ULTRASOUND ALGORITHM DERIVATION FOR SOIL MOISTURE CONTENT ESTIMATION

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ABSTRACT

Soil moisture content can be estimated by evaluating the velocity at which sound waves travel through a known volume of solid material. This research involved the development of three soil algorithms relating the moisture content to the velocity at which sound waves moved through dry and moist media. Pressure and shear wave propagation equations were used in conjunction with soil property descriptions to derive algorithms appropriate for describing the effects of moisture content variation on the velocity of sound waves in soils with and without complete soil pore water volumes. An elementary algorithm was used to estimate soil moisture contents ranging from 0.08 g/g to 0.5 g/g from sound wave velocities ranging from 526 m/s to 664 m/s. Secondary algorithms were also used to estimate soil moisture content from sound wave velocities through soils with pores that were filled predominantly with air or water.

INTRODUCTION

Ultrasound methods involve the mechanical vibration and propagation of waves above about 20,000 cycles/s through various materials (Dull et al., 1964). The velocity of these propagating waves is affected by the nature of the material through which it is passing. Curtis (1982), Szilard (1982), and Kinsler et al. (1982) describe several equations and input parameters used in estimating the velocity of sound waves traveling through different media. Sound waves travel through air and water at velocities of approximately 330 m/s and 1660 m/s, respectively. These waves travel through solid materials such as aluminum and steel at velocities of 6,300 m/s and 6,100 m/s, respectively. They also travel through porous materials such as concrete, ice, and cork at velocities of 3,100 m/s, 3,200 m/s, and 500 m/s, respectively (Kinsler et al., 1982). Kinsler et al. (1982) also reported a number of other solid material properties associated with sound wave velocities including density (kg/m³), Young’s Modulus (Pa), Shear Modulus (Pa), Adiabatic Bulk Modulus (Pa), Poisson’s Ratio, and Characteristic Impedance (Pa*s/m).

In-situ soils have some properties that are similar and different than those of many materials classically evaluated using ultrasonic sound wave velocity measurements. Soils are quite heterogeneous and the classically evaluated solid materials are homogeneous (Brady, 1990; Szilard, 1982). Soils can be compacted to a greater degree than materials such as steel and iron and have porosities around the range of materials such as cork and oak wood. Moisture content variation in soils occurs differently than in solid materials such as steel and iron, or glass and quartz. Texture, organic matter content, bulk density and porosity are major contributors to the soil water holding capacity (Brady, 1990). Additional contributions may be made by the presence of soil microbes, rocks, and other chemical and physical parameters. The purpose of this study is to perform initial investigations of the derivation of elementary and secondary ultrasound algorithms useful in evaluating soil moisture content variation. The study will attempt to incorporate soil moisture content, porosity, organic matter content, bulk density, and texture into existing equations that are used to estimate the velocity at which ultrasonic waves propagate through soils with pores that are filled predominantly with air or water.

ALGORITHMS

Sound waves moving through media with no boundary effects can be described by the sound velocity, c, which can be described by shear and pressure waves (Szilard, 1982).

\[ C_{\text{shear}} = \sqrt{\frac{E}{\rho (1 + \nu)}} = \frac{G}{\rho} \]  

(1)
and
\[ C_{\text{pressure}} = \sqrt{\frac{E}{\rho} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} \]  

where velocity is determined by the density, \( \rho \), the modulus or elasticity, \( E \), and Poisson’s ratio, \( \sigma \), and \( G \) is the modulus of rigidity (also called the shear modulus or torsional modulus). The density of a given volume of solid material is described similarly to the density of a given volume of soil, \( \rho_B \); density of soils is equal to the mass of the dry soil, \( M_S \), divided by the entire or bulk volume occupied by the material as shown in Equation (3)

\[ \rho_B = \frac{M_S}{V_{\text{TOTAL}}} \]  

where \( M_S \) is the soil mass and \( V_{\text{TOTAL}} \) is the total volume of a given soil sample. The total velocity, \( C_{\text{TOTAL}} \), of sound waves through a medium may be described as

\[ C_{\text{TOTAL}} = C_{\text{press}} + C_{\text{shear}} \]  

Therefore,

\[ C_{\text{TOTAL}} = \sqrt{\frac{E}{\rho} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} + \sqrt{\frac{G}{\rho}} \]  

Equation (5) suggests that the velocity of sound waves moving through solid media decreases with positive increases in \( \rho \).

Soil moisture content, \( \theta \), is defined as

\[ \theta = \frac{M_w}{M_S} \quad \text{and} \quad M_S = \frac{M_w}{\theta} \]  

Substituting Equation (6) into Equation (3) results in

\[ \rho_B = \frac{M_w}{\theta_m V_{\text{TOTAL}}} \]  

Substituting Equation (7) into Equation (5) results in

\[ C_{\text{TOTAL}} = \sqrt{\frac{E\theta_m V_{\text{TOTAL}}}{M_w} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} + \sqrt{\frac{G\theta_m V_{\text{TOTAL}}}{M_w}} \]  

\( C_{\text{TOTAL}} \) can be further defined as

\[ C_T = \sqrt{\frac{E\theta_m}{M_w} \alpha} + \sqrt{\theta_m \beta} \]  

where

\[ \alpha = \frac{V_{\text{TOTAL}}}{(1 - \sigma)(1 - 2\sigma)} \]
\[ \beta = \frac{g V_{\text{TOTAL}}}{M_w} \]  

(SC) 

Since 

\[ \theta_v = \rho_B \theta_m \]  

(9) 

Equation (9) can be substituted into Equation (8) and results in 

\[ C_{\text{TOTAL}} = \sqrt{\frac{E \theta_v V_{\text{TOTAL}}}{\rho_B M_w} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} + \sqrt{\frac{G \theta_v V_{\text{TOTAL}}}{\rho_B M_w}} \]  

(10) 

Equations (8) and (10) are the elementary algorithms (EI and EII) used for estimating mass and volumetric soil moisture content, respectively, from sound wave velocity through soils. The effects of porosity on \( C_{\text{TOTAL}} \) can be described using a modification of either of the elementary algorithms as shown in Equation (11) and (12). 

\[ C_{\text{TOTAL}} = \sqrt{\frac{E \theta_m V_{\text{SOLID}}}{\rho B 1 - \phi} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} + \sqrt{\frac{G \theta_m V_{\text{SOLID}}}{\rho B 1 - \phi}} \]  

(11) 

\[ C_{\text{TOTAL}} = \sqrt{\frac{E \theta_v V_{\text{SOLID}}}{\rho B 1 - \phi} \frac{1 - \sigma}{(1 + \sigma)(1 - 2\sigma)}} + \sqrt{\frac{G \theta_v V_{\text{SOLID}}}{\rho B 1 - \phi}} \]  

(12) 

where \( V_{\text{SOLID}} \) is the volume of solid portion of the soil system. Organic matter and texture effects can also be evaluated through the measurement and evaluation of the volume of soil organic and mineral fractions within \( V_{\text{SOLID}} \). 

Szilárd (1982) further describes the velocity of ultrasound waves through air and water as 

\[ C_{\text{AIR}} = \sqrt{\frac{\gamma P_o}{\rho_o}} \]  

(13) 

and 

\[ C_{\text{FLUID}} = \sqrt{\frac{k}{\rho}} \]  

(14) 

where \( C_{\text{FLUID}} \) is the speed of sound waves through air, \( \gamma \) is the specific heat ratio, \( P_o \) is the static pressure, \( \rho_o \) is the static density, \( C_{\text{FLUID}} \) is the speed of sound waves through water, \( k \) is the bulk stiffness modulus, and \( \rho \) is the density. Soil particles are considered to be in air until the water in the pore space flows or has similar properties as a complete pore volume of water. Equation (7) can therefore be substituted into Equations (13) and (14) to result in 

\[ C_{\text{AIR}} = \sqrt{\frac{\gamma P_o \theta_m V_{\text{TOTAL}}}{M_w}} \]  

(15) 

and
Equations (15) and (16) are the secondary algorithms (S1 and S11) used for evaluating c through porous media in air and water, respectively. $C_{AIR}$ and $C_{FLUID}$ can be further defined as

$$C_{AIR} = \sqrt{\frac{k\theta_m V_{TOTAL}}{M_w}}$$  \hspace{1cm} (15a)

and

$$C_{FLUID} = \sqrt{\frac{k\theta_m V_{TOTAL}}{M_w}}$$  \hspace{1cm} (16a)

where

$$A = \frac{\gamma P_0 V_{TOTAL}}{M_w}$$  \hspace{1cm} (15b)

and

$$B = \frac{k V_{TOTAL}}{M_w}$$  \hspace{1cm} (16b)

RESULTS AND DISCUSSION

The EI elementary algorithm (Equations 8a, b, and c) for estimating sound wave velocity from mass soil moisture content was used for the algorithm simulation. 100 gm of a well granulated soil and the preliminary data from Choi et al. (1996) were used as input data. The value of a was assigned a value of 0.3 and the shear propagation through the media was assumed to be negligible. The mass moisture content for the simulation ranged from 0.08 (wilting point) to 0.5 (saturation) and the numerical value for the modulus of elasticity, E, ranged from 9.25E5 (moist) to 1.47E6 (dry). Figure 1 shows the results of plotting $\theta_m$ versus $c_{TOTAL}$ for the algorithm simulation. The velocity of the sound waves decreased from 664 m/s to 527 m/s with a slope of -312 as the soil moisture content increased from 0.08 g/g to 0.5 g/g.

Soil moisture contents ranging from 0.08 g/g (wilting point) to 0.5 g/g (saturation) were estimated from the velocity of sound waves moving through the soil media using the secondary algorithms S1 and S11 (Equations 15a and b and 16a and b). The results of plotting mass moisture content, $\theta_m$, versus sound wave velocity in soils with little water in the soil pores, $C_{AIR}$, and in soils with predominantly water-filled pores, $C_{FLUID}$, are shown in Figure 2. The S1 algorithm estimates soil moisture content ranging approximately zero to 0.3 g/g and the S11 algorithm estimates soil moisture content ranging from approximate y 0.3 to that of saturation.

CONCLUSION

The ultrasound algorithms developed for estimating soil moisture content from sound wave velocities are viable tools which can be used in this regard. The EI and EII elementary algorithms are capable of estimating soil mass and volumetric moisture contents as well as other soil properties such as porosity, organic matter, and texture. The S1 and S11 secondary algorithms facilitate the estimation of soil moisture content from sound wave velocities at moisture contents ranging from approximate y zero to that of saturation. The major considerations and limitations associated with the algorithms include the fact that the equations described by Szilard were developed for solid, homogeneous materials as opposed to soils which are porous, heterogeneous materials. Material lattice description and consistency and the modulus of elasticity are parameter considerations that vary greatly from one material to the next. The described algorithms suggest a decrease in the porous material elasticity with increased moisture content.
Figure 1. Mass moisture content versus soundwave velocity using the elementary algorithm, EI.

Figure 2. Soundwave velocity through soils containing predominantly air and water using the secondary algorithms, SI and SII.
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