Influence of Tooth Spacing Error on Gears With and Without Profile Modifications

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A computer simulation was conducted to investigate the effectiveness of profile modification for reducing dynamic loads in gears with different tooth spacing errors. The simulation examined varying amplitudes of spacing error and differences in the span of teeth over which the error occurs. The modification considered included both linear and parabolic tip relief. The analysis considered spacing error that varies around most of the gear circumference (similar to a typical sinusoidal error pattern) as well as a shorter span of spacing errors that occurs on only a few teeth. The dynamic analysis was performed using a revised version of a NASA gear dynamics code, modified to add tooth spacing errors to the analysis.

Results obtained from the investigation show that linear tip relief is more effective in reducing dynamic loads on gears with small spacing errors but parabolic tip relief becomes more effective as the amplitude of spacing error increases. In addition, the parabolic modification is more effective for the more severe error case where the error is spread over a longer span of teeth. The findings of this study can be used to design robust tooth profile modification for improving dynamic performance of gear sets with different tooth spacing errors.

Keywords: Gears, Spur gears, Dynamic loads, Spacing errors, Profile modification

INTRODUCTION

Errors are inherent in gears. They are called inherent because they are basic, cannot be entirely avoided, and are therefore present to some extent in all gears [1]. Tooth spacing errors (also called pitch errors), one of the basic inherent gear errors, are important because they affect transmission velocity, and introduce vibration and noise. These effects are often appreciably more significant than the actual resulting speed variation, though whether fluctuations in velocity transmission is important largely depends upon the application of the gears. For gears running at high speeds minimizing the pitch errors is essential so that noise and dynamic loading effects are kept within acceptable limits.

Many researchers have studied the effect of pitch error on gear dynamic behavior. Houser and Seireg [2,3] conducted experimental and analytical investigations to determine dynamic factors in spur and helical gears with the combined influence of profile modification and pitch errors of different magnitudes. Their studies were limited to gear systems operated at nonresonant speed conditions. A semi-empirical formula was developed for determining the dynamic factor. Umezawa et al. [4] examined rotational vibration of gears with pitch errors on every other tooth and also on every third tooth. Their results show that gears with an integer contact ratio were little affected by pitch error. They also found that low-contact-ratio gears are more sensitive to pitch error than high-contact-ratio gears. Rakhit [5] reported the significance of the form of the pitch error variation on the vibrations of an epicyclic gear reducer of a turbo-generator. Pitch error curves showing more peaks and valleys created higher vibration than did smoother pitch error curves. Velex et al. [6] performed dynamic analysis of gears with conventional (linear) profile modification for a specific amount of pitch error. They show that low contact ratio gears are much more sensitive to pitch errors than high contact ratio gears.

Profile modification has been recognized as an effective way to reduce gear dynamic load. However, the influence of profile modification on gears with different spacing errors has not been thoroughly investigated. The distribution and cumulative amount of tooth spacing error varies from gear to gear. Their effects on the dynamics of gears with different types of profile modifications (linear and parabolic) are examined in this work.

In a typical gear spacing error chart, the error curve has an approximately sinusoidal shape with a span covering most of the teeth and the error ranges from positive to negative values. It is difficult to establish firmly the sources and causes of position error because of their small magnitude. Except for runout errors of the gear blank, generating tool, work arbor, and tool arbor, the other error sources are random contributions associated with vibration (including tool chatter), material deflection, tool imperfections, and the like. In our study, we considered a simplified version of the typical case in which the sinusoidal distribution is approximated by a triangular pattern that extends over most of the teeth. We call this a "long span case." This case could represent a typical tooth spacing error pattern of gears after the hobbing process. We also considered another case that has the same magnitude of error but distributed over a significantly shorter span. This case could represent a distinctive tooth spacing error pattern of gears after index grinding. These two cases cause different dynamic excitations to the gear system. The influence of the tooth spacing error magnitude was evaluated based on
the tolerance for precision gears. Results from the study show how the dynamic load factor is affected by different amounts and distributions of tooth spacing error for gears with and without profile modification.

**THEORY**

**Tooth Spacing Error**

A geometrically ideal gear has identical involute profile teeth that are equally spaced around the circumference. For a gear with z teeth, any reference circle of the gear is intersected by right-hand and left-hand sets of z tooth profiles at exactly equal angular increments. If a particular elemental error, the pitch error, is now introduced, then with respect to one selected (reference) profile, any other tooth profile may be displaced from its theoretical position. The angular displacement of a profile on the reference circle is converted to the arc distance around that circle, and this distance is the pitch error of that profile. The gear pitch errors are the sequence of z values of these displacements. There are two sets of pitch errors, one each for the left and right hand tooth profiles.

Conventional pitch error testers actually determine the tooth-to-tooth adjacent pitch error. This is measured as the deviation of the actual chordal distance existing between reference points at the reference radius on successive tooth profiles from the theoretical value. The cumulative pitch error is obtained as the sequence formed by algebraically summing the individual adjacent pitch error values, starting at an arbitrary tooth. The resulting sequence of z values starts at and should return to zero; though measurement difficulties sometimes result in a nonzero residual. The cumulative pitch error sequence is often adjusted to eliminate this residual.

The cumulative tooth spacing error typically has a sine wave distribution, spread over a certain span of gear teeth [7]. The magnitude of cumulative tooth spacing error for precision cut gears, lies in the range of 0.00005 in. (constrained by the least count of the measuring instrument) to 0.0004 in. [8,9]. For this study, we define two cases of spacing error distribution: "Long" spacing errors extend over most of the teeth, and "Short" errors extend over just a few teeth, closely resemble the tooth spacing error distribution after index grinding.

The study considers spacing error magnitudes varying between 0.00005 and 0.0002 in. This range is typical for high-precision gears. For this study, all of the spacing error is assumed to be on the driving gear. However, gear pairs with tooth spacing error on both the driver and the driven gears can be analyzed by similar methods. The total tooth spacing error $E_s$ of a gear pair is the difference of the tooth spacing error of the driver, $E_{s1}$, and the driven gears, $E_{s2}$, which is given mathematically as,

$$E_s = E_{s1} - E_{s2}$$  \hspace{1cm} (1)

Hence, the effect of tooth spacing error of the driven gear on the gear transmission error of the gear pair is equivalent to the effect of the same amount of tooth spacing error but with an opposite sign, on the corresponding tooth of the driver. The NASA gear dynamics code DANST [10] was used to determine gear transmission error. DANST incorporates the effect of extended tooth contact due to tooth flexibility in the transmission error calculation [11]. The influence on transmission error due to combined effects of corner contact (when tooth pairs come into mesh), profile modifications, and pitch errors are all considered in the analysis. Derivation of the equations for transmission error and meshing stiffness for the gear pair can be found in Ref. 11.

Profile modification can be an effective tool in reducing gear dynamic load. In previous work [12,13], we have described three different types of profile modifications that are simulated in our analysis. These are designated linear, (parabolic-I, and parabolic-II). For parabolic-I modification, the trace on a profile chart has a zero slope (tangent to involute) at the start of modification. For parabolic-II modification, the trace has infinite slope (vertical) at the tooth tip. The three types of modification are shown in Fig. 1. The object of this study was to find the most effective profile modifications for reducing the dynamic load in low-contact-ratio spur gears with tooth spacing errors. For all types of profile modification, the modification starts at the highest point of single tooth contact. The amount of modification at the tip of the teeth considers both the largest value of spacing error simulated in this study and deflection due to loading applied at the highest point of single tooth contact of the mating gear teeth.

**Gear Dynamic Load**

The dynamic load calculation is based on the NASA gear dynamics code DANST. This code has been validated with experimental data for high-accuracy gears at NASA Glenn Research Center [10]. DANST considers the influence of gear inertia, meshing stiffness of gear teeth, tooth profile modification, and system natural frequencies in its dynamic calculations. The analytical model of DANST employs four torsional degrees of freedom to represent a typical gear transmission. The model includes driving pinion and driven gear, connecting shafts, motor, and load. The equations of motion were derived from basic gear geometry and elementary vibration principles.

In this study, we consider two sample low-contact-ratio spur gear sets, the first having an equal number of teeth, 28/28 and the other with an unequal number of teeth, 20/36. The number of tooth pairs $N$ involved in the gear mesh cycle that must be considered for dynamic analysis depends on the numbers of teeth in the gears. For a gear pair with equal number of teeth on the driver and driven gears, $N$ is equal to the number of teeth on one gear. And for a gear pair with unequal numbers of teeth $N_1$ and $N_2$ on the driver and driven gears respectively, $N$ is equal to the hunting tooth period which is determined from the least common factors of $N_1$ and $N_2$. For example, with a combination of 20/36 teeth the factors are 4*5 and 4*9. The hunting period is thus 5 revolutions of the 36-tooth gear (180 teeth) or 9 revolutions of the 20-tooth pinion (also 180 teeth).
The dynamics of gear systems can be influenced considerably by the stiffness of the meshing gear teeth. A principal excitation for gear dynamics and vibration is the variation of this stiffness caused by teeth entering and leaving mesh. This meshing stiffness variation is related to the time-varying component of gear transmission error. The meshing stiffness variation due to such effects as pitch error, profile modification, and extended tooth contact can be determined and used as excitation input to the system equations of motion. The equations of motion can then be solved to determine the dynamic response of the gear system.

After the gear dynamic load is found, the dynamic load factor can be determined as the ratio of the maximum dynamic load to the applied load. The applied load equals the torque divided by the base circle radius. This ratio indicates the maximum relative instantaneous gear tooth load. Note: some researchers define the dynamic load factor (sometimes called dynamic factor) differently as the maximum dynamic load divided by the maximum static load.

RESULTS AND DISCUSSIONS

The effect of tooth spacing error on dynamic load factor was investigated for two sample gear sets, one with 28 teeth on both gears (28/28 teeth) and the other with 20/36 teeth. Both gear sets have a diametral pitch of 8 and a pressure angle of 20°. Table 1 shows the more detailed data for the two gear sets used in the following studies.

### Low Contact Ratio Gears with 28/28 Teeth

We consider a sample gear set with equal (28/28) numbers of teeth, solid gear bodies, diametral pitch of 8.0, face width of 0.25 in., with applied torque of 500 lb-in. (equivalent tooth load 1220 lb/in.) and at speeds ranging from 1000 to 9000 rpm. This speed range includes the critical speed of the gear system calculated from the given parameters. The gear pair has a theoretical contact ratio of 1.638.

The dynamic analysis was performed on gear pairs with the two different cases of tooth spacing error distributions on the driver. We considered maximum cumulative tooth spacing errors of 0.0005, 0.0001, 0.00015, and 0.0002 in., respectively. The driven gear is taken to be perfect with no tooth spacing error. The span of spacing error distribution is 4 teeth for the “short” case, and 16 teeth for the “long” case. Figure 2 shows these two cases for an example of maximum cumulative error of 0.0001 in.

### Influence of Profile Modification

The dynamic load factors for the 28/28 teeth gear set with the “short” cumulative tooth spacing error distribution and different profile modifications (linear, parabolic-I, and parabolic-II) are shown in Fig. 3(a). The amount of maximum cumulative spacing error over the entire 4-tooth span varies between 0 and 0.0002 in. From the figure, the following points can be observed:

![Figure 2.—Long and short types of tooth spacing error distributions on the driver of a sample 28/28 gear set.](image-url)
Low Contact Ratio Gears with 20/36 Teeth

For this section, we consider a sample gear set with 20 and 36 teeth and solid gear bodies. The diametral pitch is 8.0, face width is 0.25 in., the applied torque is 500 lb-in. (equivalent tooth load 1700 lb/in.) and at speeds ranging from 1000 to 9000 rpm. The gear pair has a theoretical contact ratio of 1.579. Different types of profile modifications (linear, parabolic-I, and parabolic-II) were considered for the investigation.

The dynamic analysis was again conducted on the gears considering the different tooth spacing error distributions ("short" and "long") on the driver, with maximum cumulative tooth spacing errors of 0.00005, 0.0001, 0.00015, and 0.0002 in. The driven gear is taken to be perfect with no tooth spacing error. The spacing error distributions are similar to those shown in Fig. 2 except, in this case, the pinion has only 20 teeth.

Influence of Profile Modification

The dynamic load factors of 20/36 tooth gear sets with the "short" tooth spacing error distribution and various profile modifications (no modification, linear, parabolic-I, and parabolic-II) are shown in Fig. 4. From the figure, the following points can be observed:

1. Profile modification can significantly decrease the dynamic load factor of a gear set with any error amplitude.
2. The gear set with linear profile modification has the least dynamic load factor for most of the error amplitudes studied (between 0 and 0.00017 in.).
3. The slope of the dynamic load factor vs. spacing error curve for a gear set with linear modification is steeper than that for gears with parabolic profile modification.
4. The gear set with parabolic-II modification has lower dynamic load factor compared with the gear set with parabolic-I modification, and the slopes of the curves in both cases are similar (the curves are parallel to each other).

Similar variations in dynamic load factor can be observed in the cases of gear sets with long tooth spacing error distribution as shown in Fig. 3(b). The above observations imply that linear modification is better for gear sets with little or no tooth spacing error, and parabolic modification is better for gears with a substantial amount of tooth spacing error.
Similar variations in dynamic load factor can be observed in the cases of gear sets with “long” cumulative tooth spacing error distributions. Just as observed above, linear modification is better for reducing dynamic loads in gear sets with little tooth spacing error, and parabolic modification is better in the case of gear sets with a substantial amount of tooth spacing error.

**Effect of Tooth Spacing Error on a Speed Survey of Dynamic Load Factor**

The variation of dynamic load factor with speed for the sample gears with 28/28 teeth and a maximum cumulative tooth spacing error of 0.0001 in. was investigated over a speed range of 1000 to 9000 rpm. The peak values of dynamic load factor for different profile modifications and spans of tooth spacing error are summarized in Table 2. To illustrate the dynamic effect, a plot showing the static and dynamic tooth loads for one of the cases listed in Table 2 is displayed in Fig. 5. In this figure the gears have linear tip modification and the short spacing error distribution. The gear set operates at a speed of 7500 rpm (near the critical speed of 7429 rpm) and the maximum dynamic load occurs at tooth number 7 with a magnitude of 413 lbs versus a static load of 304 lbs.

For illustrative purpose, contour diagrams for two of the cases from Table 2 are presented in Fig. 6. These diagrams show how the dynamic load factor varies with both the speed of the gears and the meshing tooth. They can help determine whether positive or negative pitch error produces higher dynamic effect. Figure 6(a) displays the dynamic load factor for gears with a short span of spacing error and linear profile modification. Figure 6(b) illustrates the long span of spacing error and no modification. Six more figures similar to these two were generated for all cases described in Table 2. The following observations were found from analyzing these eight contour diagrams.

1. The peak value of dynamic load factor occurs at speeds near the highest natural frequency or critical speed.
2. The peak value of dynamic load factor for gear sets with linear profile modification occurs on the tooth having maximum negative cumulative tooth spacing error or on the tooth after it, depending on the span of the errors. The peak value occurs on the tooth having maximum negative cumulative tooth spacing error for the “short” error distribution but is shifted to the following tooth for the “long” error distribution. The peak value of dynamic load factor for gear sets with no profile modification occurs on the tooth before the tooth having maximum positive cumulative tooth spacing error for all the error distributions considered.

<table>
<thead>
<tr>
<th>Cumulative tooth spacing error distribution</th>
<th>Max. pos. CTSE is on tooth number</th>
<th>Max. neg. CTSE is on tooth number</th>
<th>Tooth modification</th>
<th>Highest critical speed</th>
<th>Peak dyn. load factor</th>
<th>Speed at which Max. DLF occurs</th>
<th>Teeth on which Max. DLF occurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>“Short”</td>
<td>5</td>
<td>7</td>
<td>no-modif.</td>
<td>9594.20</td>
<td>1.931</td>
<td>8500.00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>linear</td>
<td>7429.70</td>
<td>1.358</td>
<td>7500.00</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>parabolic-I</td>
<td>7977.20</td>
<td>1.513</td>
<td>8000.00</td>
<td>5,7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>parabolic-II</td>
<td>7934.00</td>
<td>1.353</td>
<td>7500.00, 8000.00</td>
<td>5</td>
</tr>
<tr>
<td>“Long”</td>
<td>5</td>
<td>12</td>
<td>no-modif.</td>
<td>9607.00</td>
<td>1.973</td>
<td>8500.00</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>linear</td>
<td>7436.30</td>
<td>1.460</td>
<td>8000.00</td>
<td>13,14</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>parabolic-I</td>
<td>7985.10</td>
<td>1.605</td>
<td>7500.00</td>
<td>5,6</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>parabolic-II</td>
<td>7940.7</td>
<td>1.462</td>
<td>7500.00</td>
<td>5,6</td>
</tr>
</tbody>
</table>

Figure 5.—Static and dynamic (7500 rpm) loads for 28/28 tooth gears with short span of tooth spacing error, linear tip relief.
Figure 6.—Contour diagrams for the dynamic factor of sample 28/28 gear set. (a) Short span of spacing errors and linear modification. (b) Long span of spacing errors and no modification.
The peak value of dynamic load factor for gear sets with parabolic-I and parabolic-II profile modifications occurs on the tooth having maximum positive cumulative spacing error or the tooth following, for all types of tooth spacing error distribution.

3. We observed minor dynamic load factor peaks for speeds of 1/2, 1/4, and 1/8 of the maximum critical speed. These peak loads occur on the tooth having the greatest spacing error. The size of these peaks generally decreases at lower speeds.

SUMMARY AND CONCLUSIONS

A computer simulation was conducted to investigate how various types of profile modification can be used to minimize the dynamic load of gears having tooth spacing errors. The errors were simulated as triangular error distributions extending over different spans of teeth. The following conclusions were obtained:

1. Linear profile modification is more effective for minimizing spur gear dynamic load for gears with small tooth spacing errors.
2. Parabolic profile modification is best for minimizing spur gear dynamic load on gears where tooth spacing errors are larger or where the error is spread over a longer span of teeth.
3. Gears with parabolic-II profile modification have a lower dynamic load factor than similar gears with parabolic-I profile modification.
4. The dynamic load factor of spur gears with linear profile modification increases exponentially with an increase in the cumulative tooth spacing error while the dynamic load factor of similar spur gears with parabolic profile modification increases linearly.
5. The peak value of dynamic load factor occurs either on the tooth having maximum cumulative tooth spacing error (either positive or negative error) or the adjacent tooth, depending on the type of profile modification.

REFERENCES

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