Magnetic control of convection in electrically nonconducting fluids

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Abstract

Inhomogeneous magnetic fields exert a body force on electrically nonconducting, magnetically permeable fluids. This force can be used to compensate for gravity and to control convection. The effects of uniform and nonuniform magnetic fields on a laterally unbounded fluid layer heated from below or above are studied using a linear stability analysis of the Navier-Stokes equations supplemented by Maxwell's equations and the appropriate magnetic body force. For a uniform oblique field, the analysis shows that longitudinal rolls with axes parallel to the horizontal component of the field are the rolls most unstable to convection. The corresponding critical Rayleigh number and critical wavelength for the onset of such rolls are less than the well-known Rayleigh-Bénard values in the absence of magnetic fields. Vertical fields maximize these deviations, which vanish for horizontal fields. Horizontal fields increase the critical Rayleigh number and the critical wavelength for all rolls except longitudinal rolls. For a nonuniform field, our analysis shows that the magnetic effect on convection is represented by a dimensionless vector parameter which measures the relative strength of the induced magnetic buoyancy force due to the applied field gradient. The vertical component of this parameter competes with the gravitational buoyancy effect, and a critical relationship between this component and the Rayleigh number is identified for the onset of convection. Therefore, Rayleigh-Bénard convection in such fluids can be enhanced or suppressed by the field. It also shows that magnetothermal convection is possible in both paramagnetic and diamagnetic fluids. Our theoretical predictions for paramagnetic fluids agree with experiments. Magnetically driven convection in diamagnetic fluids should be observable even in pure water using current technology.

I. Introduction

In a recent experiment, Beaugnon and Tournier[1] have successfully levitated various diamagnetic solids and liquids using a strong nonuniform static magnetic field. Recent experiments [2,3] also observe the strong enhancing and suppressing effects of an applied inhomogeneous static magnetic field on thermal transport in a gadolinium nitrate solution heated from below, indicating that the thermal gradient induced buoyancy-driven convection in this paramagnetic fluid is controllable via the applied magnetic field. We provide here the theory of magnetically controlled convection in a horizontal, electrically nonconducting fluid layer heated from either above or below. We show that the convective fluid flow can be effectively controlled by placing the layer in a non-uniform magnetic field, which can promote or inhibit convection for both upward and downward thermal gradients. This phenomenon has a great potential to be utilized to enhance or to suppress the gravitational effect in terrestrial experiments and to control the flow of nonconducting fluids in a microgravity environment. This effect can be utilized to increase the efficiency of heat-transfer devices.

When a magnetically permeable fluid is placed in a static magnetic field $\vec{H}$, Landau and Lifshitz [4] calculate the volume forces on the fluid [Eq. (34.3) in Ref. 4 converted to SI units],

$$\vec{f} = -\nabla p_0 + \frac{1}{2}\nabla\left[H^2\rho\left(\frac{\partial\mu}{\partial\rho}\right)_T\right] - \frac{H^2}{2}\nabla\mu + \mu\vec{j}\times\vec{H}\ , \quad (1)$$

where p_0 is the pressure in the absence of the field, ρ the density of the fluid, T the temperature, μ the magnetic permeability of the fluid, and $\vec{j}$ the electric current density in the fluid. For electrically nonconducting fluids, $\vec{j} = 0$, and therefore the last term vanishes. As $\mu = \mu_0(1+\chi)$, $\vec{M} = \chi\vec{H}$, and $\nabla\times\vec{H} = 0$ in nonconducting diamagnetic or paramagnetic fluids, we can rewrite Eq. (1) as

$$\vec{f} = -\nabla p_0' + \mu_0\vec{M}\cdot\nabla\vec{H}\ , \quad (2)$$

where μ_0 is the permeability of free space, χ the volumetric susceptibility of the fluid, p_0' the modified pressure including magnetic contribution, and $\vec{M}$ the magnetization (the magnetic moment per unit volume). The first term on the right side of Eq. (2) has no contribution to convection since $\nabla\times\nabla p_0' = 0$. Paramagnetic fluids contain atoms or molecules that have intrinsic magnetic moment, and their magnetic susceptibilities satisfy Curie's law [5], *i.e.*, $\chi = C\rho/T$, where C is a positive constant. Unlike paramagnetic fluids, diamagnetic fluids contain atoms or molecules that have no intrinsic magnetic moment. When a static magnetic field is applied to these fluids, the change of the field induces a magnetic moment for each atom or molecule. Diamagnetic susceptibilities satisfy $\chi = \chi_m\rho$, where χ_m is the susceptibility per unit mass, a negative constant. The last term in Eq. (2) is the Kelvin body force [6] $\vec{f}_m = \mu_0(\vec{M}\cdot\nabla)\vec{H}$, which arises from the interaction between the local magnetic field $\vec{H}$ within the fluid and the molecular magnetic moments characterized by the magnetization $\vec{M}$. An imposed thermal gradient produces a spatial variation in the magnetization through the temperature-dependent magnetic susceptibility for paramagnetic fluids and through the temperature-dependent mass density for diamagnetic fluids, and therefore renders the Kelvin body force density $\vec{f}_m$ nonuniform spatially. This thermal gradient induced inhomogeneous magnetic body force density $\vec{f}_m$ can promote or inhibit convection in a manner similar to the gravitational body force.

II. Governing Equations

To study magnetically controlled convection in electrically nonconducting fluids, we consider an incompressible horizontal layer of such fluids heated on either top or bottom in the presence of an external nonuniform magnetic field. We choose our coordinate system by defining $|z| < d/2$ with $\hat{\mathbf{z}}$ pointing up, where d is the layer thickness. We assume that the external field satisfies $\vec{\mathbf{H}}^{ext} = \vec{\mathbf{H}}_0 + (\vec{\mathbf{r}} \cdot \nabla)\vec{\mathbf{H}}^{ext}$, where $\vec{\mathbf{r}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}}$ is the position vector. Here the vector $\vec{\mathbf{H}}_0$ is the field at the center of the layer, and the field gradient $\nabla\vec{\mathbf{H}}^{ext}$ is a constant tensor. Maxwell's equations require this tensor to be symmetric and traceless.

The fluid flow is governed by the Navier-Stokes equations in addition to Maxwell's equations for the magnetic field $\vec{\mathbf{H}}$ and magnetic induction $\vec{\mathbf{B}} \equiv \mu_0(\vec{\mathbf{M}} + \vec{\mathbf{H}})$. Under the Oberbeck-Boussinesq approximation, which allows density variations only in the large gravity term of the Navier-Stokes equations, we write the dimensionless governing equations for the convective flow [7],

$$\frac{1}{Pr}\left(\frac{\partial \vec{\mathbf{v}}}{\partial t} + \vec{\mathbf{v}} \cdot \nabla \vec{\mathbf{v}}\right) = -\nabla p + (R\hat{\mathbf{z}} - \vec{\mathbf{R}}_m)\theta + K \sin^2 \phi \theta \hat{\mathbf{z}}$$

$$+K(z - \theta)\hat{H}_0 \cdot \nabla \vec{\mathbf{h}} + \nabla^2 \vec{\mathbf{v}} \; , \tag{3}$$

$$\frac{\partial \theta}{\partial t} + \vec{\mathbf{v}} \cdot \nabla \theta - \hat{\mathbf{z}} \cdot \vec{\mathbf{v}} = \nabla^2 \theta + \Phi \; , \tag{4}$$

$$\nabla \cdot \vec{\mathbf{h}} - \hat{H}_0 \cdot \nabla \theta = 0 \; , \tag{5}$$

$$\nabla \cdot \vec{\mathbf{v}} = 0 \; . \tag{6}$$

Here, $\vec{\mathbf{v}}$, p, θ, and $\vec{\mathbf{h}}$ represent the respective departures of velocity, pressure, temperature, and magnetic field from the static thermal conduction state. In these equations, $\hat{H}_0 = \vec{\mathbf{H}}_0/H_0$ is the unit vector in the $\vec{\mathbf{H}}_0$ direction, ϕ the angle between $\vec{\mathbf{H}}_0$ and the horizontal, and Φ the viscous dissipation. Equation (3) involves the Prandtl number $Pr = \nu/D_T$, the Rayleigh number $R = \alpha g d^3 \Delta T/\nu D_T$, the Kelvin number

$$K = \frac{\mu_0 \chi_0^2 \Delta T^2 d^2 H_0^2}{\rho_0 \nu D_T} \times \begin{cases} 1/(1+\chi_0)T_0^2 & \text{paramagnetic;} \\ \alpha^2 & \text{diamagnetic,} \end{cases} \tag{7}$$

and the vector control parameter

$$\vec{\mathbf{R}}_m = \frac{\mu_0 \chi_0 d^3 \Delta T}{\rho_0 \nu D_T} \left(\vec{\mathbf{H}} \cdot \nabla \vec{\mathbf{H}}\right)^{ext}_{\vec{\mathbf{r}}=0} \times \begin{cases} 1/T_0 & \text{paramagnetic;} \\ \alpha & \text{diamagnetic,} \end{cases} \tag{8}$$

where α is the thermal expansion coefficient, ν the kinematic viscosity, D_T the thermal diffusivity, T_0 the average temperature of the layer, ΔT the temperature difference between the bottom and the top, χ_0 the susceptibility at T_0, and ρ_0 the density at T_0.

III. Results and Implications for Experiments

The Rayleigh number R in Eq. (3) measures the strength of gravitational buoyancy relative to dissipation. In the absence of magnetic fields, the thermal convective instability in a fluid layer heated from below is determined by this parameter R, and Rayleigh-Bénard convection sets in for $R > R_c \approx 1708$. In the presence of a uniform magnetic field ($K \neq 0$ but $\vec{\mathbf{R}}_m = 0$), the magnetic effect on convection is determined by the Kelvin number K and the angle ϕ. For ordinary diamagnetic fluids such as water, our linear stability analysis shows that the difference for the marginal state due to the magnetic effect is less than 0.1% for a field up to 30 Tesla, and therefore the uniform field effect on convection in these fluids might be negligible. For paramagnetic fluids, our linear stability analysis [8] shows that longitudinal rolls with axes parallel to the horizontal component of the field are the rolls most unstable to convection. The corresponding critical Rayleigh number and critical wavelength for the onset of such rolls are less than the well-known Rayleigh-Bénard values in the absence of magnetic fields. Vertical fields maximize these deviations, which vanish for horizontal fields. Horizontal fields increase the critical Rayleigh number and the critical wavelength for all rolls except longitudinal rolls.

The vector parameter $\vec{\mathbf{R}}_m$ in Eq. (3) measures the relative strength of the magnetic buoyancy force due to the applied field gradient. Since this parameter is the only one containing the external field gradient $\nabla\vec{\mathbf{H}}^{ext}$ in the governing equations (3-6), the effect of the field gradient on convection in a nonconducting fluid layer is completely characterized by this vector parameter. The combination of the vertical component of $\vec{\mathbf{R}}_m$ with R in Eq. (3) shows that the gravitational effect on the convective flow can be balanced by this component of $\vec{\mathbf{R}}_m$. Therefore, convection in electrically nonconducting fluids can be controlled by an inhomogeneous magnetic field. The application of this theory to experiments [2,3] yields a good agreement [9]. Our analysis also shows that magnetically controlled convection in diamagnetic fluids should be observable even in pure water using current technology [10].

This work shows that thermal convection in electrically nonconducting fluids can be controlled by an external inhomogeneous magnetic field through the vector parameter $\vec{\mathbf{R}}_m$. The inhomogeneous field exerts a magnetic body force on these fluids, and this force can balance the gravitational body force in terrestrial experiments. This magnetic field induced body force can be utilized to control the flow of nonconducting fluids in a microgravity environment with possible applications in mixing, heat transfer, and materials processing.

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