

# Piecewise Linear Membership Function Generator - Divider Approach

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**Abstract-** In this paper a simple, inexpensive, membership-function circuit for fuzzy controllers is presented. The proposed circuit may be used to generate a general trapezoidal membership function. The slope and horizontal shift are fully programmable parameters.

## 1. Introduction

There exist numerous examples of industry proven fuzzy-logic solutions in the domain of control, expert systems, and pattern recognition. A typical Mamdani fuzzy controller consists of a fuzzifier block, an inference-engine block, a rulebase block, and a defuzzification block [1]. The hardware realization of the membership-function generator circuit is the subject of this paper.

The proposed circuit is characterized by a generalized trapezoidal membership function defined in terms of horizontal shifting parameters. The authors realized a parametric class of membership functions based on piecewise linear functions. In this paper, they present a basic mathematical background of membership functions, second, simulation results for the current design, and the results for hardware realization.

## 2. Proposed Algorithm

We propose to realize the following function.

$$F(v) = \begin{cases} 0 & \text{if } v < V_a \\ \frac{v - V_a}{V_b - V_a} & \text{if } V_a \leq v < V_b \\ 1 & \text{if } V_b \leq v \leq V_c \\ \frac{v - V_d}{V_c - V_d} & \text{if } V_c < v \leq V_d \\ 0 & \text{if } V_d < v \end{cases} \quad (1)$$

where  $V_a$ ,  $V_b$ ,  $V_c$ , and  $V_d$  are real numbers determining the shape of the trapezoid illustrated in Figure 1.

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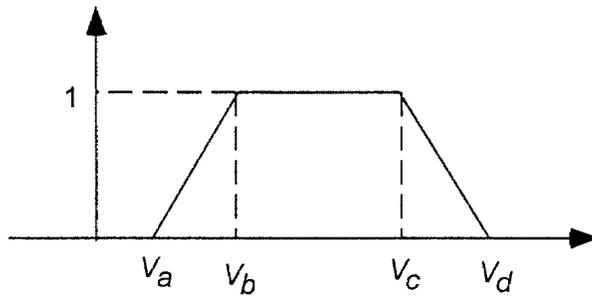


Figure1 A Trapezoidal Membership Function

To realize expression (1), two analog multiplexers, two summers, and a divider are needed. A block diagram of the proposed circuit is shown in Figure 2. Threshold voltages are multiplexed then combined with the input signal according to equation (1). To realize the required voltage ratio, a divider circuit consisting of an AD534JD is used.

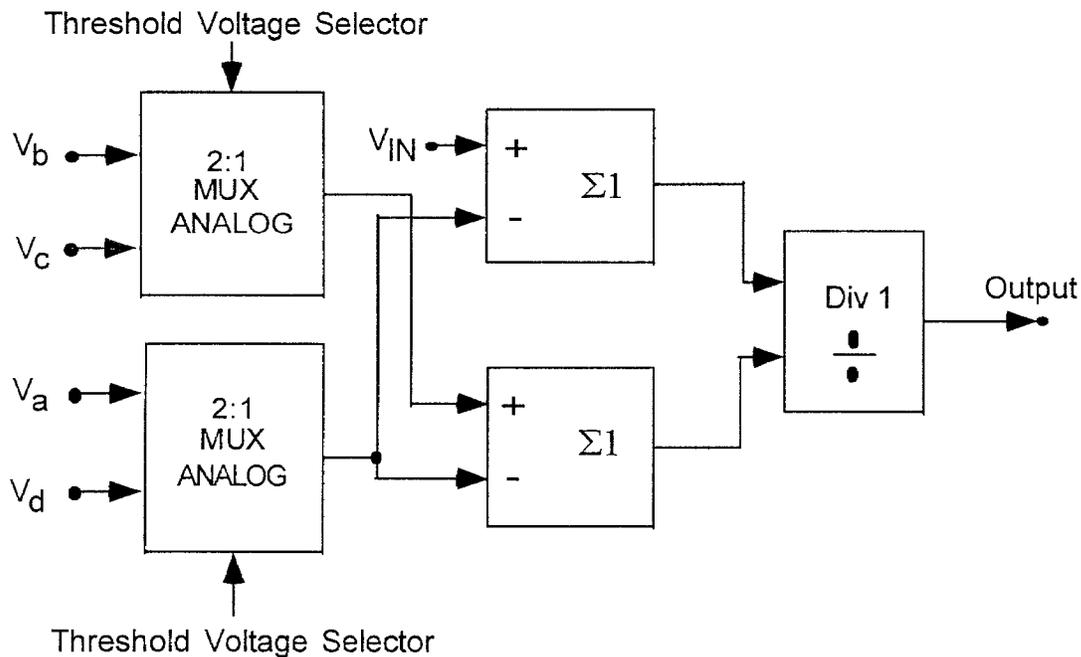


Figure 2 Block diagram of the proposed membership function circuit.

To validate this approach prior to hardware realization, computer simulations were performed. These are discussed in the next section.

### 3. Simulation Results

In our simulations, the user friendly software package TESLA is used [2]. This is a modular software system consisting of analog and digital components needed to perform this simulation. The simulation block diagram with numbered nodes is shown in Figure 3.

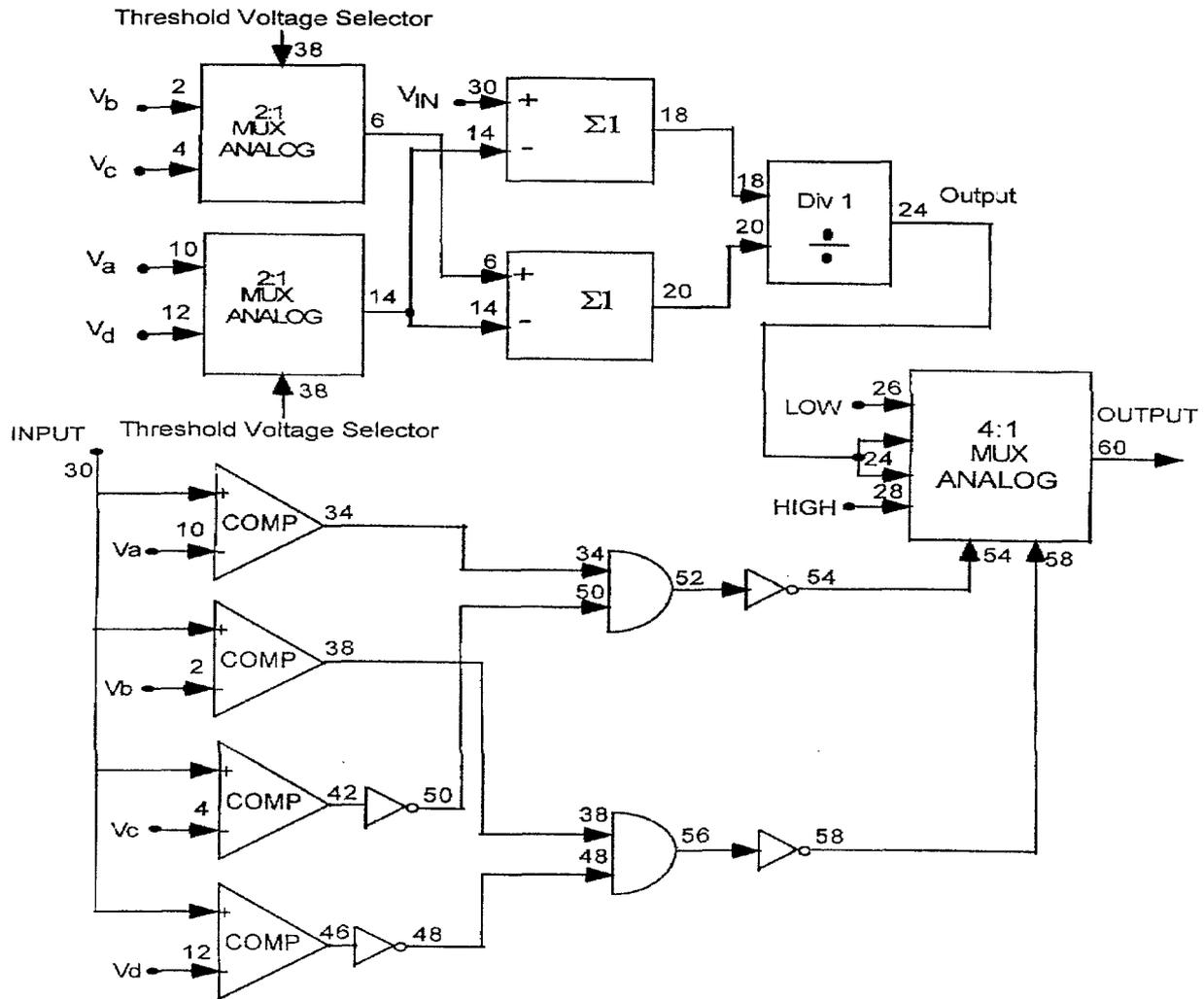


Figure 3 Simulation Block Diagram

The numbers found in the net-list in Table 1 correspond to the nodal connections in the block diagram.

```
X 0 0 30 0 FCNGEN FCN=3 F=.05 V=5
Va 10 PWR V=set(Va)=
Vb 2 PWR V=SET(Vb)=
Vc 4 PWR V=SET(Vc)=
Vd 12 PWR V=SET(Vd)=
MUX1 38 2 4 6 MUX
MUX2 38 10 12 14 MUX
```

```

SUM1 30 14 18 SUM G1=1 G2=-1
SUM2 6 14 20 SUM G1=1 G2=-1
DIV1 18 20 24 DIV
MUX4 54 58 26 24 24 28 60 MUX4
LOW 26 PWR V=0
HIGH 28 PWR V=1
COMP1 30 10 34 COMP
COMP2 30 2 38 COMP
COMP3 30 4 42 COMP
COMP4 30 12 46 COMP
NAND1 34 50 52 NAND
INV1 52 54 INV
NAND2 38 48 56 NAND
INV2 56 58 INV
INV3 42 50 INV
INV4 46 48 INV

```

Table 1: Simulation net list

The input signal was applied interactively along the horizontal voltage axis. For one set of simulations, horizontal parameters  $V_a$ ,  $V_b$ ,  $V_c$ , and  $V_d$  are kept constant. Figure 4 presents the result of a point to point simulation for threshold voltages  $V_a = 1V$ ,  $V_b = 2V$ ,  $V_c = 3V$ , and  $V_d = 5V$ .

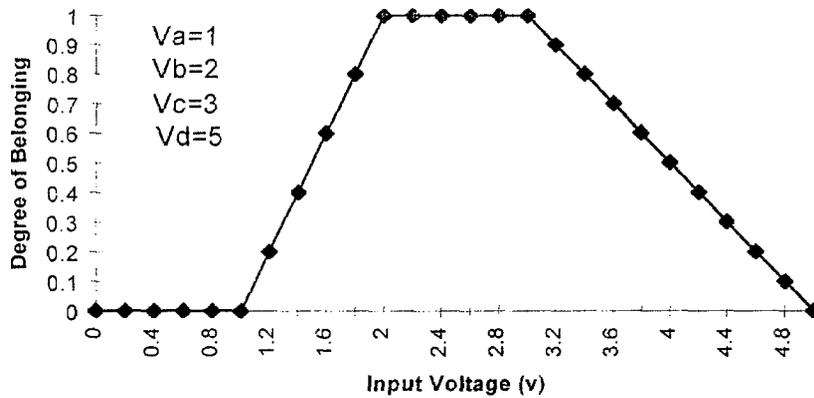


Figure 4 Simulated Trapezoidal Membership Function

The trapezoidal membership function can be considered as a general shape. For a triangular function, for example,  $V_b = V_c$ . Except, for horizontal shifting, the slope of each straight line can be changed by altering values of the threshold voltages as well. Figure 5 presents simulation results for some special trapezoidal shapes.

## 4. Realization Results

The block diagram of Figure 6 is a representation of the hardware realization of the simulated circuit shown in Figure 3. To verify our idea, inexpensive off shelf components were used to implement the circuit. As comparators we used UA741CP amplifiers. Analog Multiplexers are CD4066B bilateral switches. The divider circuit is the AD534JD. The threshold voltages  $V_a$ ,  $V_b$ ,  $V_c$ , and  $V_d$  are adjusted to realize any symmetrical or asymmetrical shape. In Figure 7, two oscilloscope diagrams for different values of threshold voltages  $V_a$ ,  $V_b$ ,  $V_c$ , and  $V_d$  are illustrated.

## 5. Conclusion

In many applications, real time fuzzy processors are needed. This paper presented an attempt to realize a simple inexpensive membership-function circuit. To verify our idea we performed simulations first, then realized the membership function generator using inexpensive off shelf components.

## 6. References

- [1] Mamdani, E. E. [1974], "Applications of fuzzy algorithms for control of simple dynamic plant." Proc. IEE 121 (22), pp. 1585 - 1588.
- [2] Tesoft, Inc. 205 Crossing Creek Ct, Roswell, GA 30076

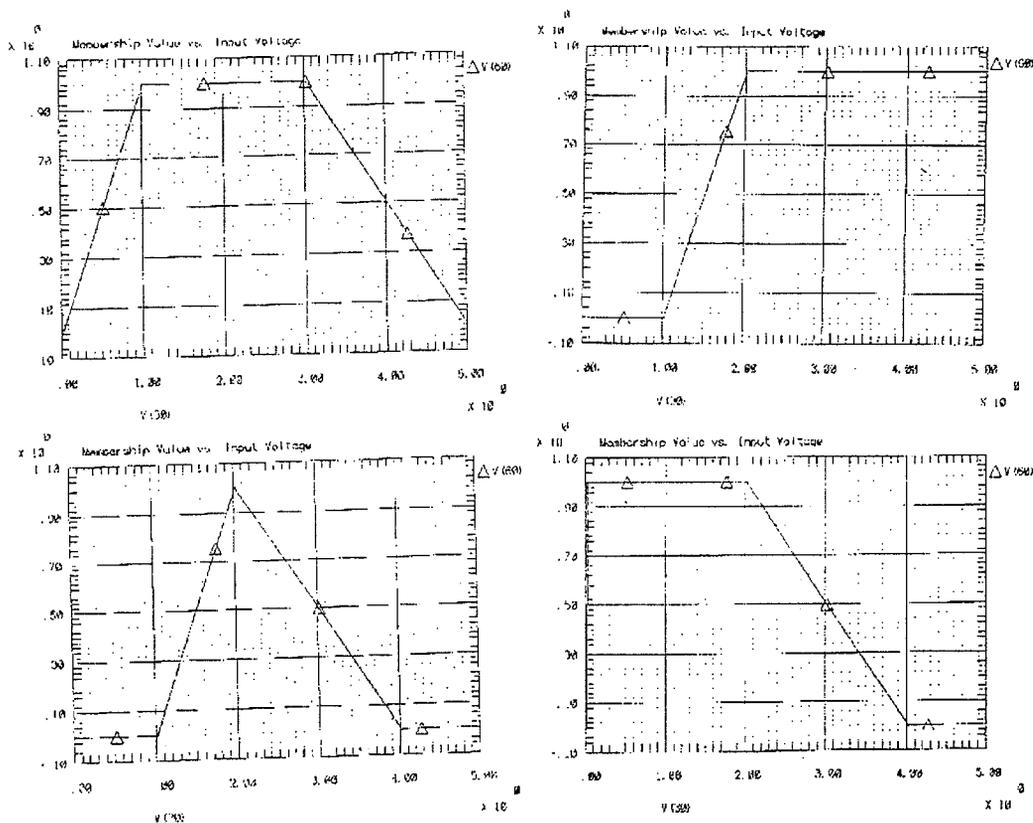


Figure 5 Plots of Simulation

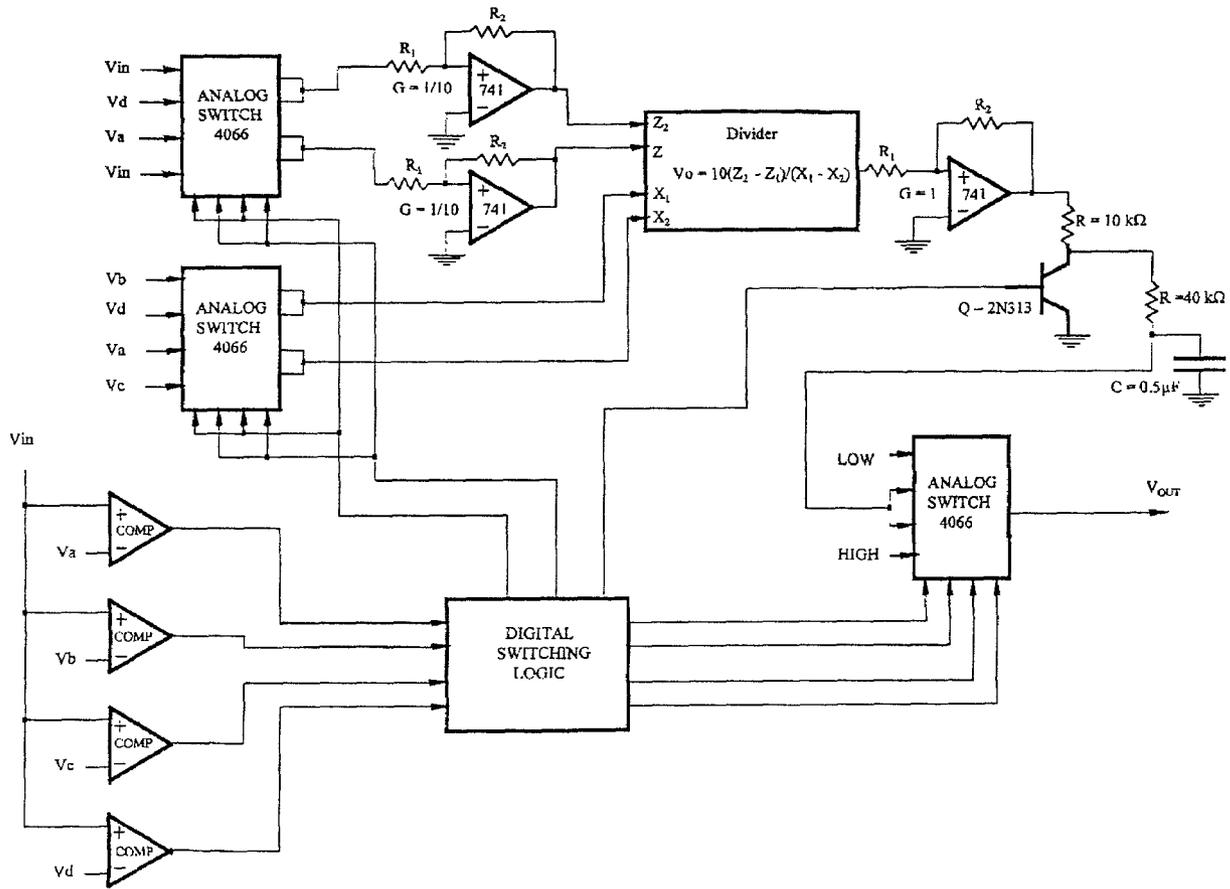


Figure 6 Circuit Block Diagram

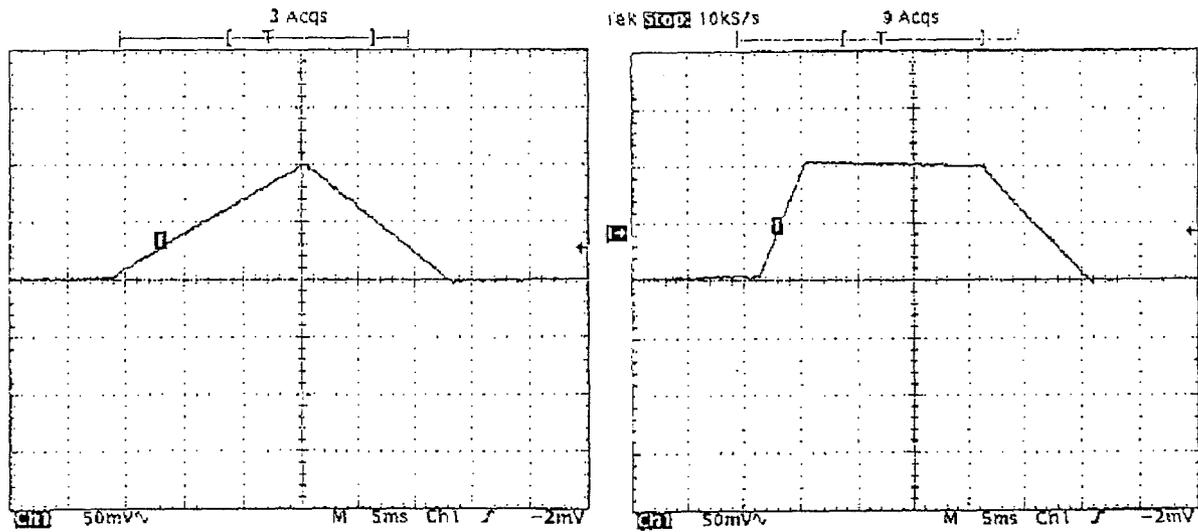


Figure 7 Realization Diagrams of Membership Function