INTELLIGENT CONTROL OF FLEXIBLE-JOINT ROBOTIC MANIPULATORS

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Abstract
This paper considers the trajectory tracking problem for uncertain rigid-link, flexible-joint manipulators, and presents a new intelligent controller as a solution to this problem. The proposed control strategy is simple and computationally efficient, requires little information concerning either the manipulator or actuator/transmission models, and ensures uniform boundedness of all signals and arbitrarily accurate task-space trajectory tracking.

1. Introduction
The problem of controlling the motion of robotic manipulators in the presence of incomplete information concerning the system model has received considerable attention during the past decade, and much progress has been made in this area. However, most of the controllers proposed as solutions to this problem have been designed by neglecting any flexibility associated with the actuator/transmission system and assuming that the actuators are rigidly connected to the manipulator links. As demonstrated in [e.g., 1], joint flexibilities constitute an important component of the complete manipulator dynamic model and thus should be addressed in the controller development process. Recognizing the potential difficulties associated with ignoring the effects of joint flexibility, several researchers have recently considered the problem of controlling rigid-link, flexible-joint (RLFJ) manipulators [e.g., 2-11]. In much of this work, the controller development requires full knowledge of the complex dynamic models for the manipulator and actuator systems [e.g., 2-5]. Research in which controllers are designed with the capability to compensate for uncertainty in the manipulator/actuator system includes adaptive schemes developed using a singular perturbation approach [6,7], which can be used if the joints are sufficiently stiff, and more recent work on robust control strategies and adaptive schemes [8-11] which is valid for arbitrary joint stiffnesses. It is noted that implementation of most of these robust and adaptive controllers requires the calculation of very complex, manipulator-specific quantities, which limits the generality and applicability of these strategies.

This paper introduces a new trajectory tracking controller for uncertain RLFJ manipulators. In contrast with existing schemes, the present strategy is developed using an intelligent control approach which combines ideas from robust control and the recently developed performance-based adaptive control methodology [12,13]. This approach effectively exploits the underlying mechanical system structure of the manipulator dynamic model to permit reliance on information regarding this model to be eliminated, and as a consequence overcomes the difficulties associated with previous control methods. Thus the proposed tracking controller possesses a simple and modular structure, is easy to implement, and requires virtually no information regarding either the mechanical or actuator models. It is shown that the controller ensures uniform boundedness of all signals and provides arbitrarily accurate tracking control.

2. Preliminaries
Let \( \mathbf{p} \in \mathbb{R}^n \) define the position and orientation of the robot end-effector relative to a fixed user-defined reference frame and \( \mathbf{\theta} \in \mathbb{R}^n \) denote the vector of robot link coordinates. Then the forward kinematic and differential kinematic maps between the robot link
coordinates $\theta$ and the end-effector coordinates $p$ can be written as

$$p = h(\theta), \quad \dot{p} = J(\theta)\dot{\theta}$$

where $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$ is smooth and $J \in \mathbb{R}^{m \times n}$ is the end-effector Jacobian matrix.

Observe that there are numerous advantages to formulating the manipulator control problem directly in terms of the end-effector coordinates $p$. For example, these coordinates are typically more task-relevant than the link coordinates $\theta$, so that developing the controller in terms of $p$ can lead to improved performance, efficiency, and implementability. If the manipulator is nonredundant (so that $m = n$) and is in a region of the workspace where $J$ has full rank, then $p$ and $\dot{p}$ are diffeomorphic and this formulation presents no difficulties. A task-space formulation can also be realized if the manipulator is kinematically redundant (so that $m < n$) by utilizing, for example, the configuration control approach [e.g., 14, 15]. In what follows, we shall consider nonredundant and redundant robots together and introduce a set of $n$ task-space coordinates $x$ obtained by augmenting $p$ with $n - m$ kinematic functions that define some auxiliary task to be performed by the manipulator [14, 15]. To retain generality, we shall require only that the kinematic relationship between $\theta$ and $x$ is known and smooth and can be written in a form analogous to (1):

$$x = h_a(\theta), \quad \dot{x} = J_a(\theta)\dot{\theta}$$

where $h_a : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $J_a \in \mathbb{R}^{n \times n}$. Observe that for $x$ to be a valid task-space coordinate vector the elements of $x$ must be independent in the region of interest: thus it will be assumed in our development that $J_a$ is of full rank.

Consider an $n$ link RLFJ manipulator with actuator coordinates $\phi \in \mathbb{R}^n$ and actuator torques $u \in \mathbb{W}$. The task-space dynamic model for this manipulator system is a 4th order differential equation relating the end-effector coordinates $x$ and the system control input $u$:

$$F = H\ddot{x} + V_{cc}\dot{x} + G, \quad T = J_a^T F$$

$$u = J_m\dot{\phi} + K_m T$$

where $F \in \mathbb{R}^n$ is the generalized force associated with $x$, $H(x) \in \mathbb{R}^{n \times n}$ is the manipulator inertia matrix, $V_\& (x, \dot{x}) \in \mathbb{R}^{n \times n}$ quantifies Coriolis and centripetal acceleration effects, $G(x) \in \mathbb{R}^n$ is the vector of gravity forces, $P \in \mathbb{R}^n$ is the vector of forces and moments exerted by the environment, and $J_m, K_m \in \mathbb{R}^{n \times n}$ are positive, constant, diagonal matrices which characterize the actuator dynamics. Note that in obtaining the RLFJ manipulator model (3) we have scaled $H$, $V_{cc}$, and $G$ by the joint stiffness, introduced the definition $T = \phi - \theta$, and assumed that actuator rotor motion is a pure rotation relative to an inertial frame. It is well known that the rigid-link manipulator dynamics (3a) possesses considerable structure. For example, for any set of generalized coordinates $x$, the dynamic model terms $H$, $G$ are bounded functions of $x$ whose time derivatives $\dot{H}$, $\dot{G}$ are also bounded in $x$ and depend linearly on $\dot{x}$, the matrix $H$ is symmetric and positive-definite, the matrix $V_{cc}$ is bounded in $x$ and depends linearly on $x$, and the matrices $H$ and $V_{cc}$ are related according to $H = V_{cc} + V_{cc}^T$. Additionally, $V_\&(x; x) y = V_\&(x; x) x \dot{x}$ holds for all $y$, and if $y$ and $y$ are bounded then $V_\&(x; y)$ is bounded and $V_{cc}(x, y)$ grows linearly with $x$.

In this paper we shall address the trajectory tracking problem. The control objective for tracking is to ensure that the manipulator/actuator system (3) evolves from its initial state to the desired final state along some specified task-space trajectory $x_d(t) \in \mathbb{R}^n$ (where $x_d$ is bounded with bounded derivatives). In what follows, it is assumed that the manipulator/actuator system state $\theta, \dot{\theta}, \phi,$ and $\dot{\phi}$ is measurable. Observe that the dynamic
model (3) consists of two cascaded dynamical systems. One consequence of this structure is that the rigid-link manipulator input $T$ cannot be commanded directly, as is assumed in the design of controllers at the link torque input level, and instead must be realized as the output of the actuator dynamics (3b) through proper specification of the actuator control input $u$. The structure of the RLFJ manipulator dynamics (3) suggests partitioning the control system design problem into two subproblems: regard $T$ as the control input for the subsystem (3a) and specify the desired evolution of this variable $T_d(t)$ in such a way that if $T = T_d$ then accurate tracking would be achieved, and then specify the actual control input $u$ so that $T$ closely tracks $T_d$. This approach to controller design is adopted in this paper, so that the proposed control system consists of two subsystems: an adaptive strategy that provides the (fictitious) control input $T_d$ required to ensure that the system (3a) pm-forms as desired, and a robust control scheme that determines the (actual) control input $u$ which guarantees that the system (3b) evolves with $T$ closely tracking $T_d$.

3. Tracking Control Scheme

Let $e = x_d - x$ denote the task-space trajectory tracking error and $E = T_d - T$ represent the link torque tracking error. Consider the following tracking controller for RLFJ manipulators:

$$F_d = A(t)x_d + B(t)x_d + f(t) + k_1\gamma^2w + k_2\gamma^2e$$

$$w = -2\gamma w + \gamma e$$

$$T_d = J_a^TF_d$$

$$u = f_0(t) + [b(t)\text{sat}(\frac{s}{\epsilon})] + k_au$$

where the notation $[gh] = [g_1h_1, g_2h_2, \ldots, g_nh_n] \in \mathbb{R}^n$ (for any two n-vectors $g, h$) and $\text{sat}(g) = [\text{sat}(g_1), \text{sat}(g_2), \ldots, \text{sat}(g_n)]^T \in \mathbb{R}^n$ (with $\text{sat}(\cdot)$ the standard saturation function) is introduced, $s = \dot{E} + \lambda E$ is the weighted torque-torque rate tracking error, $f_0(t), b(t) \in \mathbb{R}^n$ are robust control terms, $f(t), b(t) \in \mathbb{R}^n$ are smooth vector functions which are defined in the proof of the Theorem below, and the adaptive gains $f, A, B$ are adjusted according to the following simple update laws:

$$\dot{f} = -\sigma_1f + \beta_1q$$

$$\dot{A} = -\sigma_2A + \beta_2q\dot{x}_d^T$$

$$\dot{B} = -\sigma_3B + \beta_3q\dot{x}_d^T$$

where $q = e + k_2\epsilon/k_1\gamma - w/\gamma$ is the weighted and filtered position-velocity tracking error and the $\sigma_i$ and $\beta_i$ are positive scalar adaptation gains.

The stability properties of the proposed tracking strategy (4),(5) are summarized in the following theorem.

**Theorem:** The control scheme (4),(5) ensures that (3) evolves with all signals (semiglobally) uniformly bounded provided $\gamma$ is chosen sufficiently large and $b$ is properly defined. Moreover, the trajectory tracking error $e, \epsilon$ is guaranteed to converge exponentially to a compact set which can be made arbitrarily small.

**Proof:** Observe first that the actuator dynamics (3b) can be written

$$J_m\dot{s} = f_m(\theta, \dot{\theta}, \phi, \dot{\phi}) - u$$

$$177$$
where $f_m(\theta, \dot{\theta}, \phi, \dot{\phi})$ is a smooth function obtained through routine manipulation. Applying the control law (4) to the manipulator dynamics (3) yields the closed-loop error dynamics

\[ H\ddot{e} + V_{cc}\dot{e} + k_1\gamma^2w + k_2\gamma^2e + \Phi_f + \Phi_A\ddot{x}_d + \Phi_B\dot{x}_d + V_{cd}\dot{e} - J_e^{-T}E = 0 \]

\[ J_m\dot{s} + k_a\dot{s} + [bsat(\frac{s}{\epsilon})] + f_0 - f_m = 0 \]

(7)

where $\Phi_f = f - G$, $\Phi_A = A - H$, $\Phi_B = B - V_{cd}$, and the notation $V_{cd} = V_{cc}(x, \dot{x}_d)$ is introduced.

Consider the Lyapunov function candidate

\[ V = \frac{1}{2}e^TH\dot{e} + \frac{1}{2}k_2\gamma^2e^TEe + \frac{1}{2}k_1\gamma^2e^TW\dot{e} + \frac{1}{2}\beta_1\Phi_f^T\Phi_f + \frac{1}{2}\beta_2\Phi_A^T\Phi_A + \frac{1}{2}\beta_3\Phi_B^T\Phi_B \]

(8)

and note that $V$ is a positive-definite and proper function of the closed-loop system state if $\gamma$ is chosen sufficiently large. Computing the derivative of (8) along (7) and simplifying permits the following upper bound on $\dot{V}$ to be established:

\[ \dot{V} \leq -\lambda_m(Q^*)\|z_1\|^2 - k_a\|\dot{E}\|^2 - k_a\lambda^2\|E\|^2 + \frac{3}{\sigma_{min}}\|z_1\|\|E\| - \frac{\eta_1}{\beta_{min}}\|\Phi\|^2 + \frac{k_2k_{cc}}{k_1\gamma}\|e\|\|\dot{e}\|^2 + \frac{k_{cc}}{\gamma}\|w\|\|\dot{w}\|^2 + \eta_2\epsilon + \frac{\eta_3}{\beta_{min}} \]

(9)

where $\lambda_m(\cdot)$, $\lambda_M(\cdot)$ denote the minimum and maximum eigenvalue of the matrix argument, respectively, $k_{cc}$ satisfies $\|V_{cc}\|_F \leq k_{cc}\|\dot{x}\|_F \forall x, k_{cd}$ is an upper bound on $V_{cd}, n_M$ is an upper bound on $\|\dot{x}_d\|$, the $\eta_i$ are positive scalar constants which do not increase as $\epsilon$ is decreased and the $\beta_i$ are increased, $f_0$ is any (nominal) estimate for $f_m$ (for example, $f_0 = 0$ can be used), $b$ is chosen so that $b_i \geq \max[1, (f_{mi} - f_0)^2]$ for $i = 1, 2, ..., n$, $\gamma$ is chosen so that $\gamma \geq \max[1, k_2/k_1]$, $z_1 = [\|e\| \|e\| \|\dot{e}\| \|w\| \|\dot{w}\|]$, $\Phi = [\|\Phi_f\| \|\Phi_A\| \|\Phi_B\| \|\dot{E}\|]$, $J_a$ (recall that $J_a$ is assumed to be nonsingular in the region of interest, so that $\sigma_{min}$ is nonzero), and

\[ Q^* = \begin{bmatrix}
\frac{k_2\gamma}{k_1} & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & 0 \\
\frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & \frac{(\gamma - 1)}{2}\frac{k_2\gamma}{k_1}\lambda_m(H) - k_{cd} - \lambda_M(H) - \frac{k_{cd}}{2\gamma} - \frac{k_{cc}v_M}{2\gamma} & -\frac{k_2\gamma}{k_1}\lambda_m(H) - k_{cd} - \frac{k_{cc}v_M}{2\gamma} & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) \\
0 & \frac{-k_2\gamma}{k_1}\lambda_m(H) - k_{cd} - \frac{k_{cc}v_M}{2\gamma} & \frac{(\gamma - 1)}{2}\frac{k_2\gamma}{k_1}\lambda_m(H) - k_{cd} - \lambda_M(H) - \frac{k_{cd}}{2\gamma} - \frac{k_{cc}v_M}{2\gamma} & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) \\
0 & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & \frac{-k_2}{2k_1\gamma}(k_{cc}v_M + k_{cd}) & \frac{(\gamma - 1)}{2}\frac{k_2\gamma}{k_1}\lambda_m(H) - k_{cd} - \lambda_M(H) - \frac{k_{cd}}{2\gamma} - \frac{k_{cc}v_M}{2\gamma}
\end{bmatrix} \]

(note that $Q^*$ is positive-definite if $\gamma$ is chosen large enough). Next let $2 = [\|z_1\| \|\dot{E}\| \|\dot{E}\|]^T$ and

\[ Q = \begin{bmatrix}
\lambda_m(Q^*) & -3/2\sigma_{min} & 0 \\
-3/2\sigma_{min} & k_2\lambda^2 & 0 \\
0 & 0 & k_3\lambda^3
\end{bmatrix} \]

and notice that $Q$ is positive-definite if $k_a$ is chosen large enough. If $\epsilon$ is chosen to be inversely proportional to $\beta_{min}$ and $\beta_{max}/\beta_{min}$ is fixed, then there exist positive scalar
constants $\eta_4, \eta_5$ that does not increase as $\gamma$ and $\beta_{min}$ increase, and positive scalar constants $\lambda_i$ independent of $\gamma$ and $\beta_{min}$, such that $V$ and $V$ in (8) and (9) can be bounded as

$$
\lambda_1 \| z^* \|^2 + \frac{\lambda_2}{\beta_{min}} \| \Psi \|^2 \leq V \leq \lambda_3 \| z_2 \|^2 + \frac{\lambda_4}{\beta_{min}} \| \Psi \|^2
$$

$$
\dot{V} \leq -\left( \lambda_m(Q) - \frac{\eta_4}{\gamma} V^{1/2} \right) \| z^* \|^2 - \frac{\lambda_5}{\beta_{min}} \| \Psi \|^2 + \gamma
$$

Now choose two scalar constants $V_M, V_m$ so that $V_M > V_m \geq V(0)$, and define $c_M = \lambda_m(Q) - \frac{\eta_4 V_m^{1/2}}{\gamma}$; then choose $\gamma$ large enough so that $c_M > 0$ (this is always possible). Let $\delta = \max(\lambda_3/c_M, \lambda_4/\lambda_5)$ and choose $\beta_0$ so that $\eta_5/\beta_0 < V_m$ (this is always possible). Then selecting $\beta_{min} \geq \beta_0$ ensures that if $V_m \leq V < V_M$ then $V < 0$. This condition together with $V_M > V_m \geq V(0)$ implies that $V(t) \leq V_M \forall t$, so that $c(t) = \lambda_m(Q) - \frac{\eta_4 V^{1/2}(t)}{\gamma} > c_M > 0 \forall t$ and

$$
\dot{V} \leq -c_M \| z_2 \|^2 - \frac{\lambda_5}{(\beta_0 + \Delta\beta)} \| \Psi \|^2 + \frac{\eta_5}{(\gamma + \Delta\beta)}
$$

where $\Delta\beta = \beta_{min} - \beta_0$ and it is assumed that $\beta_{min}$ is chosen so that $\Delta\beta > 0$. The ultimate boundedness results developed in [13,14] are now directly applicable and permit the conclusion that $\| z_2 \|, \| \Psi \|$ are uniformly bounded and that $\| z_2 \|, \| \Psi \|$ converge exponentially to the closed balls $B_{r_1}, B_{r_2}$, respectively, where

$$
r_1 = \left( \frac{\delta \eta_4}{\lambda_1 (\beta_0 + \Delta\beta)} \right)^{1/2}
$$

$$
r_2 = \left( \frac{\delta \eta_4}{\lambda_2} \right)^{1/2}
$$

Observe that the radius of the ball to which $\| z_2 \|^2$ is guaranteed to converge can be decreased as desired simply by increasing $\Delta\beta$. \[ \blacksquare \]

4. Conclusions

This paper presents a new solution to the motion control problem for uncertain RLFJ manipulators. The proposed control strategy is simple and computationally efficient, requires little information concerning either the manipulator or actuator/transmission models, and ensures uniform boundedness of all signals and arbitrarily accurate task-space trajectory tracking. Future research will involve the implementation of the controllers for robotic applications in hazardous and unstructured environments.

5. Acknowledgments

The research described in this paper was supported in part by the Army Research Office, the Department of Energy (WERC), and the NASA Center for Autonomous Control Engineering at the University of New Mexico.
6. References