Rolling Bearing Life Prediction—Past, Present, and Future

E.V. Zaretsky
Glenn Research Center, Cleveland, Ohio

J.V. Poplawski
J.V. Poplawski and Associates, Bethlehem, Pennsylvania

C.R. Miller
Williams International, Walled Lake, Michigan

November 2000
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Space Administration

Glenn Research Center

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Comparisons were made between the life prediction formulas of Lundberg and Palmgren, Ioannides and Harris, and Zaretsky and full-scale ball and roller bearing life data. The effect of Weibull slope on bearing life prediction was determined. Life factors are proposed to adjust the respective life formulas to the normalized statistical life distribution of each bearing type. The Lundberg-Palmgren method resulted in the most conservative life predictions compared to Ioannides and Harris and Zaretsky methods which produced statistically similar results. Roller profile can have significant effects on bearing life prediction results. Roller edge loading can reduce life by as much as 98 percent. The resultant predicted life not only depends on the life equation used but on the Weibull slope assumed, the least variation occurring with the Zaretsky equation. The load-life exponent $p$ of $10/3$ used in the ANSI/ABMA/ISO standards is inconsistent with the majority roller bearings designed and used today.

Keywords: Rolling-Element Bearings; Life Prediction Methods; Rolling-Element Fatigue

**INTRODUCTION**

Rolling bearing technology has evolved over 4000 years to the present. H.T. Morton [1] in his 1965 book, “Anti-Friction Bearings,” describes the evolution of rolling bearing technology. Although rolling bearing technology continued to develop throughout the first half of the 19th century, it is not until the invention of the pedal bicycle that the rolling-element bearing industry becomes established. In 1868, A.C. Cowper made a bicycle with ball bearings. According to Morton, W. Bown of Coventry, England, appears to be the most successful bearing manufacturer. In 1880, he had a contract to produce 12 ball bearings a day for Singer and Company, a bicycle manufacturer. By 1920, most types of rolling-element bearings used today were in production.

By the close of the 19th century, the bearing industry began to focus on the sizing of bearings for specific applications and determining bearing life and reliability. In 1896, R. Stribeck [2] began fatigue testing full-scale bearings. In 1912, J. Goodman [3] published formulas based on fatigue data that would compute safe loads on ball and cylindrical roller bearings.

The most influential person in developing life prediction methods for ball and roller bearings was A. Palmgren [4] in Sweden. His work with G. Lundberg [5,6] published in 1947 and 1952, modified and developed the theoretical basis for Palmgren's 1945 formulas [7] by using the Weibull distribution function [8,9]. Their work resulted in the International organization for Standardization (ISO) and the American National Standards Institute (ANSI)/Anti-Friction Bearing Manufacturers Association (AFBMA) (now American Bearing Manufacturers Association (ABMA)) standards for the load ratings and life of rolling-element bearings [10 to 12].

### NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>material-life factor</td>
</tr>
<tr>
<td>$C$</td>
<td>dynamic load capacity, N (lbf)</td>
</tr>
<tr>
<td>$c$</td>
<td>critical shear stress-life exponent</td>
</tr>
<tr>
<td>$e$</td>
<td>Weibull slope</td>
</tr>
<tr>
<td>$F$</td>
<td>probability of failure, fraction or percent</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>probability of survival function</td>
</tr>
<tr>
<td>$h$</td>
<td>exponent</td>
</tr>
<tr>
<td>$L$</td>
<td>life, number of stress cycles or hr</td>
</tr>
<tr>
<td>$L_{10}$</td>
<td>10-percent life or life at which 90 percent of a population survives, number of stress cycles or hr</td>
</tr>
<tr>
<td>$N$</td>
<td>life, number of stress cycles</td>
</tr>
<tr>
<td>$N_n$</td>
<td>maximum Hertz stress-life exponent or number of components, elemental volumes</td>
</tr>
<tr>
<td>$P$</td>
<td>normal or equivalent radial load, N, (lbf)</td>
</tr>
<tr>
<td>$p$</td>
<td>load-life exponent</td>
</tr>
<tr>
<td>$S$</td>
<td>probability of survival, fraction or percent</td>
</tr>
<tr>
<td>$s_{max}$</td>
<td>maximum Hertz stress, GPa (ksi)</td>
</tr>
<tr>
<td>$V$</td>
<td>stressed Hertz stress, GPa (ksi)</td>
</tr>
<tr>
<td>$X$</td>
<td>load, time, or stress</td>
</tr>
<tr>
<td>$Z$</td>
<td>depth to maximum critical shear stress, m (in.)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>stress or strength, GPa (ksi)</td>
</tr>
<tr>
<td>$\tau$</td>
<td>critical shear stress, GPa (ksi)</td>
</tr>
<tr>
<td>$\tau_s$</td>
<td>fatigue limit, GPa (ksi)</td>
</tr>
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</table>

**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>act.</td>
<td>actual life based on laboratory or field data</td>
</tr>
<tr>
<td>$n$</td>
<td>number of components or elemental volumes</td>
</tr>
<tr>
<td>pred.</td>
<td>predicted life based on designated life prediction method</td>
</tr>
<tr>
<td>$\beta$</td>
<td>designates characteristic life or stress</td>
</tr>
</tbody>
</table>
presented by Palmgren [13] in 1924. This paper is the first to propose the $L_{10}$ life as the basis for life prediction and also the "linear damage rule" to determine the effect of varying load.

As rolling bearing design and steels improved, the ISO standard and the Lundberg-Palmgren equations were found to underpredict bearing lives. In order to account for these differences in life, ASME published "Life Adjustment Factors" in 1971 [14]. These life adjustment factors were modified and expanded by the STLE in 1991 [15]. In 1985, E. Ioannides and T. Harris [16] proposed a modification of the Lundberg-Palmgren theory based upon a material fatigue limit and discrete finite elements. The concept of a fatigue limit was first introduced by Palmgren in 1924 [13] and then subsequently disregarded by him.

In 1987, E.V. Zaretsky [17] proposed a Weibull-based life theory using discrete finite elements that accounts for deviations in the accepted material Hertz stress-life relation. Zaretsky, together with J.V. Poplawski and S. Peters, applied his life theory to ball and roller bearings [18,19].

Other researchers have proposed modifications of the Lundberg-Palmgren theory based upon both a fatigue limit and debris contamination [20 to 23]. T.E. Tallian [24 to 26] has developed a mathematical model also based on Lundberg-Palmgren but which is fitted to a bearing data base.

The work reported and discussed herein is based in part on our work presented in [18,19]. We summarize and compare the life prediction formulas of Lundberg and Palmgren, Ioannides and Harris, and Zaretsky to each other and the ISO standards as well as with full-scale ball- and roller-bearing life data.

**LIFE THEORIES**

**Weibull Equation**

**Fracture Strength**—In 1939 W. Weibull [8,9] published two papers that describe a statistical approach to determine the strength of solids. Weibull postulated that the dispersion in material strength for a homogeneous group of test specimens could be expressed according to the following relation:

$$\ln \ln(1/S) = \alpha \ln\left[ X/ X_\beta \right]$$  \hspace{1cm} (1)

where $X = \sigma$ and $X_\beta = \sigma_\beta$.

Equation (2) relates specimen survival $S$ to the fracture (or rupture) strength $\sigma$. When $\ln \ln(1/S)$ is used as the ordinate and $\ln \sigma$ as the abscissa and fracture (and fatigue) data are assumed to plot as a straight line. The slope (tangent) of this line is referred to as the Weibull slope or Weibull modulus usually designated by the letter $\alpha$ or $m$. The plot itself is referred to as a Weibull plot.

By using a Weibull plot, it becomes possible to estimate a cumulative distribution of an infinite population from an extremely small sample size. The Weibull slope is indicative of the dispersion of the data and its density (statistical) distribution. Weibull slopes of 1, 2, and 3.57 are indicative of exponential, Rayleigh, and normal (Gaussian) distributions, respectively [8].

The scatter in the data is inversely proportional to the Weibull slope, that is, the lower the value of the Weibull slope, the larger the scatter in the data and vice versa. The Weibull slope is also liable to statistical variation depending on the sample size (data base) making up the distribution [27]. The smaller the sample size the greater the statistical variation in the slope.

Weibull [8,9] related the material strength to the volume of the material subjected to stress. If we imagine the solid to be divided in an arbitrary manner into $n$ volume elements, the probability of survival for the entire solid can be obtained by multiplying the individual survivabilities together as follows:

$$S = S_1 \cdot S_2 \cdot S_3 \ldots S_n$$  \hspace{1cm} (2)

where the probability of failure $F$ is

$$F = 1 - S$$  \hspace{1cm} (3)

Weibull [8,9] further related the probability of survival $S$, the material strength $\sigma$, and the stressed volume $V$, according to the following relation:

$$\ln \frac{1}{S} = \int_v f(X) dV$$  \hspace{1cm} (4)

where

$$f(X) = \sigma^{\tau}$$  \hspace{1cm} (5)

For a given probability of survival $S$,

$$\sigma \sim \left[ \frac{1}{V} \right]^{1/\tau}$$  \hspace{1cm} (6)

From Eq. (6) for the same probability of survival the components with the larger stressed volume will have the lower strength (or shorter life).

**Fatigue Life**—In conversations with E.V. Zaretsky on January 22, 1964, W. Weibull related how he had suggested to his contemporaries A. Palmgren and G. Lundberg in Gothenburg, Sweden, to use his equation to predict bearing (fatigue) life where

$$f(X) = \tau^{\tau} N^\tau$$  \hspace{1cm} (7)

and where $\tau$ is the critical shear stress and $N$ is the number of stress cycles to failure.

In the past we have credited this relation to Weibull. However, there is no documentation of the above nor any publication to the authors' knowledge of the application of Eq. (7) by Weibull in the open literature. However, in [18] we did apply Eq. (7) to Eq. (4) where

$$N = \left[ \frac{1}{\tau} \right]^{\tau/\tau} \left[ \frac{1}{V} \right]^{1/\tau}$$  \hspace{1cm} (8)
The parameter \( c/e \) is the stress-life exponent. This implies that the inverse relation of life with stress is a function of the life scatter or data dispersion.

From Hertz theory, \( V \) and \( \tau \) can be expressed as a function of \( S_{\text{max}} \) [18] and substituting \( L \) for \( N \)

\[
L = A \left( \frac{1}{\tau} \right)^{c/e} \left( \frac{1}{V} \right)^{1/e} \sim \frac{1}{S_{\text{max}}^n} \quad (9)
\]

From [18], solving for the value of the exponent \( n \) for line contact (roller on raceway) from Eq. (9) gives

\[
n = \frac{c + 1}{e} \quad (10a)
\]

and for point contact

\[
n = \frac{c + 2}{e} \quad (10b)
\]

Using the value of \( e = 1.11 \) and \( c/e = 9.3 \) from Lundberg and Palmgren [5], \( n \) equals 10.2 and 11.1 for line and point contact, respectively.

**Lundberg-Palmgren Equation**

In 1947, G. Lundberg and A. Palmgren [5] applied Weibull analysis to the prediction of rolling-element bearing fatigue life. The Lundberg-Palmgren theory expressed \( f(X) \) in Eq. (4) as

\[
f(X) = \frac{\tau^{c/e} N^e}{Z^h} \quad (11)
\]

where \( \tau \) is the critical shear stress, \( N \) is the number of stress cycles to failure, and \( Z \) is the depth to the maximum critical shear stress in a concentrated (Hertzian) contact. From Eqs. (4) and (11)

\[
N \sim \left[ \frac{1}{\tau} \right]^{c/e} \left[ \frac{1}{V} \right]^{1/e} [Z]^{h/e} \quad (12)
\]

From Hertz theory, \( V \), \( \tau \), and \( Z \) can be expressed as a function of \( S_{\text{max}} \) and substituting \( L \) for \( N \)

\[
L = A \left( \frac{1}{\tau} \right)^{c/e} \left( \frac{1}{V} \right)^{1/e} [Z]^{h/e} \sim \frac{1}{S_{\text{max}}^n} \quad (13)
\]

Solving for the exponent \( n \) for line contact gives

\[
n = \frac{c + 1 - h}{e} \quad (14a)
\]

and for point contact

\[
n = \frac{c + 2 - h}{e} \quad (14b)
\]

From Lundberg and Palmgren [5], using the values of \( c \) and \( e \) previously discussed and \( h = 2.33 \), then from Eqs. (14a) and (14b), \( n \) equals 8.1 and 9 for line and point contact, respectively.

**Ioannides-Harris Equation**

Ioannides and Harris [16], using Weibull [8,9] and Lundberg and Palmgren [5,6] introduced a fatigue-limiting stress where from Eq. (4)

\[
f(X) = \frac{\tau - \tau_u}{Z^h} \quad (15)
\]

From Eqs. (4) and (15)

\[
N \sim \left( \frac{1}{\tau - \tau_u} \right)^{c/e} \left[ \frac{1}{V} \right]^{1/e} [Z]^{h/e} \quad (16)
\]

Equation (16) is identical to that of Lundberg and Palmgren (Eq. (12)) except for the introduction of a fatigue-limiting stress. Equation (16) can be expressed as a function of \( S_{\text{max}} \) where

\[
L = A \left( \frac{1}{\tau - \tau_u} \right)^{c/e} \left[ \frac{1}{V} \right]^{1/e} [Z]^{h/e} \sim \frac{1}{S_{\text{max}}^n} \quad (17)
\]

Ioannides and Harris [16] use the same values of Lundberg and Palmgren for \( e \), \( c \), and \( h \). If \( \tau_u = 0 \), then the values of the exponent \( n \) are identical to those of Lundberg and Palmgren (Eqs. (14a) and (14b)). However, for values of \( \tau_u > 0 \), \( n \) is also a function of \( \tau - \tau_u \).

**Zaretsky Equation**

Both the Weibull and Lundberg-Palmgren equations above relate the critical shear stress-life exponent \( c \) to the Weibull slope \( e \). The parameter \( c/e \) thus becomes, in essence, the effective critical shear stress-life exponent, implying that the critical shear stress-life exponent depends on bearing life scatter or dispersion of the data. A search of the literature for a wide variety of materials and for nonrolling-element fatigue reveals that most stress-life exponents vary from 6 to 12. The exponent appears to be independent of scatter or dispersion in the data. Hence, Zaretsky [28] has rewritten the Weibull equation to reflect that observation by making the exponent \( c \) independent of the Weibull slope \( e \), where

\[
f(X) = \tau^{c/e} N^e \quad (18)
\]
From Eqs. (4) and (18)

$$N \sim \left[ \frac{1}{\bar{S}_j} \right] \left[ \frac{1}{V} \right]^{1/e}$$

(19)

Equation (19) differs from the Weibull Eq. (8) and the Lundberg-Palmgren Eq. (12) in the exponent of the critical stress. Zaretsky assumes based upon experience that the value of the stress-exponent \(e = 9\).

Lundberg and Palmgren (5) assumed that once initiated, the time a crack takes to propagate to the surface and form a fatigue spall is a function of the depth \(Z\) to the critical shear stress. Hence, by implication, bearing fatigue life is crack propagation time dependent. However, rolling-element fatigue life can be categorized as “high-cycle fatigue.” (It should be noted that at the time dependent. However, rolling-element fatigue life can be categorized as “high-cycle fatigue.” (It should be noted that at the time (1947) Lundberg and Palmgren published their theory, the concepts of “high cycle” and “low cycle” fatigue were only beginning to be formulated.) Crack propagation is an extremely small time fraction for the total life or running time of the bearing. The Lundberg-Palmgren relation implies that the bearing fatigue life is crack propagation life for the elemental stressed volume is assumed to be infinite. For critical stresses less than the fatigue-limiting stress, the approach is entirely different from that of Ioannides and Harris (6). For critical stresses less than the fatigue-limiting stress, the theoretical lives were normalized to a maximum Hertz stress of 4.14 GPa (600 ksi) and subsequently normalized to the calculated ANSI/ABMA/ISO standards at each stress level. For the Ioannides-Harris comparison shown in Fig. 1, a fatigue limiting stress of 276 MPa (40 ksi) was assumed. For ball bearings, the ANSI/ABMA/ISO standard and the Lundberg-Palmgren equation give identical results. For roller bearings, the results are not identical. From Lundberg and Palmgren (5), the \(L_{10}\) life of a bearing can be determined from the equation:

$$L_{10} = \left[ \frac{C}{P} \right]^n$$

(22)

where \(L_{10}\) is the bearing life in millions of race revolutions, \(C\) is the dynamic load capacity of the bearing or the theoretical load that will produce a life of 1-million race revolutions with a 90 percent probability of survival, \(P\) is the applied equivalent radial load and \(p\) is the load-life exponent. For point contact, \(p\) equals \(n/3\). For line contact, \(p\) equals \(n/2\).

The ANSI/ABMA/ISO standards use a load-life exponent of 10/3 (3.33) for line contact. This results in a value of \(n\) equal to 6.6 and can account in part for the lower life predictions than that experienced in the field.

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$$L_{10} = \left[ \frac{C}{P} \right]^n$$

(22)
and Weibull slope. The value of a slight variation of the maximum Hertz stress-life exponent critical shear stress-life relation and the Weibull slope shows only relation no longer matches reality. Significantly decreases with increases in Weibull slope whereby the as shown in Table 1, the stress-life (load-life) exponent significan
tly or lower. If the slope were factored into the equations then, Zaretsky [18] reflect slopes in the range of 1 to 2 and some cases existing rolling-element fatigue data reported by Parker and relations of Weibull, Lundberg and Palmgren, and Ioannides and bearing life data base.

For line and point contact, the maximum Hertz stress-life and load-life exponents were determined for the Weibull, Lundberg-Palmgren, Ioannides-Harris, and Zaretsky equations as a function of the Weibull slope or Weibull modulus. These results are summarized in Table 1. Both the load-life and stress-life relations are based upon the value of the Weibull slope which for rolling-element bearings is assumed to be 1.11. For Lundberg and Palmgren this assumption resulted in their analysis matching preexisting life equations of Palmgren [7] and their nonpublished bearing life data base.

As shown in Table 1, both the load-life and stress-life relations of Weibull, Lundberg and Palmgren, and Ioannides and Harris reflect a strong dependence on the Weibull slope. The existing rolling-element fatigue data reported by Parker and Zaretsky [18] reflect slopes in the range of 1 to 2 and some cases higher or lower. If the slope were factored into the equations then, as shown in Table 1, the stress-life (load-life) exponent significantly decreases with increases in Weibull slope whereby the relation no longer matches reality.

The Zaretsky equation that decouples the dependence of the critical shear stress-life relation and the Weibull slope shows only a slight variation of the maximum Hertz stress-life exponent \( n \) and Weibull slope. The value of \( n \) varies between 9.5 and 9.9 for line contact and 10 and 10.8 for point contact for Weibull slopes between 2 and 1.11.

These results would indicate that for a ninth power Hertz stress-life exponent for ball bearings, the Lundberg-Palmgren formula best predicts life. However, for a 12th power relation reflected by modern bearing steels, the Zaretsky formula based on the Weibull equation is superior.

Poplawski, Zaretsky, and Peters [19] analyzed the effect of roller profile on cylindrical bearing life. The results of their analysis are summarized in Table 2 for the Lundberg-Palmgren method and the Zaretsky method.

With a closed form solution and not considering edge or stress concentrations, the flat roller profile has the longest predicted life followed by the end-tapered profile, the aerospace profile and the crowned profile, respectively. The full-crowned profile produces the lowest lives. While there are life differences between the end-tapered profile and the aerospace profile, these differences may not be significant. The effect of edge loading on the flat roller profile is to reduce life at higher loads by as much as 98 and 82 percent at lower loads. The actual percentage calculated depends on the analysis used. The effect of roller profile needs to be incorporated into the life prediction methodology.

**Comparison With Bearing Life Data**

Harris [30,31] analyzed 62 rolling-element bearing endurance sets. These data were obtained from four bearing manufacturers, two helicopter manufacturers, three aircraft engine manufacturers, and U.S. Government agency-sponsored technical reports. The data sets comprised deep-groove radial ball bearings, angular-contact ball bearings, tapered roller bearings and spherical roller bearings totaling 7935 bearings. Data for the tapered roller bearings were not included by Harris [30,31] because of insufficient bearing internal geometry for analysis. Also, data for spherical roller bearings and some cylindrical roller bearings were not included because the operating conditions were too complex for accurate analysis. Of the 62 data sets 51 were analyzed. We applied to these 51 data sets the Ioannides-Harris life method (Eq. (23)) and the Zaretsky life method (Eq. (26)). We used the results from Harris [30] for the Lundberg-Palmgren method (Eq. (19)) and the modified Harris method without any changes. Using Weibull statistics we evaluated these data for each bearing type at the \( L_{10} \) life level and compared the results in Table 3.

Table 3 compares both the mean and median values of the ratio of the actual \( L_{10} \) life to that predicted for angular-contact ball bearings, deep-groove ball bearings and cylindrical roller bearings. Ideally, a life ratio \( (L_{10}\text{act.}/L_{10}\text{pred.}) \) of 1 is desirable. This indicates that the actual \( L_{10} \) life (act.) is that which was normalized.

### Table 1—Maximum Hertz Stress-Life Exponent as Function of Weibull Slope for Four Life Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Weibull slope</th>
<th>Stress-life exponent, ( n )</th>
<th>Load-life exponent, ( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSI/ABMA/ISO</td>
<td>1.11</td>
<td>6.6</td>
<td>9</td>
</tr>
<tr>
<td>Weibull eq. (9)</td>
<td>1.11</td>
<td>10.2</td>
<td>11.1</td>
</tr>
<tr>
<td>Lundberg-Palmgren, eq. (13)</td>
<td>1.11</td>
<td>8.1</td>
<td>9</td>
</tr>
<tr>
<td>Ioannides-Harris, eq. (17)</td>
<td>1.11</td>
<td>8.1</td>
<td>9</td>
</tr>
<tr>
<td>Zaretsky, eq. (20)</td>
<td>1.11</td>
<td>9.9</td>
<td>10.3</td>
</tr>
</tbody>
</table>

1No fatigue limit assumed, \( u \) equal 0.

it is certainly not consistent with the vast majority of cylindrical roller and tapered roller bearings designed and used today.

<table>
<thead>
<tr>
<th>Maximum Hertz stress, GPa, (ksi)</th>
<th>ANSI/ABMA/ISO standard</th>
<th>Closed form solution without edge loading</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lundberg-Palmgren (eq. (13))</td>
<td>Flat</td>
<td>End tapered</td>
</tr>
<tr>
<td>Zaretsky (eq. (20))</td>
<td>Flat</td>
<td>End tapered</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
<th>Life</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4 (200)</td>
<td>40</td>
<td>6.6</td>
<td>209</td>
<td>8.1</td>
<td>90</td>
<td>8.1</td>
<td>75</td>
<td>8.1</td>
<td>20</td>
<td>8.1</td>
<td>1509</td>
<td>9.9</td>
<td>537</td>
<td>9.9</td>
<td>431</td>
<td>9.9</td>
<td>86</td>
</tr>
<tr>
<td>1.9 (275)</td>
<td>4.9</td>
<td>6.6</td>
<td>16</td>
<td>8.1</td>
<td>95</td>
<td>8.1</td>
<td>83</td>
<td>8.1</td>
<td>34</td>
<td>8.1</td>
<td>65</td>
<td>9.9</td>
<td>34</td>
<td>9.9</td>
<td>29</td>
<td>9.9</td>
<td>97</td>
</tr>
<tr>
<td>2.4 (350)</td>
<td>4.1</td>
<td>6.6</td>
<td>12</td>
<td>8.1</td>
<td>92</td>
<td>8.1</td>
<td>89</td>
<td>8.1</td>
<td>34</td>
<td>8.1</td>
<td>65</td>
<td>9.9</td>
<td>34</td>
<td>9.9</td>
<td>29</td>
<td>9.9</td>
<td>97</td>
</tr>
</tbody>
</table>

1Normalized to maximum Hertz stress of 4.14 GPa (600 ksi).

2Normalized to maximum Hertz stress of 2.4 GPa (350 ksi).
over prediction occurs with the Lundberg-Palmgren method. For median ratio of 1 would mean that one-half of the predictions values were lower than the actual values. For life ratios less than 1, for each life method and bearing type whereby the predicted parameters of Table 3, life adjustment factors were determined calculations. Based upon a statistical analysis using the Weibull adjustment factors need to be configured into the bearing life engineering certainty. In order to accomplish this, additional life of all bearing types greater than 1 at reasonable levels of approaches infinity and the median life ratio is 1.

The Weibull modulus indicates the dispersion in the \( L_{10\text{act.}} / L_{10\text{pred.}} \) data. The higher the value of the Weibull modulus, the less scatter in the life ratios. The lower the value, the higher the scatter. For each bearing type, the dispersion in the life predictions approximates an exponential distribution. The most desirable result would be one where the Weibull modulus approaches infinity and the median life ratio is 1.

It would be desirable to have the \( L_{10\text{act.}} / L_{10\text{pred.}} \) life ratios of all bearing types greater than 1 at reasonable levels of engineering certainty. In order to accomplish this, additional life adjustment factors need to be configured into the bearing life calculations. Based upon a statistical analysis using the Weibull parameters of Table 3, life adjustment factors were determined for each life method and bearing type whereby the predicted \( L_{10} \) life would be equal to or less than the actual \( L_{10} \) life at a designated confidence level. These statically determined life adjustment factors are shown in Table 4.

Table 4 is based upon the bearing life data of Harris [30,31]. While it is probable that all design and operating parameters necessary to more accurately calculate bearing life were not available to Harris [30,31], these bearing life data are the best compilation in the open literature. The use of these life factors will result in very conservative values of predicted life \( (L_{10\text{pred.}}) \). A calculated \( L_{10} \) life multiplied by a life adjustment factor at a confidence level of 90 percent should result in an actual life \( (L_{10\text{act.}}) \) that will be greater than the predicted life \( (L_{10\text{pred.}}) \) 90 out of 100 times regardless of whose life prediction formula is used. At a confidence level of 99 percent, when using the life adjustment factors, the ratio of \( L_{10\text{act.}} / L_{10\text{pred.}} \geq 1 \) should occur 99 out of 100 times. Whether the designers and users of bearings should use these life adjustment factors or variants of them needs careful consideration and deliberation. That is for the future to decide.

**SUMMARY OF RESULTS**

Comparisons were made between the life prediction formulas for Lundberg and Palmgren, Ioannides and Harris, and Zaretsky and their application to full-scale ball and roller bearing life data. The effect of Weibull slope on bearing life prediction was determined. Life factors are proposed to adjust the respective life formulas in order to normalize them to the data base of each bearing type. The following results were obtained:

1. For the three bearing types studied, the Lundberg-Palmgren method resulted in the most conservative life predictions compared to that of Ioannides-Harris and Zaretsky. The methods of Ioannides-Harris and Zaretsky produced statistically similar results.

2. For a ninth power Hertz stress-life life exponent for ball bearings, the Lundberg-Palmgren formula best predicts life. However, for a 12th power relation reflected by modern bearing steels, the Zaretsky formula based on the Weibull equation is superior.

3. For cylindrical roller bearings, the flat roller profile without considering edge or stress concentrations has the longest predicted life followed by the end-tapered profile, the aerospace profile and the fully crowned profile, respectively. The fully crowned profile produces the lowest lives. While there are life

### TABLE 3.—STATISTICAL COMPARISON OF FIELD AND LABORATORY BEARING LIFE DATA TO THAT PREDICTED

<table>
<thead>
<tr>
<th>Bearing type</th>
<th>( L_{10\text{act.}} / L_{10\text{pred.}} )</th>
<th>Weibull modulus</th>
<th>Probability of over predicting life, percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Median</td>
<td></td>
</tr>
<tr>
<td>Lundberg-Palmgren method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular-contact ball brgs</td>
<td>14.5</td>
<td>6.4</td>
<td>0.68</td>
</tr>
<tr>
<td>Deep-groove ball brgs</td>
<td>3.5</td>
<td>3</td>
<td>1.41</td>
</tr>
<tr>
<td>Cylindrical roller brgs</td>
<td>20.1</td>
<td>14.1</td>
<td>0.98</td>
</tr>
<tr>
<td>Ioannides-Harris method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular-contact ball brgs</td>
<td>5.7</td>
<td>2</td>
<td>0.70</td>
</tr>
<tr>
<td>Deep-groove ball brgs</td>
<td>4.4</td>
<td>3.3</td>
<td>1.14</td>
</tr>
<tr>
<td>Cylindrical roller brgs</td>
<td>12.4</td>
<td>7.8</td>
<td>0.89</td>
</tr>
<tr>
<td>Modified Ioannides-Harris method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular-contact ball brgs</td>
<td>4.6</td>
<td>1.9</td>
<td>0.65</td>
</tr>
<tr>
<td>Deep-groove ball brgs</td>
<td>2.7</td>
<td>2.4</td>
<td>1.55</td>
</tr>
<tr>
<td>Cylindrical roller brgs</td>
<td>4.7</td>
<td>3.2</td>
<td>0.98</td>
</tr>
<tr>
<td>Zaretsky method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular-contact ball brgs</td>
<td>3.1</td>
<td>1.6</td>
<td>0.75</td>
</tr>
<tr>
<td>Deep-groove ball brgs</td>
<td>2.7</td>
<td>2.2</td>
<td>1.36</td>
</tr>
<tr>
<td>Cylindrical roller brgs</td>
<td>7.1</td>
<td>5.0</td>
<td>1.02</td>
</tr>
</tbody>
</table>

*14 bearing data sets.
*26 bearing data sets
*11 bearing data sets
*3From Harris [30]. Contains STLE life factors [15].
*4From Eq. (17). Contains STLE life factors [15]. Does not contain life factors for material and processing.
*5From Harris [30]. Contains life factors for lubricant contamination only.

Life prediction (pred.). Ratios greater than 1 mean that the predicted values were lower than the actual values. For life ratios less than 1, the predicted values are higher than the actual values. A median ratio of 1 would mean that one-half of the predictions were high and the other one-half were low. The least incidence of prediction occurs with the Lundberg-Palmgren method. For both the Ioannides-Harris method and the Zaretsky method, the incidence of over prediction is statistically equivalent.

### TABLE 4.—ROLLING-ELEMENT BEARING LIFE ADJUSTMENT FACTORS FOR LIFE PREDICTION METHODS BASED ON \( L_{10\text{act.}} / L_{10\text{pred.}} = 1 \)

<table>
<thead>
<tr>
<th>Confidence level, percent</th>
<th>Angular-contact ball bearings</th>
<th>Deep-groove ball bearings</th>
<th>Cylindrical roller bearings</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>0.39</td>
<td>0.78</td>
<td>2.1</td>
</tr>
<tr>
<td>95</td>
<td>0.14</td>
<td>0.47</td>
<td>0.99</td>
</tr>
<tr>
<td>99</td>
<td>0.012</td>
<td>0.15</td>
<td>0.19</td>
</tr>
<tr>
<td>99.9</td>
<td>----</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Modified Ioannides-Harris method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.11</td>
<td>0.70</td>
<td>0.47</td>
</tr>
<tr>
<td>95</td>
<td>0.04</td>
<td>0.44</td>
<td>0.22</td>
</tr>
<tr>
<td>99</td>
<td>0.003</td>
<td>0.15</td>
<td>0.042</td>
</tr>
<tr>
<td>99.9</td>
<td>----</td>
<td>0.04</td>
<td>0.004</td>
</tr>
<tr>
<td>Zaretsky method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>90</td>
<td>0.13</td>
<td>0.55</td>
<td>0.88</td>
</tr>
<tr>
<td>95</td>
<td>0.05</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>99</td>
<td>0.006</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>99.9</td>
<td>----</td>
<td>0.02</td>
<td>0.01</td>
</tr>
</tbody>
</table>
differences between the end tapered profile and the aerospace profile, these differences may not be significant.

4. The effect of edge loading on the flat roller profile is to reduce life at high loads by as much as 98 and 82 percent at the low loads. The actual percentage calculated depends on the analysis used.

5. The resultant predicted life at each stress condition not only depends on the life equation used but also on the Weibull slope assumed. The smallest change in predicted life with Weibull slope comes with the Zaretsky equation. For Weibull slopes of 1.5 and 2, both the Lundberg-Palmgren and Ioannides-Harris (where \( r_0 = 0 \)) equations predict lower lives than the ANSI/ABMA/ISO standard.

6. Based upon the Hertz stresses for line contact, the load-life exponent \( p \) of 10/3 for roller bearings results in a maximum Hertz stress-life exponent \( n \) equal to 6.6. This value is inconsistent with that experienced in the field. A value of \( n \) equal 8.1 for roller bearings should be used with the Lundberg-Palmgren method and 9.9 for the Zaretsky method whereby \( p \) equals 4.05 and 4.95, respectively.

REFERENCES

Comparisons were made between the life prediction formulas of Lundberg and Palmgren, Ioannides and Harris, and Zaretsky and full-scale ball and roller bearing life data. The effect of Weibull slope on bearing life prediction was determined. Life factors are proposed to adjust the respective life formulas to the normalized statistical life distribution of each bearing type. The Lundberg-Palmgren method resulted in the most conservative life predictions compared to Ioannides and Harris, and Zaretsky methods which produced statistically similar results. Roller profile can have significant effects on bearing life prediction results. Roller edge loading can reduce life by as much as 98 percent. The resultant predicted life not only depends on the life equation used but on the Weibull slope assumed, the least variation occurring with the Zaretsky equation. The load-life exponent \( p \) of 10/3 used in the ANSI/ABMA/ISO standards is inconsistent with the majority roller bearings designed and used today.