroutine which writes out specific data at regular intervals to unit 15 when *IO15* = 1. The data written to unit 15 are: time(hrs), surface concentrations *AL(0)* and *CR(0)*, the rate of Al consumption *WMDOT*, and the total Al consumption *WMINT*. The data are written in increasing intervals of 0.01, 0.1, 1.0, 10, 100 hrs. Unit 15 is assigned in the MAIN program to file "OUT15". The subroutine is useful for tracking the time dependence of specific parameters. Sample output printed by WRT15 is given in Appendix D.

#### **Subroutine MAXOUT**

Subroutine called when *MAXLOOP* iterations are exceeded in MAIN program. Message is printed to unit 6 as well as current time *TIME* and time increments *DELT1*, *DELT2*. Subroutine WRT6F is called.

#### **Subroutine CRITSB**

Subroutine called when surface Al concentration *AL(0)* decreases below *CRITAL* (for *CRITAL*  $\geq 0.0$  specified in COSIM.INP. Message is printed to unit 6 as well as current time value for *CRITAL*. Subroutine WRT6F is called.

#### **Subroutine OX\_INIT**

Subroutine to initialize variables for *COSP* oxide growth and spalling model. For *MODEL* = 1 (parabolic oxide growth), *KP* is redefined in terms of *AGROW* and *BGROW*.

#### **Subroutine UNITS**

Subroutine to convert units to a consistent set used throughout the program. Parameters converted are:

<i>VBAR</i>	Molar volume of Al from $\text{cm}^3/\text{mole}$ to $(\text{at}\%)^{-1}$
<i>DXCOAT</i>	Microns to cm
<i>DXSUB</i>	Microns to cm
<i>THICK</i>	Microns to cm
<i>DENSITY</i>	$\text{g}/\text{cm}^3$ to $\text{mg}/\text{cm}^3$
<i>ALWCOAT</i>	Al concentration of the coating from at% to wt% (used in converting <i>WMDOT</i> , rate of Al consumption in $\text{mg}/\text{cm}^2/\text{hr}$ predicted by <i>COSP</i> oxide spalling model to $\text{at}\%/\text{cm}/\text{s}$ used in COSIM)
<i>TMAX(I)</i>	All times in hrs converted to sec.
<i>MAXDT</i>	Time in minutes converted to sec.

#### **Subroutine INIT1**

Subroutine to set the initial concentration profile for the outer (oxidation) and inner (coating/substrate interdiffusion) diffusion zones. Concentrations for outer diffusion zone, nodes 0 to *NCOAT*, and inner diffusion zone, nodes *NSUBL-1* to *NSUBH+1* are assigned initial values as discussed above. Subroutine calls the Function DERF to generate an error function solution for the inner diffusion zone.

#### **Subroutine DERF(X)**

Function which returns the error function for parameter *X*.

#### **Subroutine D\_C(*Alx*,*Cr<sub>x</sub>*)**

Subroutine which returns values of the four interdiffusion coefficients and derivatives at the concentration *Al<sub>x</sub>*, *Cr<sub>x</sub>*. Since the concentration dependence of the diffusivities for  $\gamma$  NiCrAl alloys is known in polynomial form (ref. 12), derivatives with respect to concentration (i.e.,  $\partial D/\partial C$ , see eq. (4)) are calculated and evaluated at the specific concentration *Al<sub>x</sub>*, *Cr<sub>x</sub>*.

### Subroutine JEXT

Subroutine version of the COSP oxide growth and spalling program (ref. 3). Subroutine returns values for the rate of Al consumption, *WMDOT* for the outer boundary condition. Subroutine tracks oxide growth each iteration for time increment *DELX*, the total weight of Al consumed *TOTWM*, and the amount of oxide spall each thermal cycle.

### Subroutine M\_L(*N1,N2,DELX,DX,AN*)

Subroutine to shift concentrations when a zone is expanded or contracted. Al and Cr concentrations are shifted as explained in the text.

*N1* the node number where the maximum shift in node spacing occurs  
*N2* the node number where the minimum shift in node spacing occurs  
*DELX* the node spacing  
*DX* the amount the zone is to be shifted (negative to the left, positive to the right)  
*AN* the number of nodes in the zone

### Subroutine FSL(*N1,N2,DELX,DELX*)

Subroutine to perform Fick's second law calculations on the indicated nodes according to equation (4).

*N1* the minimum node number upon which to operate  
*N2* the maximum node number upon which to operate  
*DELX* the current node spacing  
*DELX* the current time increment

### Subroutines SPLINE(*M*), CUBIC, EVALUAT

Subroutines to read concentration/distance data for both inner and outer diffusion zones from unit 12, fit to a single smooth curve using natural splines, and redetermine concentrations at equal node spacings (ref. 10)

*M* number of nodes in the new zone (equal to *NCOAT+NSUB*)

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3. C.E. Lowell, C.A. Barrett, R.W. Palmer, J.V. Auping, and H.B. Probst, *Oxid. Met.*, **36**, 81, 1991.
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5. K.W. Chan, *Met. and Mat. Trans.*, **28A**, 411, 1997.
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12. J.A. Nesbitt and R.W. Heckel, *Met. Trans.*, **18A**, 2075, 1987.
13. J.A. Nesbitt, NASA TM 83738, Cleveland, OH, 1984.

## Appendix A

### Unit 11 Data Input to the Program (File: COSIM.INP)

COSIM (Thick=100um, Kp=0.002, Qo=0.008, 1100C) 1/28/98  
Coating: Ni-20Cr-13Al Substrate: Ni-10.0Cr-5.0Al  
1100C DIFFUSION DATA FROM: MET. TRANS.,18A, (1987) p2075 (TABLE V).

&INPUT

ALCOAT=13.0, CRCOAT=20.0,  
ALSUB=5.0, CRSUB=10.0,  
MW\_AL=26.98, MW\_CR=52.0, MW\_NI=58.71,  
DENSITY= 7.754,  
THICK=100.0, TEMP=1100.0,  
TMAX(1)=1.0,  
TMAX(2)=100.0,  
TMAX(3)=1000.0,  
NCOAT=30, NSUB=40,  
DXCOAT=0.1, DXSUB=0.1,  
VBAR=7.1,  
MAXLOOP=100000,  
MAXDT=1.0,  
CRITAL=-1.0,  
/

&OX\_INP

MODEL=1,  
KP=0.002,  
AGROW=0.0, BGROW=0.0, A0=1.1242,  
TCYCLE=1.0, QO=0.008, ETA=1.0,  
/

&INP15

IO15=1,  
/

&D\_COEF

A110=1.229, A111=0.0731, A112=-0.0083, A113=0.0101,  
A114=0.00016, A115= 1.0D-10,  
A120=0.0116, A121=0.0923, A122=-0.0010, A123=0.00016,  
A124=0.000017, A125= 1.0D-10,  
A210=0.0766, A211=-0.0153, A212= 0.0837, A213=0.00062,  
A214=-0.0015, A215= 1.0D-10,  
A220=0.783, A221=-0.0123, A222= 0.0247, A223=0.00096,  
A224=-0.00057, A225= 1.0D-10,  
/

**Appendix B**  
Output to Unit 6 (File: OUT6)

COSIM (Thick=100um, Kp=0.002, Qo=0.008, 1100C) 1/28/98  
Coating: Ni-20Cr-13Al Substrate: Ni-10.0Cr-5.0Al  
1100C DIFFUSION DATA FROM: MET. TRANS., 18A, (1987) p2075 (TABLE V).

TEMPERATURE = 1100.0

OUTPUT TIMES (HRS) =       1.0  
                              100.0  
                              1000.0

AL, CR COATING COMPOSITION (at%) = 13.00 20.00  
AL, CR SUBSTRATE COMPOSITION (at%) = 5.00 10.00

MOLECULAR WEIGHT: AL, CR, NI = 26.98 52.00 58.71  
COATING DENSITY (g/cm<sup>3</sup>) = 7.75  
COATING THICKNESS (MICRONS) = 100.00  
INITIAL # OF NODES IN COATING = 30  
INITIAL # OF NODES IN SUBSTRATE = 40  
INITIAL DIFFUSION ZONE WIDTH IN COATING (MICRONS) = .10  
INITIAL DIFFUSION ZONE WIDTH IN SUBSTRATE (MICRONS) = .10  
PARTIAL MOLAR VOLUME OF AL = 7.100  
MAXIMUM ITERATIONS ALLOWED = 100000  
MAXIMUM DELT (MINUTES) = 1.0

OXIDE GROWTH MODEL IS 1  
KP (MODEL=1) = .002000  
A (MODEL=2) = .000000  
B (MODEL=2) = .000000  
A0 RATIO = 1.124200  
TCYCLE = 1.000000  
QO SPALLING PARAMETER = .008000  
ETA SPALLING PARAMETER = 1.000000

DIFFUSION COEFFICIENT PARAMETERS

	0	1	2	3	4	5
A11=	.123E+01	.731E-01	-.830E-02	.101E-01	.160E-03	.10E-09
A12=	.116E-01	.923E-01	-.100E-02	.160E-03	.170E-04	.10E-09
A21=	.766E-01	-.153E-01	.837E-01	.620E-03	-.150E-02	.10E-09
A22=	.783E+00	-.123E-01	.247E-01	.960E-03	-.570E-03	.10E-09

DATA IS WRITTEN TO FILE OUT15

TIME (HRS)= 1.00  
DELT (SEC)= 7.41  
ITERATIONS= 6306

SURFACE RECESSION (MICRONS)= .13  
TOTAL WEIGHT OF AL CONSUMED AS OXIDE (WMINT)= .05 (MG/CM<sup>2</sup>)

Node #	Distance (microns)	Al at. %	Cr	Ni
0	.1	11.96	20.84	67.19
1	1.3	12.06	20.70	67.24
2	2.5	12.16	20.56	67.28
3	3.7	12.25	20.43	67.31
4	4.9	12.34	20.32	67.34
5	6.1	12.42	20.22	67.36
6	7.2	12.49	20.14	67.37
7	8.4	12.55	20.08	67.38
8	9.6	12.60	20.03	67.37
9	10.8	12.65	19.99	67.36
10	12.0	12.69	19.97	67.34
11	13.2	12.73	19.95	67.32
12	14.3	12.76	19.95	67.29
13	15.5	12.79	19.94	67.27
14	16.7	12.81	19.95	67.24
15	17.9	12.84	19.95	67.22
16	19.1	12.86	19.95	67.19
17	20.3	12.87	19.96	67.17
18	21.4	12.89	19.96	67.15
19	22.6	12.91	19.97	67.13
20	23.8	12.92	19.97	67.11
21	25.0	12.93	19.98	67.09
22	26.2	12.94	19.98	67.08
23	27.4	12.95	19.98	67.06
24	28.6	12.96	19.99	67.05
25	29.7	12.97	19.99	67.04
26	30.9	12.98	19.99	67.03
27	32.1	12.98	19.99	67.02
28	33.3	12.99	20.00	67.01
29	34.5	13.00	20.00	67.01
32	51.4	13.00	20.00	67.00
33	53.6	12.99	20.00	67.01
34	55.8	12.99	20.00	67.02
35	58.1	12.98	19.99	67.03
36	60.3	12.96	19.99	67.05
37	62.5	12.94	19.98	67.08
38	64.7	12.91	19.97	67.11
39	66.9	12.87	19.96	67.17
40	69.1	12.82	19.94	67.24
41	71.3	12.75	19.91	67.34
42	73.5	12.65	19.88	67.47
43	75.7	12.54	19.84	67.63
44	78.0	12.39	19.78	67.83
45	80.2	12.21	19.71	68.09
46	82.4	11.98	19.62	68.40
47	84.6	11.72	19.51	68.77
48	86.8	11.41	19.36	69.23
49	89.0	11.05	19.17	69.77
50	91.2	10.67	18.89	70.44
51	93.4	10.28	18.45	71.27
52	95.6	9.92	17.78	72.30
53	97.9	9.63	16.82	73.56

54	100.1	9.38	15.62	75.00
55	102.3	9.16	14.32	76.52
56	104.5	8.90	13.10	78.00
57	106.7	8.55	12.11	79.35
58	108.9	8.08	11.42	80.50
59	111.1	7.54	11.01	81.46
60	113.3	6.97	10.78	82.25
61	115.6	6.44	10.64	82.93
62	117.8	5.99	10.51	83.50
63	120.0	5.65	10.38	83.97
64	122.2	5.41	10.25	84.34
65	124.4	5.24	10.15	84.61
66	126.6	5.13	10.08	84.78
67	128.8	5.07	10.04	84.89
68	131.0	5.03	10.02	84.95
69	133.2	5.01	10.01	84.98
70	135.5	5.00	10.00	84.99
71	137.7	5.00	10.00	85.00

SPLINE CALLED AT LOOP 6434

TIME (HRS)= 100.01  
 DELT (SEC)= 60.00  
 ITERATIONS= 12719

SURFACE RECESSION (MICRONS)= 1.42  
 TOTAL WEIGHT OF AL CONSUMED AS OXIDE (WMINT)= .54 (MG/CM^2)

Node #	Distance (microns)	Al at. %	Cr at. %	Ni at. %
0	1.4	6.89	18.09	75.03
1	7.9	6.99	17.95	75.07
2	14.4	7.08	17.79	75.13
3	20.9	7.17	17.62	75.21
4	27.5	7.26	17.43	75.31
5	34.0	7.34	17.23	75.44
6	40.5	7.41	17.01	75.58
7	47.0	7.48	16.79	75.74
8	53.5	7.54	16.55	75.92
9	60.0	7.59	16.30	76.11
10	66.5	7.64	16.04	76.32
11	73.0	7.69	15.77	76.55
12	79.5	7.73	15.49	76.78
13	86.0	7.76	15.21	77.03
14	92.6	7.78	14.93	77.29
15	99.1	7.80	14.64	77.56
16	105.6	7.81	14.36	77.83
17	112.1	7.82	14.08	78.11
18	118.6	7.81	13.80	78.39
19	125.1	7.80	13.53	78.67
20	131.6	7.78	13.27	78.95

21	138.1	7.75	13.02	79.23
22	144.6	7.72	12.78	79.51
23	151.1	7.67	12.55	79.78
24	157.7	7.62	12.34	80.05
25	164.2	7.56	12.14	80.31
26	170.7	7.49	11.95	80.57
27	177.2	7.41	11.78	80.81
28	183.7	7.32	11.62	81.06
29	190.2	7.23	11.48	81.29
30	196.7	7.14	11.35	81.51
31	203.2	7.04	11.23	81.73
32	209.7	6.94	11.12	81.94
33	216.3	6.83	11.03	82.14
34	222.8	6.73	10.94	82.33
35	229.3	6.62	10.86	82.52
36	235.8	6.51	10.79	82.69
37	242.3	6.41	10.73	82.86
38	248.8	6.31	10.67	83.02
39	255.3	6.21	10.62	83.18
40	261.8	6.11	10.57	83.33
41	268.3	6.01	10.52	83.47
42	274.8	5.93	10.48	83.60
43	281.4	5.84	10.44	83.72
44	287.9	5.76	10.40	83.84
45	294.4	5.69	10.36	83.95
46	300.9	5.62	10.33	84.05
47	307.4	5.55	10.30	84.15
48	313.9	5.49	10.27	84.24
49	320.4	5.44	10.24	84.32
50	326.9	5.38	10.22	84.40
51	333.4	5.34	10.19	84.47
52	339.9	5.30	10.17	84.53
53	346.5	5.26	10.15	84.59
54	353.0	5.22	10.13	84.64
55	359.5	5.19	10.11	84.69
56	366.0	5.17	10.10	84.73
57	372.5	5.14	10.08	84.77
58	379.0	5.12	10.07	84.81
59	385.5	5.10	10.06	84.84
60	392.0	5.09	10.05	84.86
61	398.5	5.07	10.04	84.89
62	405.0	5.06	10.03	84.91
63	411.6	5.05	10.03	84.93
64	418.1	5.04	10.02	84.94
65	424.6	5.03	10.02	84.95
66	431.1	5.02	10.01	84.97
67	437.6	5.01	10.01	84.98
68	444.1	5.01	10.01	84.99
69	450.6	5.00	10.00	84.99

TIME (HRS)= 1000.01  
DELT (SEC)= 60.00  
ITERATIONS= 66719

SURFACE RECESSION (MICRONS)= 8.32  
TOTAL WEIGHT OF AL CONSUMED AS OXIDE (WMINT)= 3.16 (MG/CM<sup>2</sup>)

Node #	Distance (microns)	at. %		Ni
		Al	Cr	
0	8.3	2.48	14.88	82.64
1	24.1	2.75	14.57	82.67
2	39.8	3.01	14.28	82.71
3	55.6	3.25	14.01	82.75
4	71.3	3.47	13.75	82.78
5	87.1	3.68	13.50	82.82
6	102.8	3.87	13.27	82.86
7	118.6	4.05	13.05	82.90
8	134.3	4.21	12.85	82.94
9	150.1	4.37	12.65	82.98
10	165.8	4.51	12.47	83.02
11	181.6	4.64	12.29	83.07
12	197.3	4.76	12.13	83.11
13	213.1	4.87	11.97	83.16
14	228.8	4.97	11.83	83.21
15	244.6	5.05	11.69	83.26
16	260.3	5.13	11.56	83.31
17	276.1	5.21	11.44	83.36
18	291.8	5.27	11.33	83.41
19	307.6	5.32	11.22	83.46
20	323.3	5.37	11.12	83.51
21	339.1	5.41	11.03	83.56
22	354.8	5.45	10.95	83.60
23	370.6	5.47	10.87	83.65
24	386.3	5.50	10.80	83.70
25	402.1	5.51	10.74	83.75
26	417.8	5.52	10.68	83.80
27	433.6	5.53	10.62	83.84
28	449.3	5.54	10.57	83.89
29	465.0	5.53	10.53	83.94
30	480.8	5.53	10.49	83.98
31	496.5	5.52	10.45	84.02
32	512.3	5.51	10.42	84.07
33	528.0	5.50	10.39	84.11
34	543.8	5.49	10.36	84.15
35	559.5	5.47	10.33	84.19
36	575.3	5.46	10.31	84.23
37	591.0	5.44	10.29	84.27
38	606.8	5.42	10.27	84.31
39	622.5	5.40	10.25	84.35
40	638.3	5.38	10.24	84.39
41	654.0	5.36	10.22	84.42
42	669.8	5.34	10.20	84.46
43	685.5	5.32	10.19	84.49
44	701.3	5.30	10.18	84.52
45	717.0	5.28	10.17	84.56

46	732.8	5.26	10.15	84.59
47	748.5	5.24	10.14	84.62
48	764.3	5.22	10.13	84.64
49	780.0	5.21	10.12	84.67
50	795.8	5.19	10.11	84.70
51	811.5	5.17	10.10	84.72
52	827.3	5.16	10.09	84.75
53	843.0	5.14	10.09	84.77
54	858.8	5.13	10.08	84.79
55	874.5	5.12	10.07	84.81
56	890.3	5.11	10.06	84.83
57	906.0	5.09	10.06	84.85
58	921.8	5.08	10.05	84.87
59	937.5	5.07	10.04	84.88
60	953.3	5.06	10.04	84.90
61	969.0	5.05	10.03	84.91
62	984.8	5.05	10.03	84.92
63	1000.5	5.04	10.02	84.94
64	1016.3	5.03	10.02	84.95
65	1032.0	5.03	10.02	84.96
66	1047.8	5.02	10.01	84.97
67	1063.5	5.01	10.01	84.98
68	1079.3	5.01	10.01	84.98
69	1095.0	5.00	10.00	84.99

## Appendix C

### Output to Unit 14 (File: OUT14)

COSIM (Thick=100um, Kp=0.002, Qo=0.008, 1100C) 1/28/98

Coating: Ni-20Cr-13Al Substrate: Ni-10.0Cr-5.0Al

1100C DIFFUSION DATA FROM: MET. TRANS.,18A,(1987) p2075 (TABLE V).

DIST(UM)	AL	CR	NI	TIME(HR)
.13	11.96	20.84	67.19	1.00
1.32	12.06	20.70	67.24	1.00
2.50	12.16	20.56	67.28	1.00
3.69	12.25	20.43	67.31	1.00
4.87	12.34	20.32	67.34	1.00
6.05	12.42	20.22	67.36	1.00
7.24	12.49	20.14	67.37	1.00
8.42	12.55	20.08	67.38	1.00
9.61	12.60	20.03	67.37	1.00
10.79	12.65	19.99	67.36	1.00
11.98	12.69	19.97	67.34	1.00
13.16	12.73	19.95	67.32	1.00
14.34	12.76	19.95	67.29	1.00
15.53	12.79	19.94	67.27	1.00
16.71	12.81	19.95	67.24	1.00
17.90	12.84	19.95	67.22	1.00
19.08	12.86	19.95	67.19	1.00
20.27	12.87	19.96	67.17	1.00
21.45	12.89	19.96	67.15	1.00
22.63	12.91	19.97	67.13	1.00
23.82	12.92	19.97	67.11	1.00
25.00	12.93	19.98	67.09	1.00
26.19	12.94	19.98	67.08	1.00
27.37	12.95	19.98	67.06	1.00
28.56	12.96	19.99	67.05	1.00
29.74	12.97	19.99	67.04	1.00
30.92	12.98	19.99	67.03	1.00
32.11	12.98	19.99	67.02	1.00
33.29	12.99	20.00	67.01	1.00
34.48	13.00	20.00	67.01	1.00
51.42	13.00	20.00	67.00	1.00
53.63	12.99	20.00	67.01	1.00
55.84	12.99	20.00	67.02	1.00
58.05	12.98	19.99	67.03	1.00
60.26	12.96	19.99	67.05	1.00
62.48	12.94	19.98	67.08	1.00
64.69	12.91	19.97	67.11	1.00
66.90	12.87	19.96	67.17	1.00
69.11	12.82	19.94	67.24	1.00
71.32	12.75	19.91	67.34	1.00
73.53	12.65	19.88	67.47	1.00
75.74	12.54	19.84	67.63	1.00
77.96	12.39	19.78	67.83	1.00
80.17	12.21	19.71	68.09	1.00
82.38	11.98	19.62	68.40	1.00
84.59	11.72	19.51	68.77	1.00
86.80	11.41	19.36	69.23	1.00

89.01	11.05	19.17	69.77	1.00
91.23	10.67	18.89	70.44	1.00
93.44	10.28	18.45	71.27	1.00
95.65	9.92	17.78	72.30	1.00
97.86	9.63	16.82	73.56	1.00
100.07	9.38	15.62	75.00	1.00
102.28	9.16	14.32	76.52	1.00
104.50	8.90	13.10	78.00	1.00
106.71	8.55	12.11	79.35	1.00
108.92	8.08	11.42	80.50	1.00
111.13	7.54	11.01	81.46	1.00
113.34	6.97	10.78	82.25	1.00
115.55	6.44	10.64	82.93	1.00
117.76	5.99	10.51	83.50	1.00
119.98	5.65	10.38	83.97	1.00
122.19	5.41	10.25	84.34	1.00
124.40	5.24	10.15	84.61	1.00
126.61	5.13	10.08	84.78	1.00
128.82	5.07	10.04	84.89	1.00
131.03	5.03	10.02	84.95	1.00
133.25	5.01	10.01	84.98	1.00
135.46	5.00	10.00	84.99	1.00
137.67	5.00	10.00	85.00	1.00
DIST (UM)	AL	CR	NI	TIME (HR)
1.42	6.89	18.09	75.03	100.01
7.93	6.99	17.95	75.07	100.01
14.44	7.08	17.79	75.13	100.01
20.95	7.17	17.62	75.21	100.01
27.46	7.26	17.43	75.31	100.01
33.97	7.34	17.23	75.44	100.01
40.48	7.41	17.01	75.58	100.01
46.99	7.48	16.79	75.74	100.01
53.50	7.54	16.55	75.92	100.01
60.01	7.59	16.30	76.11	100.01
66.52	7.64	16.04	76.32	100.01
73.03	7.69	15.77	76.55	100.01
79.54	7.73	15.49	76.78	100.01
86.05	7.76	15.21	77.03	100.01
92.56	7.78	14.93	77.29	100.01
99.07	7.80	14.64	77.56	100.01
105.58	7.81	14.36	77.83	100.01
112.09	7.82	14.08	78.11	100.01
118.60	7.81	13.80	78.39	100.01
125.11	7.80	13.53	78.67	100.01
131.62	7.78	13.27	78.95	100.01
138.13	7.75	13.02	79.23	100.01
144.64	7.72	12.78	79.51	100.01
151.15	7.67	12.55	79.78	100.01
157.66	7.62	12.34	80.05	100.01
164.17	7.56	12.14	80.31	100.01
170.68	7.49	11.95	80.57	100.01
177.19	7.41	11.78	80.81	100.01
183.70	7.32	11.62	81.06	100.01
190.21	7.23	11.48	81.29	100.01
196.72	7.14	11.35	81.51	100.01

203.23	7.04	11.23	81.73	100.01
209.74	6.94	11.12	81.94	100.01
216.25	6.83	11.03	82.14	100.01
222.76	6.73	10.94	82.33	100.01
229.27	6.62	10.86	82.52	100.01
235.78	6.51	10.79	82.69	100.01
242.29	6.41	10.73	82.86	100.01
248.80	6.31	10.67	83.02	100.01
255.31	6.21	10.62	83.18	100.01
261.82	6.11	10.57	83.33	100.01
268.33	6.01	10.52	83.47	100.01
274.84	5.93	10.48	83.60	100.01
281.35	5.84	10.44	83.72	100.01
287.86	5.76	10.40	83.84	100.01
294.37	5.69	10.36	83.95	100.01
300.88	5.62	10.33	84.05	100.01
307.39	5.55	10.30	84.15	100.01
313.90	5.49	10.27	84.24	100.01
320.41	5.44	10.24	84.32	100.01
326.92	5.38	10.22	84.40	100.01
333.43	5.34	10.19	84.47	100.01
339.94	5.30	10.17	84.53	100.01
346.45	5.26	10.15	84.59	100.01
352.96	5.22	10.13	84.64	100.01
359.47	5.19	10.11	84.69	100.01
365.98	5.17	10.10	84.73	100.01
372.49	5.14	10.08	84.77	100.01
379.00	5.12	10.07	84.81	100.01
385.51	5.10	10.06	84.84	100.01
392.02	5.09	10.05	84.86	100.01
398.53	5.07	10.04	84.89	100.01
405.04	5.06	10.03	84.91	100.01
411.55	5.05	10.03	84.93	100.01
418.06	5.04	10.02	84.94	100.01
424.57	5.03	10.02	84.95	100.01
431.08	5.02	10.01	84.97	100.01
437.59	5.01	10.01	84.98	100.01
444.10	5.01	10.01	84.99	100.01
450.61	5.00	10.00	84.99	100.01
DIST (UM)	AL	CR	NI	TIME (HR)
8.32	2.48	14.88	82.64	1000.01
24.07	2.75	14.57	82.67	1000.01
39.82	3.01	14.28	82.71	1000.01
55.56	3.25	14.01	82.75	1000.01
71.31	3.47	13.75	82.78	1000.01
87.06	3.68	13.50	82.82	1000.01
102.81	3.87	13.27	82.86	1000.01
118.56	4.05	13.05	82.90	1000.01
134.31	4.21	12.85	82.94	1000.01
150.06	4.37	12.65	82.98	1000.01
165.81	4.51	12.47	83.02	1000.01
181.56	4.64	12.29	83.07	1000.01
197.31	4.76	12.13	83.11	1000.01
213.06	4.87	11.97	83.16	1000.01
228.81	4.97	11.83	83.21	1000.01

244.56	5.05	11.69	83.26	1000.01
260.31	5.13	11.56	83.31	1000.01
276.06	5.21	11.44	83.36	1000.01
291.81	5.27	11.33	83.41	1000.01
307.56	5.32	11.22	83.46	1000.01
323.31	5.37	11.12	83.51	1000.01
339.05	5.41	11.03	83.56	1000.01
354.80	5.45	10.95	83.60	1000.01
370.55	5.47	10.87	83.65	1000.01
386.30	5.50	10.80	83.70	1000.01
402.05	5.51	10.74	83.75	1000.01
417.80	5.52	10.68	83.80	1000.01
433.55	5.53	10.62	83.84	1000.01
449.30	5.54	10.57	83.89	1000.01
465.05	5.53	10.53	83.94	1000.01
480.80	5.53	10.49	83.98	1000.01
496.55	5.52	10.45	84.02	1000.01
512.30	5.51	10.42	84.07	1000.01
528.05	5.50	10.39	84.11	1000.01
543.80	5.49	10.36	84.15	1000.01
559.55	5.47	10.33	84.19	1000.01
575.30	5.46	10.31	84.23	1000.01
591.05	5.44	10.29	84.27	1000.01
606.79	5.42	10.27	84.31	1000.01
622.54	5.40	10.25	84.35	1000.01
638.29	5.38	10.24	84.39	1000.01
654.04	5.36	10.22	84.42	1000.01
669.79	5.34	10.20	84.46	1000.01
685.54	5.32	10.19	84.49	1000.01
701.29	5.30	10.18	84.52	1000.01
717.04	5.28	10.17	84.56	1000.01
732.79	5.26	10.15	84.59	1000.01
748.54	5.24	10.14	84.62	1000.01
764.29	5.22	10.13	84.64	1000.01
780.04	5.21	10.12	84.67	1000.01
795.79	5.19	10.11	84.70	1000.01
811.54	5.17	10.10	84.72	1000.01
827.29	5.16	10.09	84.75	1000.01
843.04	5.14	10.09	84.77	1000.01
858.79	5.13	10.08	84.79	1000.01
874.53	5.12	10.07	84.81	1000.01
890.28	5.11	10.06	84.83	1000.01
906.03	5.09	10.06	84.85	1000.01
921.78	5.08	10.05	84.87	1000.01
937.53	5.07	10.04	84.88	1000.01
953.28	5.06	10.04	84.90	1000.01
969.03	5.05	10.03	84.91	1000.01
984.78	5.05	10.03	84.92	1000.01
1000.53	5.04	10.02	84.94	1000.01
1016.28	5.03	10.02	84.95	1000.01
1032.03	5.03	10.02	84.96	1000.01
1047.78	5.02	10.01	84.97	1000.01
1063.53	5.01	10.01	84.98	1000.01
1079.28	5.01	10.01	84.98	1000.01
1095.03	5.00	10.00	84.99	1000.01

## Appendix D

### Output to Unit 15 (File: OUT15)

TIME (HRS)	AL (0)	CR (0)	WMDOT	WMINT
.278E-03	.124E+02	.206E+02	.419E-03	.833E-03
.556E-03	.122E+02	.207E+02	.296E-03	.118E-02
.833E-03	.122E+02	.207E+02	.242E-03	.145E-02
.111E-02	.122E+02	.207E+02	.209E-03	.167E-02
.139E-02	.121E+02	.208E+02	.187E-03	.187E-02
.167E-02	.121E+02	.208E+02	.171E-03	.205E-02
.195E-02	.121E+02	.208E+02	.158E-03	.221E-02
.223E-02	.121E+02	.208E+02	.148E-03	.237E-02
.250E-02	.121E+02	.208E+02	.140E-03	.251E-02
.278E-02	.121E+02	.208E+02	.133E-03	.265E-02
.306E-02	.121E+02	.208E+02	.126E-03	.277E-02
.334E-02	.121E+02	.208E+02	.121E-03	.290E-02
.361E-02	.121E+02	.208E+02	.116E-03	.302E-02
.389E-02	.121E+02	.208E+02	.112E-03	.313E-02
.417E-02	.121E+02	.208E+02	.108E-03	.324E-02
.445E-02	.121E+02	.208E+02	.105E-03	.335E-02
.473E-02	.121E+02	.208E+02	.102E-03	.345E-02
.500E-02	.121E+02	.208E+02	.987E-04	.355E-02
.528E-02	.120E+02	.208E+02	.961E-04	.365E-02
.556E-02	.120E+02	.208E+02	.937E-04	.374E-02
.583E-02	.120E+02	.208E+02	.915E-04	.384E-02
.612E-02	.120E+02	.208E+02	.893E-04	.393E-02
.640E-02	.120E+02	.208E+02	.873E-04	.402E-02
.668E-02	.120E+02	.208E+02	.855E-04	.410E-02
.695E-02	.120E+02	.208E+02	.838E-04	.419E-02
.722E-02	.120E+02	.208E+02	.822E-04	.427E-02
.751E-02	.120E+02	.208E+02	.806E-04	.435E-02
.778E-02	.120E+02	.208E+02	.792E-04	.443E-02
.806E-02	.120E+02	.208E+02	.778E-04	.451E-02
.834E-02	.120E+02	.208E+02	.765E-04	.459E-02
.862E-02	.120E+02	.208E+02	.752E-04	.466E-02
.890E-02	.120E+02	.208E+02	.741E-04	.474E-02
.917E-02	.120E+02	.208E+02	.729E-04	.481E-02
.945E-02	.120E+02	.208E+02	.719E-04	.488E-02
.972E-02	.120E+02	.208E+02	.708E-04	.495E-02
.100E-01	.120E+02	.208E+02	.698E-04	.503E-02
.200E-01	.120E+02	.208E+02	.494E-04	.711E-02
.300E-01	.120E+02	.208E+02	.403E-04	.871E-02
.400E-01	.120E+02	.208E+02	.349E-04	.101E-01
.500E-01	.120E+02	.208E+02	.312E-04	.112E-01
.601E-01	.120E+02	.208E+02	.285E-04	.123E-01
.701E-01	.120E+02	.208E+02	.264E-04	.133E-01
.801E-01	.120E+02	.208E+02	.247E-04	.142E-01
.900E-01	.120E+02	.208E+02	.233E-04	.151E-01
.100E+00	.120E+02	.208E+02	.221E-04	.159E-01
.200E+00	.120E+02	.208E+02	.156E-04	.225E-01
.301E+00	.120E+02	.208E+02	.127E-04	.276E-01
.400E+00	.120E+02	.208E+02	.110E-04	.318E-01
.500E+00	.120E+02	.208E+02	.988E-05	.356E-01
.601E+00	.120E+02	.208E+02	.901E-05	.390E-01

.700E+00	.120E+02	.208E+02	.835E-05	.421E-01
.802E+00	.120E+02	.208E+02	.780E-05	.450E-01
.901E+00	.120E+02	.208E+02	.736E-05	.477E-01
.100E+01	.120E+02	.208E+02	.698E-05	.503E-01
.200E+01	.120E+02	.208E+02	.494E-05	.712E-01
.301E+01	.120E+02	.208E+02	.403E-05	.872E-01
.401E+01	.119E+02	.208E+02	.350E-05	.101E+00
.501E+01	.119E+02	.208E+02	.313E-05	.113E+00
.600E+01	.118E+02	.208E+02	.286E-05	.123E+00
.701E+01	.118E+02	.208E+02	.265E-05	.133E+00
.801E+01	.117E+02	.207E+02	.248E-05	.142E+00
.901E+01	.116E+02	.207E+02	.234E-05	.151E+00
.100E+02	.114E+02	.207E+02	.223E-05	.159E+00
.200E+02	.104E+02	.203E+02	.160E-05	.226E+00
.300E+02	.949E+01	.199E+02	.134E-05	.279E+00
.400E+02	.883E+01	.196E+02	.118E-05	.324E+00
.500E+02	.833E+01	.193E+02	.109E-05	.365E+00
.600E+02	.793E+01	.190E+02	.102E-05	.403E+00
.700E+02	.760E+01	.187E+02	.972E-06	.439E+00
.800E+02	.733E+01	.185E+02	.936E-06	.473E+00
.900E+02	.709E+01	.183E+02	.908E-06	.506E+00
.100E+03	.689E+01	.181E+02	.887E-06	.539E+00
.200E+03	.564E+01	.167E+02	.813E-06	.841E+00
.300E+03	.493E+01	.160E+02	.803E-06	.113E+01
.400E+03	.441E+01	.156E+02	.802E-06	.142E+01
.500E+03	.398E+01	.154E+02	.802E-06	.171E+01
.600E+03	.362E+01	.152E+02	.802E-06	.200E+01
.700E+03	.329E+01	.151E+02	.802E-06	.229E+01
.800E+03	.300E+01	.150E+02	.802E-06	.258E+01
.900E+03	.273E+01	.149E+02	.802E-06	.287E+01
.100E+04	.248E+01	.149E+02	.802E-06	.316E+01

## Appendix E

Variables used within the program, arranged alphabetically\*:

<u>Name</u>	<u>Description</u>
	For the following 20 parameters A for diffusion coefficients, see description for READ11 subroutine in paper.
A110, A111, A112, A113, A114, A115	Parameters for $D^{Al}$
A120, A121, A122, A123, A124, A125	Parameters for $D^{Al}$
A210, A211, A212, A213, A214, A215	Parameters for $D^{Al}$
A220, A221, A222, A223, A224, A225	Parameters for $D^{Cr}$
A0	Parameter relating weight of metal in oxide to weight of oxygen
A	Local interim variable**
AB	Local interim variable
A(6)	Local array used in Function DERF to calculate error function
ADIST	Array containing equally spaced node distance positions after spline fit
AGROW	Parameter for power law oxide growth model (see description for READ11 subroutine in paper.)
AJ	Single precision equivalent to index counter J
AL	Array containing current Al concentrations for all nodes (time=t)
AL2	Array containing new Al concentrations for all nodes (time=t+ $\Delta t$ )
ALX	Subroutine arguments for D_C representing Al concentration
ALCOAT	Al concentration of coating, in atomic percent
ALPHA	Parameter relating flux towards and away from moving interface (see Eq. 7)
ALSP	Array containing Al concentrations after spline interpolation
ALSUB	Al concentration of substrate, in atomic percent
ALWCOAT	Coating concentration in weight percent
ALY	Array containing spline parameters for Al (used in SPLINE subroutine)
AM	Real variable equivalent to M (used in SPLINE subroutine)
AN	Argument in Subroutine M_L giving the number of nodes in the diffusion zone
ANCOAT	Number of nodes in coating
ANSUB	Number of nodes in inner diffusion zone (coating/substrate inter-diffusion)
ARG1	Interim variable used to produce correct shift to concentrations to concentrations in Subroutine M_L
ASUBL	Lowest node number in inner (substrate) diffusion zone (ncoat+2) (see NSUBL)
AX2	Interim variable used to determine distance coordinates for inner diffusion zone when printing to units 6, 12 and 14 (for ZONE=2)
B	Interim variable
B0	Parameter to convert weight of oxygen in scale to total weight of oxide (metal+oxygen). $B0=A0+1.0$
BAC	Local interim variable used to calculate error function in FUNCTION DERF
BGROW	Parameter for power law oxide growth model (see description for READ11 subroutine in paper.)
C	Local interim variable
CD	Local interim variable
CR	Array containing current Cr concentrations for all nodes
CR2	Array containing new Cr concentrations for all nodes (time=t+ $\Delta t$ )
CRITAL	Critical surface Al concentration (at.%)

CRSP Array containing Cr concentrations after spline interpolation  
 CRX Subroutine arguments for D\_C representing Cr concentration  
 CRCOAT Cr concentration of coating, in atomic percent  
 CRSUB Cr concentration of substrate, in atomic percent  
 D Local interim variable  
 D11 Main diffusion coefficient for Al ( $D_{Al}$ )  
 D12 Cross term diffusion coefficient for Al ( $D_{AlAl}$ )  
 D21 Cross term diffusion coefficient for Cr ( $D_{AlCr}$ )  
 D22 Main diffusion coefficient for Cr ( $D_{Cr}$ )  
 D2ALDX Second order Al concentration derivative (i.e.,  $d^2C/dX^2$ )  
 D2CRDX Second order Cr concentration derivative (i.e.,  $d^2C/dX^2$ )  
 DAL Interim local variable  
 DALDX Al concentration gradient (i.e.,  $dC/dX$ )  
 DCR Interim local variable  
 DCRDX Cr concentration gradient (i.e.,  $dC/dX$ )  
 DD111 Derivative of  $D_{Al}$  (D11) diffusion coefficient wrt Al  
 DD112 Derivative of  $D_{AlAl}$  (D11) diffusion coefficient wrt Cr  
 DD121 Derivative of  $D_{AlAl}$  (D12) diffusion coefficient wrt Al  
 DD122 Derivative of  $D_{AlCr}$  (D12) diffusion coefficient wrt Cr  
 DD211 Derivative of  $D_{AlCr}$  (D21) diffusion coefficient wrt Al  
 DD212 Derivative of  $D_{CrAl}$  (D21) diffusion coefficient wrt Cr  
 DD221 Derivative of  $D_{CrAl}$  (D22) diffusion coefficient wrt Al  
 DD222 Derivative of  $D_{CrCr}$  (D22) diffusion coefficient wrt Cr  
 DELT Time increment used each iteration  
 DELT1 Time interval for outer diffusion zone based on Eq. 13  
 DELT2 Time interval for inner diffusion zone based on Eq. 13  
 DELX Argument in subroutines M\_L and FSL representing node spacing  
 DELX1 node spacing in outer diffusion zone (calculated)  
 DELX2 node spacing in inner diffusion zone (calculated)  
 DELXI1 Surface recession in time Delt  
 DELXMN Minimum node spacing between Delx1 and Delx2  
 DEL\_X Interim distance variable used in defining the initial concentration profile for the inner diffusion zone DENOM Interim variable used in the calculation of the weight percent of Al in the coating  
 DENSITY Density of the coating (gm/cm<sup>3</sup>)  
 DF Array used to "weight" spline representation (used in SPLINE subroutine)  
 DIST Array containing node distance positions before spline fit  
 DIST1 Width of outer diffusion zone measured from the original surface to node ncoat  
 DIST2 Width of inner diffusion zone from coating/substrate interface to node nsubl-1 (i.e., the width of the inner diffusion zone extending into the coating) Used with DIST2 and THICK to determine when the diffusion zones overlap in the coating  
 DIST\_SP Interim distance array containing node positions after spline interpolation  
 DMAX Maximum diffusion coefficient used to calculate minimum Delt  
 DXCOAT Initial width of the outer diffusion zone (microns)  
 DX Argument in Subroutine M\_L indicating the amount the zone is to be shifted  
 DXSUB Initial width of the inner diffusion zone (microns)  
 ERFC Complimentary error function calculated in FUNCTION DERF  
 ETA Parameter used in COSP oxide spalling program (Ref 3)

HALF Distance variable indicating the midpoint of the overlapping diffusion zones, used when printing to unit 12 (ZONE=2) in preparation for SPLINE curve fit  
 I Local index counter  
 IC Parameter used in SPLINE subroutine  
 ILOOP Loop iteration counter  
 INT Index for DO loop in Subroutine M\_L (values of 1 or -1)  
 IO15 Flag indicating output to file15 desired  
 ITIME Integer counter for times stored in array TMAX  
 J Local index counter  
 JAL Al flux in coating  
 JOX Al flux in the oxide at the oxide/coating interface  
 KP Parabolic oxide growth constant due to weight of oxygen on surface  
 M SPLINE subroutine argument equal to NCOAT+NSUB  
 MASSFACT Factor to convert units for rate of al consumption (mg/cm<sup>2</sup>/sec) to units compatible with Al flux in program(at%al\*cm/sec)  
 MAXDT Maximum time increment per iteration (minutes)  
 MAXLOOP Maximum iterations allowed before stopping the program  
 MODEL Parameter indicating oxide growth model  
 MW\_AL Molecular weight of Al  
 MW\_CR Molecular weight of Cr  
 MW\_NI Molecular weight of Ni  
 N Local index counter  
 N1 Argument in subroutines M\_L and FSL indicating node numbers  
 N2 Argument in subroutines M\_L and FSL indicating node numbers  
 NCOAT Number of nodes in the outer diffusion zone  
 NCOATH Highest node number in outer (coating) diffusion zone (ncoat-1)  
 NICOAT Ni concentration in the coating (at.%)  
 NN local index counter  
 NSUB number of nodes in the inner diffusion zone  
 NSUBH Highest node number in inner (substrate) diffusion zone (nsubl+nsub-1)  
 NSUBL Lowest node number in inner (substrate) diffusion zone (ncoat+2)  
 QO Spalling parameter used in COSP oxide spalling program (Ref 3)  
 RATIO Interim variable used to produce correct shift to concentrations to concentrations in Subroutine M\_L  
 SAL Single precision variable for AL  
 SAMPLE 80 character run id (1 of 3, for documentation only)  
 SCR Single precision variable for CR  
 SDELTA Single precision variable for DELTA  
 SDELTA1 Single precision variable for DELTA1  
 SDELTA2 Single precision variable for DELTA2  
 SDIST Single precision variable for DIST  
 SHR Single precision variable for HR  
 SNI Single precision variable for NI  
 STIME Single precision variable for TIME  
 SWMDOT Single precision variable for WMDOT  
 SWMINT Single precision variable for WMINT  
 SXI1 Single precision variable for XI1  
 T1 Interim time variable used in COSP oxide spalling model (Ref 3)  
 T2 Interim time variable used in COSP oxide spalling model (Ref 3)  
 T15 Intermediate time counter; time between writing to File15  
 TCYCLE Thermal cycle duration used in COSP oxide spalling program (Ref 3)

TEFF "Effective" time based on current oxide thickness used in COSP oxide spalling model (Ref 3)

TEMP Oxidation temperature (C) (for documentation only)

THICK Thickness of the coating (microns)

TIME Simulated total oxidation time (TIME=TIME+DELT)

TIME15 Time intervals between writing to File15

TITLE1 80 character run id (2 of 3, for documentation only)

TITLE2 80 character run id (3 of 3, for documentation only)

TMAX Array containing times for output to File6

TMID Intermediate time variable used in COSP oxide spalling model (Ref 3)

TOTWM Total weight of metal consumed predicted by COSP oxide spalling program (Ref 3)

TOTWO Total weight of oxygen in the oxide (Ref 3)

TOTWR Total weight of oxide retained on surface, predicted by COSP oxide spalling program (Ref 3)

TSUM Intermediate time counter giving elapsed time between thermal cycles (sec)

VBAR Partial molar volume of Al in coating (cm<sup>3</sup>/mole)

WM Weight of metal (Al) in oxide used in COSP oxide spalling model (Ref 3)

WMDOT Rate of metal consumption predicted by COSP oxide spalling program (Ref 3)

WMINT Total accumulated weight of metal consumption (mg/cm<sup>2</sup>)

WO Weight of oxygen in oxide used in COSP oxide spalling model (Ref 3)

WO1 Weight of oxygen in oxide at time T1 used in COSP oxide spalling model (Ref 3)

WO2 Weight of oxygen in oxide at time T2 used in COSP oxide spalling model (Ref 3)

WR Weight of oxide retained on surface used in COSP oxide spalling model (Ref 3)

WS Weight of oxide spall following a thermal cycle used in COSP oxide spalling model (Ref 3)

X Interim distance variable used in printing node positions

XI1 Position of outer surface (i.e., amount of surface recession, cm)

YCR Array containing spline parameters for Cr (used in SPLINE subroutine)

ZONE Indicator for number of diffusion zones (zone=2 for inner and outer diffusion zones, zone=1 after SPLINE subroutine creating single combined diffusion zone)

\* Variables associated with Spline subroutines EVALUAT and CUBIC are not included. See Ref 11.

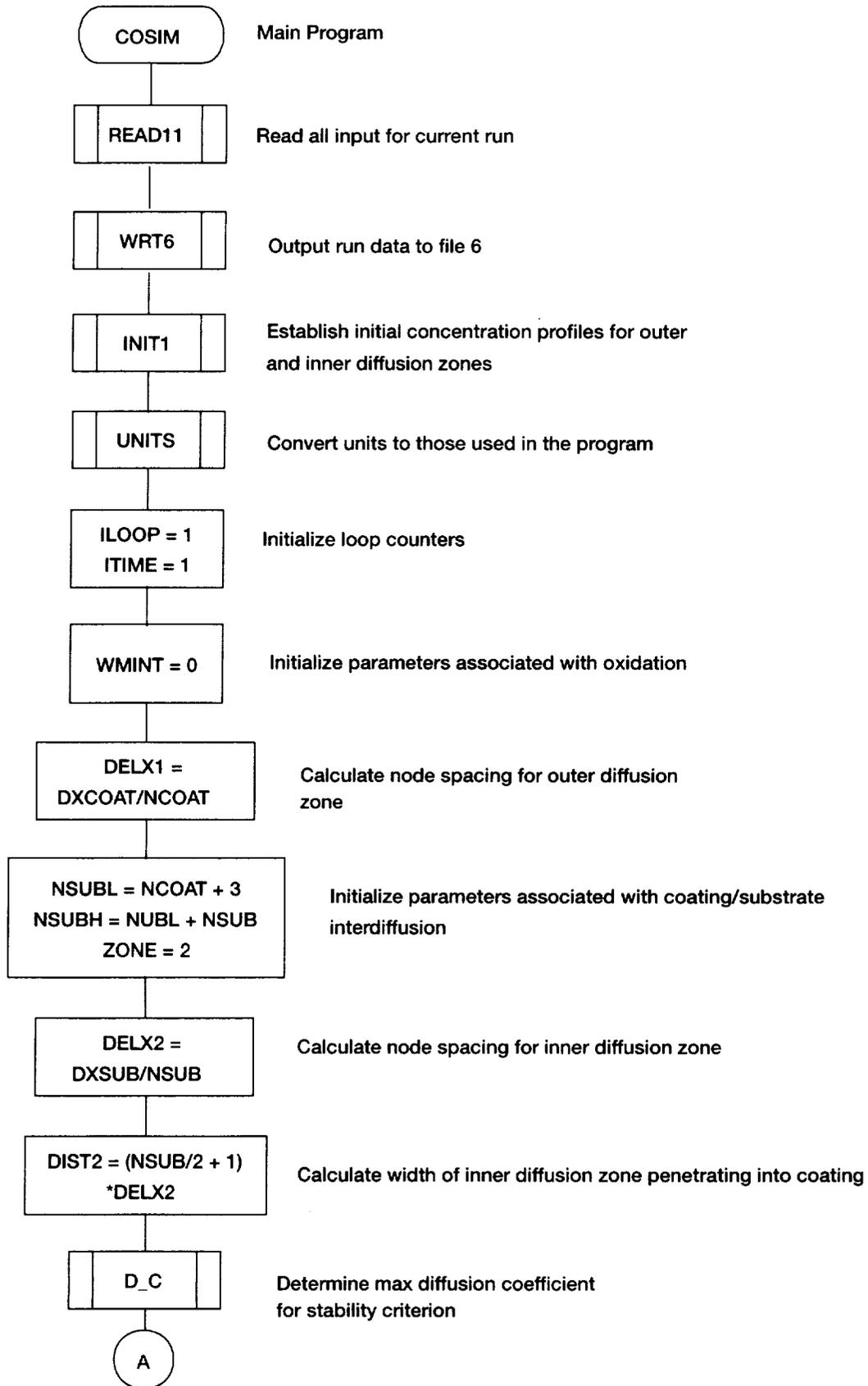
\*\* Local interim variables are used in intermediate steps in an algebraic equation, such as:

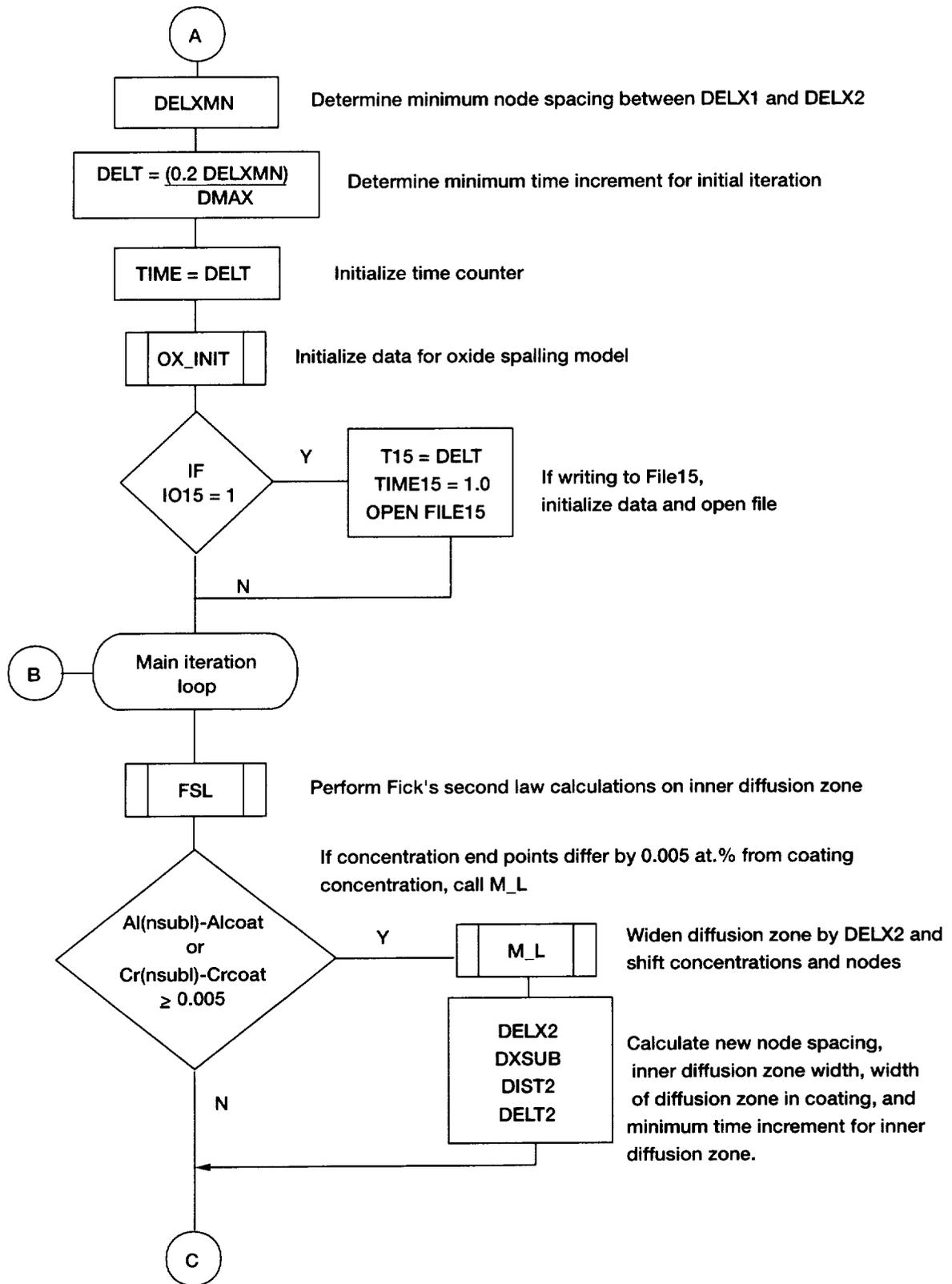
$$\begin{aligned}
 A &= DD111 * DALDX + DD112 * DCRDX \\
 B &= DD121 * DALDX + DD122 * DCRDX \\
 AB &= A * DALDX + D11 * D2ALDX + B * DCRDX + D12 * D2CRDX \\
 AL2(J) &= AL(J) + DELT * AB
 \end{aligned}$$

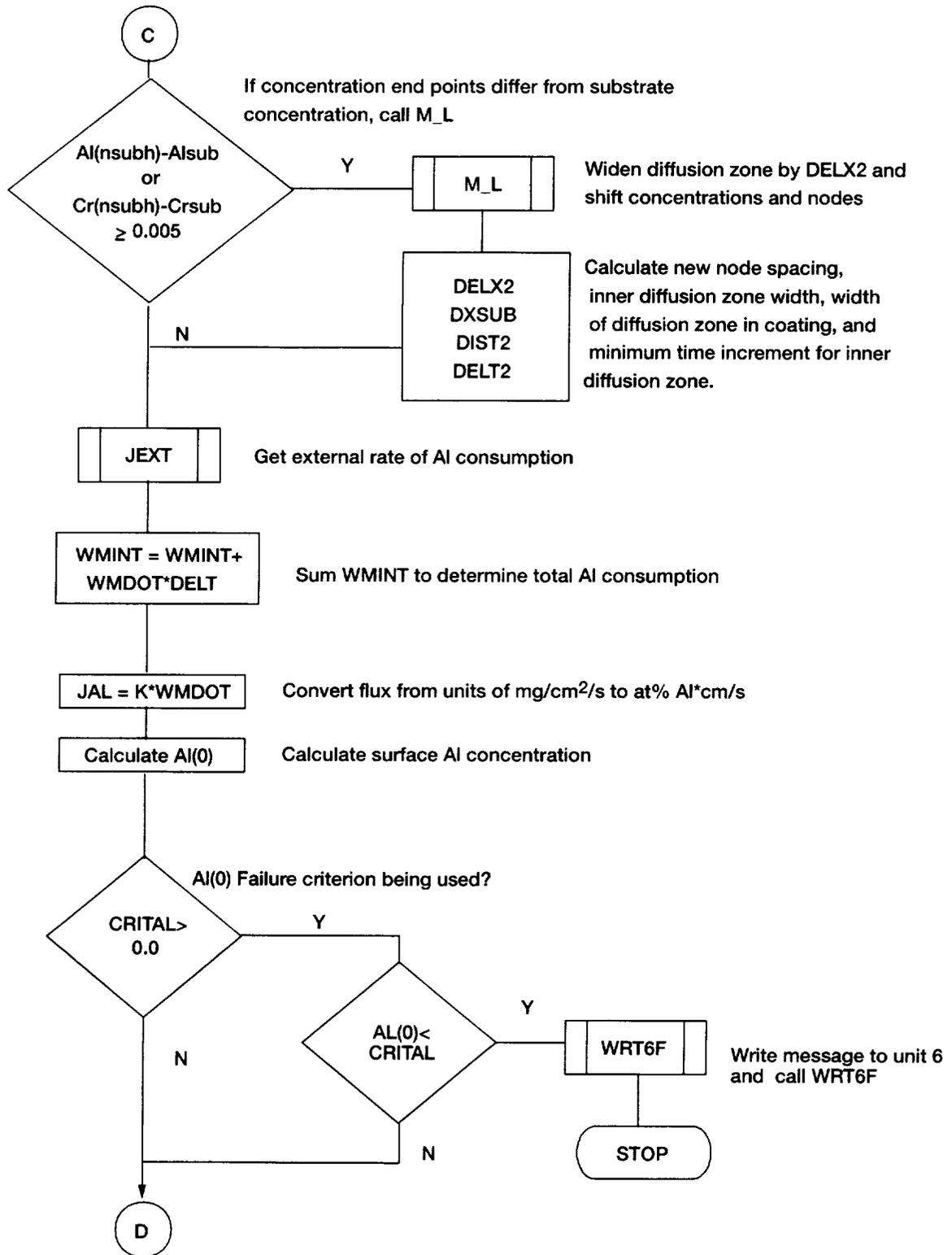
where the calculations for A, B, and AB are intermediate steps toward calculating the new Al concentration AL2(J).

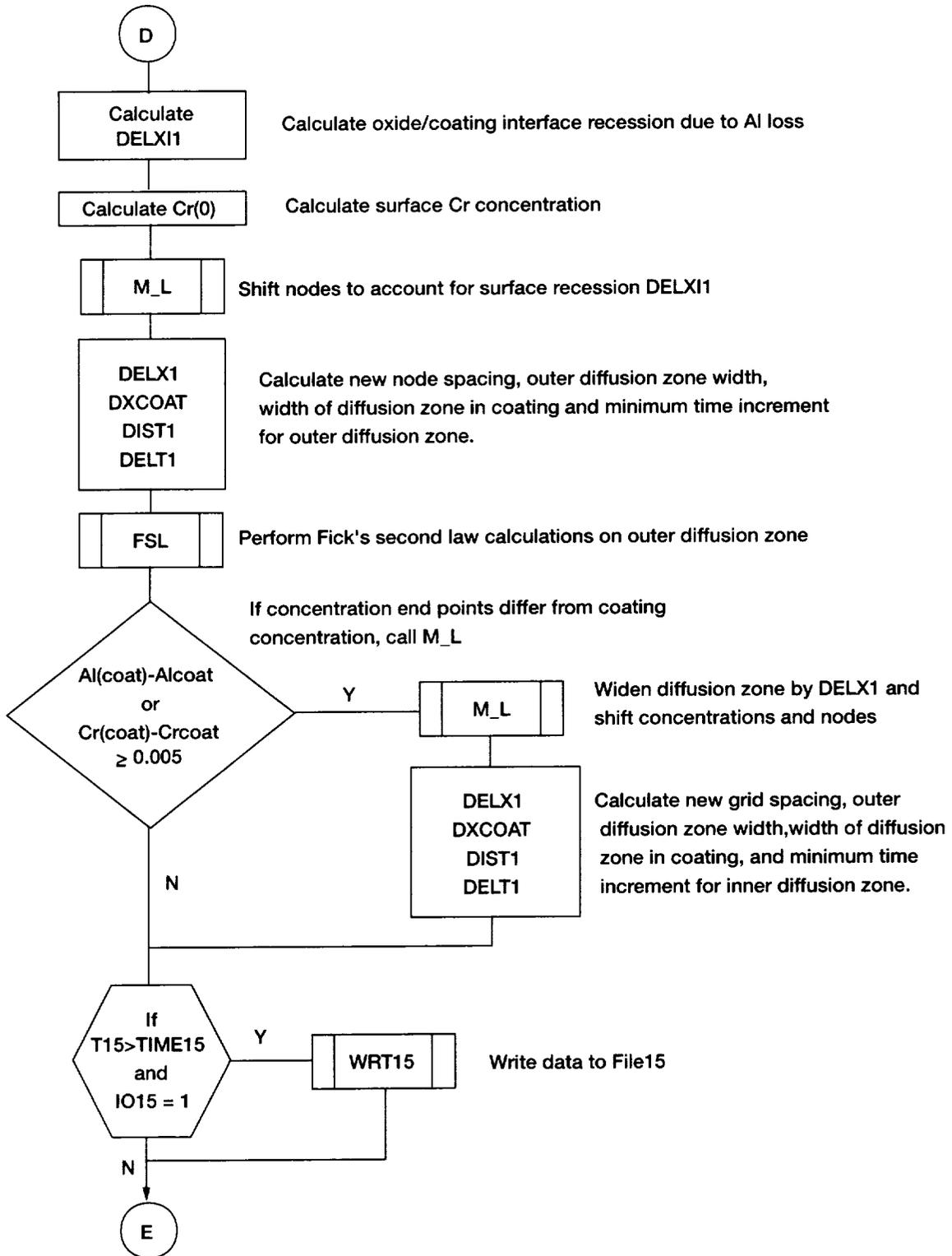
# Appendix F

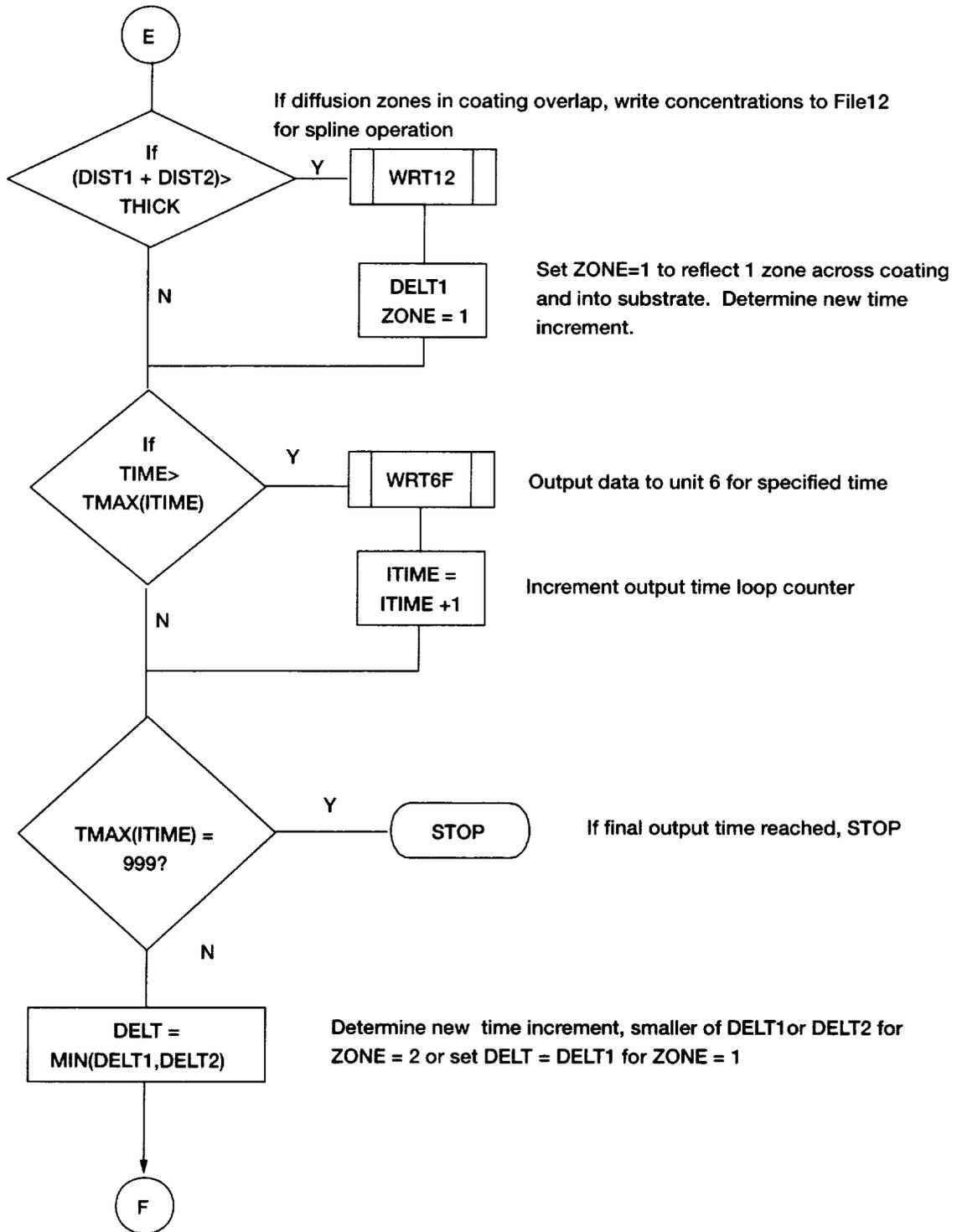
Flow charts.

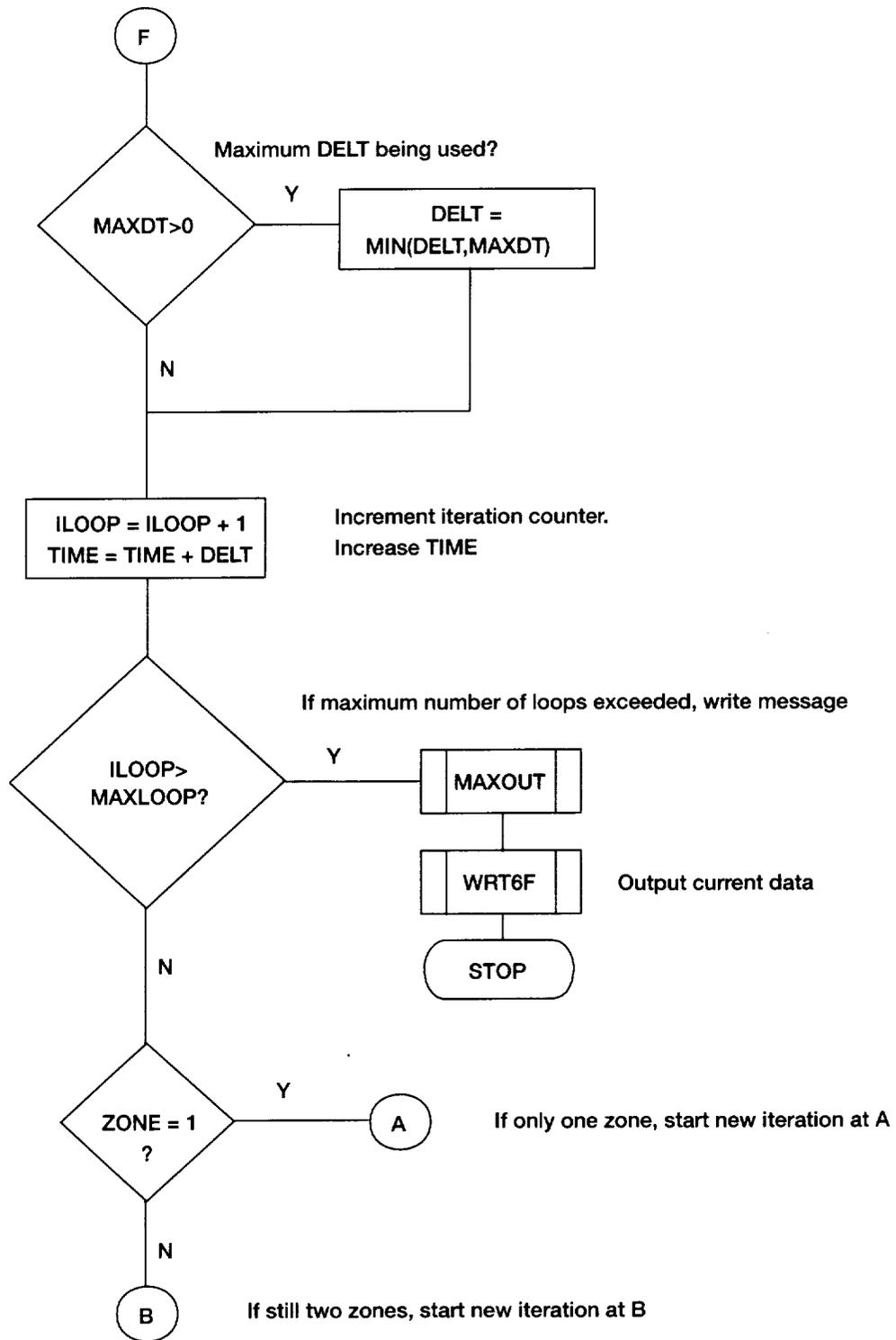


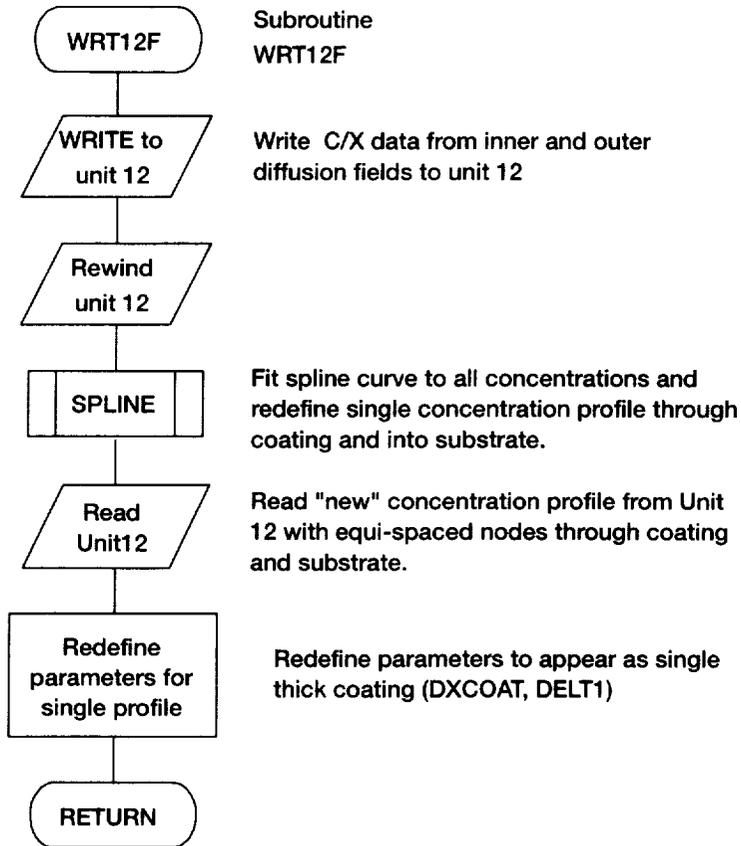


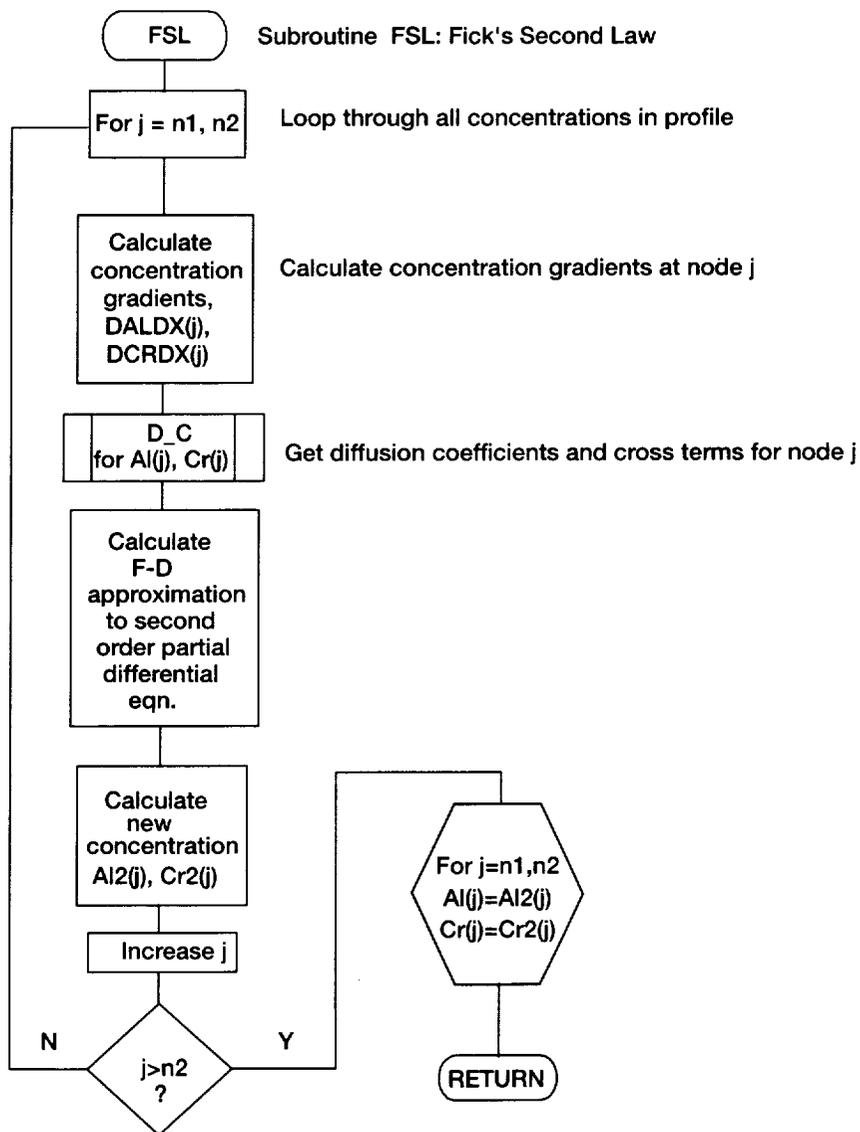


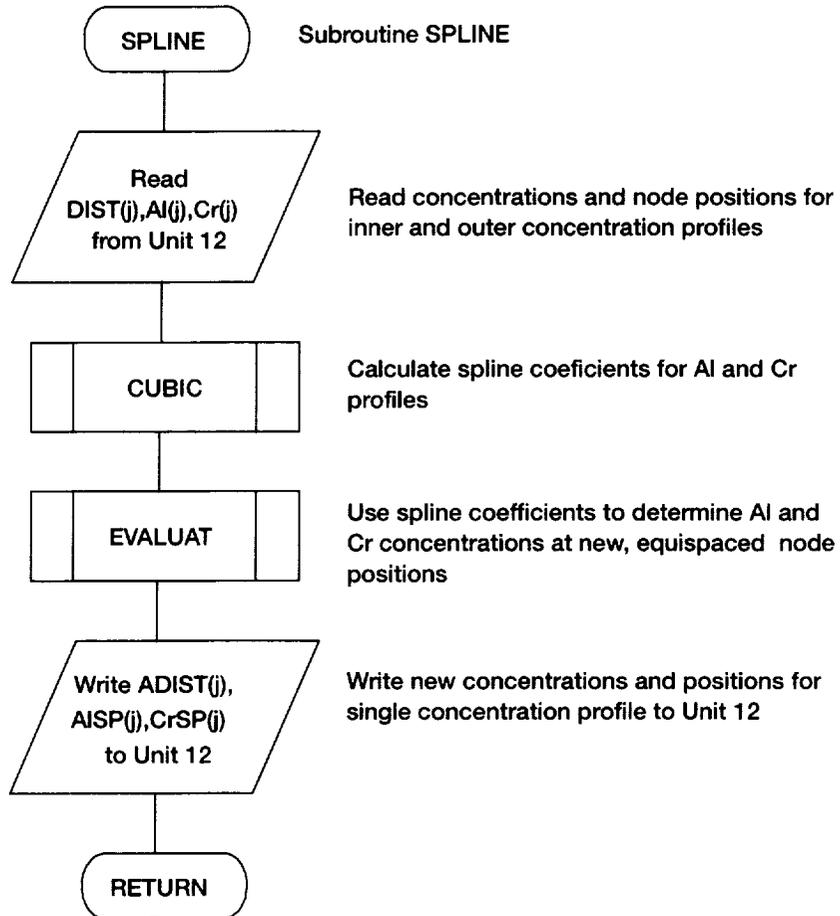














# REPORT DOCUMENTATION PAGE

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<b>13. ABSTRACT</b> ( <i>Maximum 200 words</i> )  A finite-difference computer program (COSIM) has been written which models the one-dimensional, diffusional transport associated with high-temperature oxidation and interdiffusion of overlay-coated substrates. The program predicts concentration profiles for up to three elements in the coating and substrate after various oxidation exposures. Surface recession due to solute loss is also predicted. Ternary cross terms and concentration-dependent diffusion coefficients are taken into account. The program also incorporates a previously-developed oxide growth and spalling model to simulate either isothermal or cyclic oxidation exposures. In addition to predicting concentration profiles after various oxidation exposures, the program can also be used to predict coating life based on a concentration dependent failure criterion (e.g., surface solute content drops to 2%). The computer code, written in an extension of FORTRAN 77, employs numerous subroutines to make the program flexible and easily modifiable to other coating oxidation problems.				
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