Slide Conveying of Granular Materials - Thinking out of the Glovebox[1]

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Abstract

The vibratory conveyor, routinely employed for normal-gravity transport of granular materials, usually consists of a continuous open trough vibrated sinusoidally to induce axial movement of a granular material. Motivated in part by a hypothetical application in zero gravity, we propose a novel modification of the vibratory conveyor based on a closed 2d trough operating in a "slide-conveying" mode, with the granular mass remaining permanently in contact with the trough walls. We present a detailed analysis of the mechanics of transport, based on a rigid-slab model for the granular mass with frictional (Coulomb) slip at the upper and lower walls. The form of the vibration cycle plays a crucial role, and the optimal conveying cycle is not the commonly assumed rectilinear sinusoidal motion. The conveying efficiency for the novel slide conveyor will be presented for several simple vibration cycles, including one believed to represent the theoretical optimum.

Background - Vibratory Conveying

Granular media represent an interesting class of materials that can exhibit a spectrum of complex flow behavior, ranging from solid-like to gas-like. Understanding and describing their mechanical behavior poses a scientific interesting and technologically important challenge, since a many processes involve handling and processing of granular solids. One particular interesting class of mechanical processes are those involving vibratory excitation or "fluidization" of granular masses. Following a long-standing scientific fascination with the wave-like patterns on the surface of vibrated powders and grains, dating back to the celebrated work of Faraday (1831), there has been a resurgence of activity in recent times, accompanying the growth of theoretical interest in pattern formation in non-linear dynamical systems (See, e.g., Bizon *et al.* 1999). There is an almost completely disjoint body of engineering literature on vibratory conveying of granular materials.

Vibratory conveyors, routinely employed in industry for transport of granular materials, generally consist of a continuous trough vibrated sinusoidally in time to induce axial movement of the granular material. Fig.1 presents a schematic cross-sectional view of a vibratory conveyor. Key process variables are inclination α, amplitude A , frequency f and direction β of vibration, along with frictional/mechanical properties of the conveyor surface and the granular material. In recent works, Nedderman & Harding (1990) extend the earlier analysis of Booth and McCallion (1963) and present optimization studies for horizontal and inclined sliding. In most applications, vibratory conveyors work in one of two distinct modes, *slide conveying* or *flight conveying*, accordingly as $N < 1$ or $N > 1$, respectively, where N is the nondimensional *throw number*

$$N = \Gamma \sin\beta / \cos\alpha, \quad \text{with} \quad \Gamma = A\omega^2/g, \tag{1}$$

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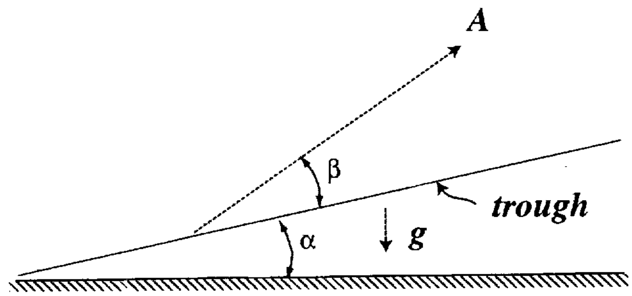

Figure 1: Definition sketch of a vibratory conveyor

in which the various symbols are defined above and g denotes gravity. Assumptions common to most existing models of vibratory transport are: 1.) The granular mass can be treated as a single rigid slab, 2.) side-wall friction and air drag are negligible, 3.) the granular mass interacts with the trough wall as a rigid body with Coulomb friction, and 4.) the trough executes a rectilinear sinusoidal motion (A in Fig. 1), This engineering model corresponds essentially to the lowest Γ states discussed in basic scientific studies (Bizon *et al.* 1999).

Present Work

It is evident that pure slide conveying and/or a closed trough would be required in a zero-g environment, and the intial phase of the current work is concerned with the theoretical analysis of the closed 2d trough with parallel walls, completely filled with a granular mass. As a starting point, we adopt the first three of the assumptions listed immediately above but consider a more general periodic motion than 4.). The basic equations (nonlinear ODEs) are but slight modifications of those given elsewhere (Nedderman and Harding, 1990) and are not repeated here. They lead to an interesting optimal control problem, involving the maximization of axial transport subject to constrained periodic forcings. The present talk will discuss a few preliminary results, including some numerical simulations for simple periodic cycles and a conjectured form of the theoretically optimal cycle.

References

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Background and Outline of Talk

- Work described is part of a program of research on the mechanics of vibrated granular layers, with objectives:

  1. Development of fluid-mechanical models to explain complex ("Faraday") patterns on vertically-vibrated layers, and

  2. Connection to engineering models of vibratory conveying, with a view to possible variable-g application (e.g. heat transfer)

- Focus of this talk is on results from recent efforts on Item 2, including

  – Review of current modes and models of conveying ("throw" and "slide") and limitations in reduced g

  – Analysis of closed-channel slide conveying, with discussion of optimal vibration cycles and discussion of recently discovered exact solution to the problem

  – Future work

"Throw" Conveying

- (a) Layer in flight - normal gravity g_0
- (b) "Solid-block" model - basic equations:

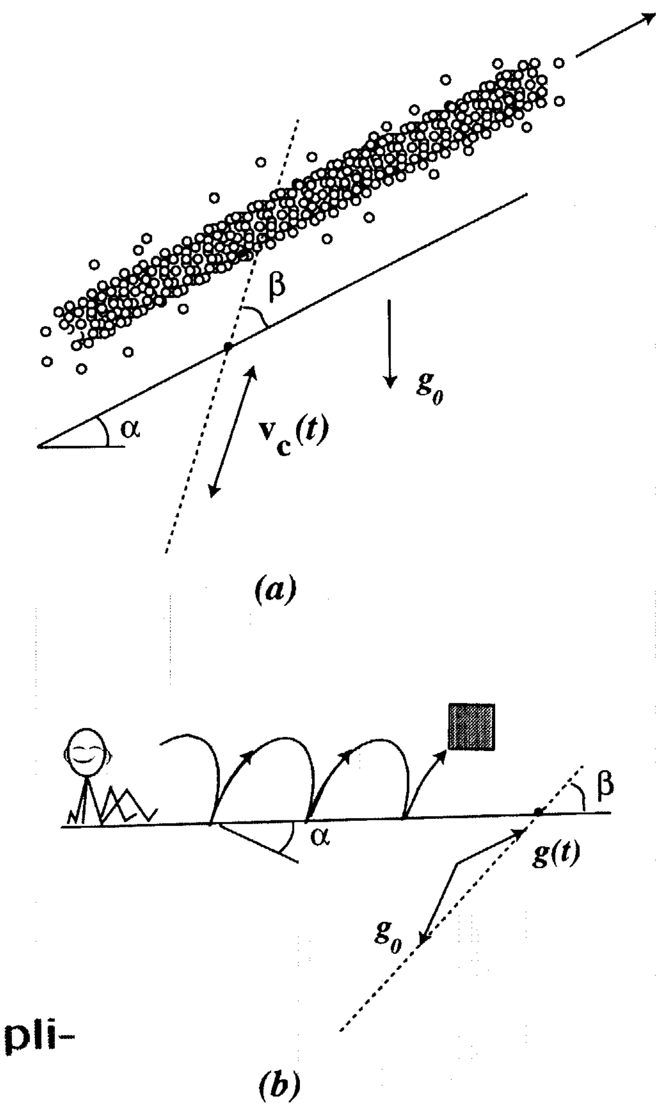
$$\frac{d\mathrm{v}(t)}{dt} = \mathrm{f}(t) + \mathrm{g}(t), \text{ with } \mathrm{g}(t) = \mathrm{g}_0 - \dot{\mathrm{v}}_c(t)$$

relative to plate, where

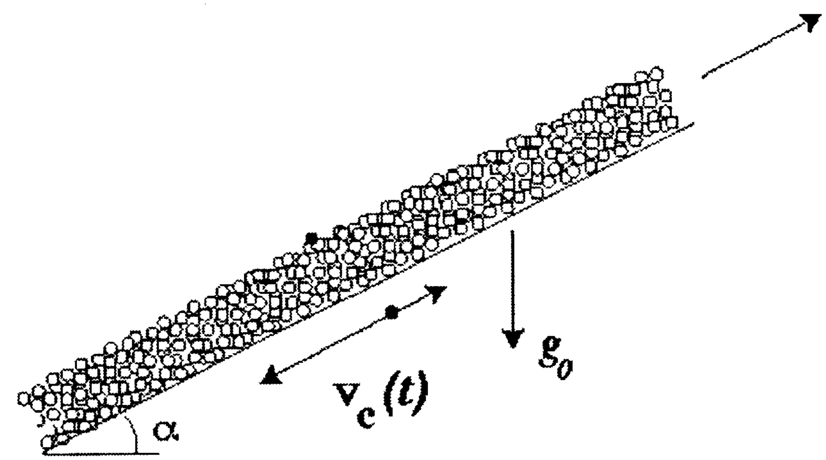
– $\mathrm{g}(t)$ is virtual gravity

– $\mathrm{f}(t)$ is specific plate-contact force (frictional-elastic, generally impulsive).

- Without a "lid", the plate-displacement amplitude and frequency A, ω give unbounded $\Gamma = A\omega^2/g_0$ in zero-g.

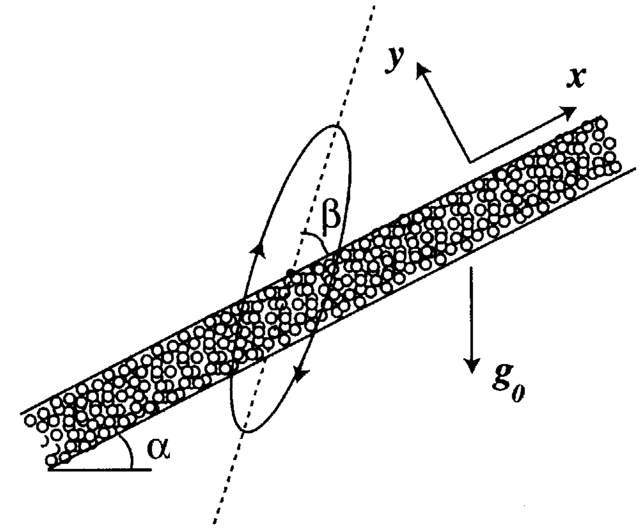


"Slide" Conveying



- Employed for fragile materials.
- Layer in permanent contact with surface and active contact force $f(t)$ is purely frictional.
- In-line ($\beta = 0$), unsymmetrical $v_c(t)$ can provide transport:
- Tilt ($\beta > 0$) enhances efficiency but amplitude is limited by magnitude of g_0

Closed-Conduit Slide Conveying



- A, ω not limited by gravity - device can work in zero g.
- "Ideal" cycles with zero frictional dissipation exist.
- A simple theory arises - with constant wall friction μ, length scaled by A and time by ω^{-1}, the x-velocity $u(t)$ satisfies

$$\frac{du}{dt} = T(t) - \mu|N(t)|\mathrm{sgn}(u), \quad \text{for} \quad |T(t)| \geq \mu|N(t)|$$

$$u = 0, \quad \text{otherwise}$$

with 2π-periodic coefficients:

$$T = -(g_0 \sin\alpha + \ddot{u}_c)/A\omega^2, \quad N = -(g_0 \cos\alpha + \dot{v}_c)/A\omega^2$$

where pressure "head" should be added to $g_0 \sin\alpha$.

Optimality

- <u>Present effort</u>:
  - establish optimality criteria and ideal cycles
  - investigate optimal real cycles (e.g. elliptical cycle in Fig. above)

  Lack of u-differentiability of ODE for u rules out standard variational methods (mitigated by exact solution below)

- As, e.g., simplest type of optimality, maximize net cyclic displacement:

$$\max_{T,N\in\mathcal{C}} X, \quad \text{where} \quad X = \int_0^{2\pi} u(t)\,dt$$

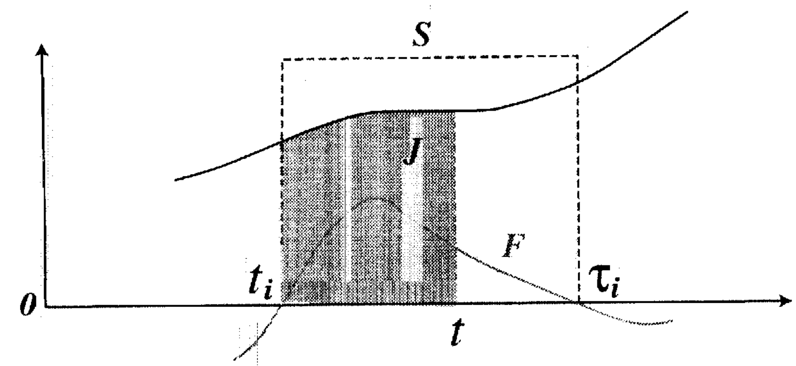
where $\mathcal{C}$ denotes constraint class of $T(t), N(t)$

Solution

With intervals of sliding in $(0, 2\pi)$:

$$F(t) := |T(t)| - \mu|N(t)| \geq 0, \;\; t_i \leq t \leq \tau_i$$

$i = 1, 2 \ldots$, the <u>exact</u> solution is



$$u(t) = \sum_i S_i(t) u_i(t)$$

where

$$u_i(t) = R\{J_i^{(+)}(t)\} - R\{-J_i^{(-)}(t)\}$$

$$J_i^{(\pm)}(t) = \int_{t_i}^{t} [T(t') \mp \mu|N(t')|] dt'$$

$$S_i(t) = H(t - t_i) - H(t - \tau_i)$$

$R(u)$, $H(u) \equiv R'(u)$, and S_i denote ramp, Heaviside-step and window functions, and $J^{(\pm)}$ are right- and left-directed "impulses", with $J^{(-)} \geq J^{(+)}$

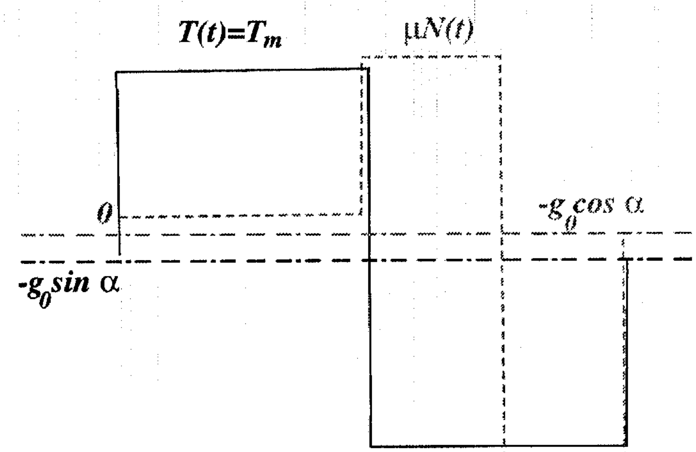
Ideal and Elliptical Cycles

Exact solution has been employed to:

- establish ideal cycle for bounded forcing,

$$\mu|N(t)|_{\max} \leq |T(t)|_{\max} \leq T_m,$$

found to be square waves, with X=1



- work out (complicated!) two-parameter algebraic expression for X for the elliptical cycle. The optimal parameter values have as yet not been determined.

Future Work

- Work out details of optimal elliptical cycle for slide conveying with view towards simple experiment
- Consider partially full channel and throw conveying, in conjunction with our other work on stability of vibrated layers