Analytic Guided-Search Model of Human Performance Accuracy in Target-Localization Search Tasks

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Summary

Current models of human visual search have extended the traditional serial/parallel search dichotomy. Two successful models for predicting human visual search are the Guided Search model and the Signal Detection Theory model. Although these models are inherently different, it has been difficult to compare them because the Guided Search model is designed to predict response time, while Signal Detection Theory models are designed to predict performance accuracy. Moreover, current implementations of the Guided Search model require the use of Monte-Carlo simulations, a method that makes fitting the model’s performance quantitatively to human data more computationally time consuming. We have extended the Guided Search model to predict human accuracy in target-localization search tasks. We have also developed analytic expressions that simplify simulation of the model to the evaluation of a small set of equations using only three free parameters. This new implementation and extension of the Guided Search model will enable direct quantitative comparisons with human performance in target-localization search experiments and with the predictions of Signal Detection Theory and other search accuracy models.

Introduction

In standard visual-search tasks, the observer looks for a target among a set of distractors. When the target differs greatly from the distractors along a single feature dimension (e.g. contrast, orientation, color, etc.), the time to find the target is relatively constant as a function of the number of elements (set size). This fact is commonly interpreted as evidence for parallel search (i.e. the simultaneous examination of potential targets). Alternatively, when the target is similar to the distractors, the distractors are heterogeneous, or the target can be differentiated from the distractors only by the combination of two or more feature dimensions (conjunctions), the response time increases drastically with set size. This finding is commonly interpreted as evidence for a temporally serial search (i.e. the sequential examination of potential targets). Although the parallel/serial dichotomy has dominated research for more than two decades (refs. 1–4), and is central to many theories of visual search (e.g., Feature Integration Theory; ref. 5), more recently, models of visual search have moved away from the original strict serial/parallel dichotomy.

There have been two traditions in studying the processes mediating visual search. One approach has been to allow extended viewing of the stimulus and to measure observer response times (e.g., refs. 6–8), while a second approach has been to use fixed duration displays and to measure detection accuracy, the probability of correctly detecting the presence of the target (e.g., refs. 9, 10). The response-time results can be predicted by a two-stage Guided Search (GS) model (refs. 6, 11) in which an initial parallel system guides a subsequent serial-search stage. The accuracy results can be predicted by a single-stage Signal Detection Theory (SDT) model (ref. 12) in which processing is parallel but noisy (e.g. refs. 9, 10, and 13–16). Even though the GS and SDT models are based on fundamentally different assumptions about human visual information processing, progress in our understanding of search has been hampered by the difficulty associated with directly comparing these two models because they were developed for different experimental paradigms and are applicable to different empirical measurements. A second difficulty is that current implementations of the GS model (ref. 11) require Monte-Carlo simulations, which make fitting the model to human data more computationally time consuming.

This report describes an analytic extension of the GS model, the Guided Search Accuracy (GSA) model, which predicts performance accuracy in a target-
localization search task as a function of set size. We develop analytic mathematical expressions that allow quantitative fitting of the model to human data in a time-efficient manner using only three free parameters. The significance of this implementation and extension is that it will allow the direct and quantitative comparison of Guided Search and Signal Detection Theory models in target-localization search tasks.

Theory

The Guided Search model

The GS model (ref. 11) was developed to predict response times as a function of set size in one type of visual search task. In this type of search experiment, there are two kinds of displays: target-present and target-absent. Each consists of N elements. In target-present trials, one element, the target, differs from the others, the distractors, while in target-absent trials, all the elements are distractors. The observer's task is to search the display to determine whether it is a target-present trial or target-absent trial and, as quickly as possible, to make a response indicating the decision. The dependence of the response times on the set size is measured.

The GS model assumes that each element is processed by broadly tuned channels that correspond to categorical features (e.g. "red", "green", "bright", etc.). The bottom-up response of each element is determined by a weighted average of the difference in output between that element and its neighbors. The top-down response is a function of the match of the element to the designated target. The final internal response associated with a given element is a weighted sum of the top-down and bottom-up responses. This final response is perturbed by the addition of internal neural noise, assumed to be Gaussian, to yield a final "activation" for each element. Finally, visual attention serially searches through those elements, whose activations are above a threshold ($\lambda$), according to a self-terminating procedure. Unlike the standard serial search model where visual attention proceeds randomly from one element to another (ref. 5), in the GS model, visual attention proceeds in an activity dependent order. It begins with the element that elicited the highest activation and continues in order of decreasing activation. The search terminates when either the target is found, or no elements remain with activation above the activation threshold. Rejected elements are not revisited. It is assumed that attending to each element requires a fixed amount of processing time, and thus that the response time is determined by the total number of elements searched, and also that if the target is attended, the observer always correctly identifies that trial as a target-present trial. If the search is terminated without processing the target (because the target did not exceed threshold), the observer guesses "target absent" 97% of the time and "target present" 3% of the time.

The Guided Search Accuracy model

We have developed an analytic extension of the GS model, the GSA model (Fig. 1), which predicts accuracy in a localization search task. In localization search tasks, a fixed-duration display containing a single target and a number of distractors is presented to the observer. The observer then reports which of the N locations contains the target. The accuracy of correctly identifying the target location is measured as a function of set size (N). The structure of the GSA model is nearly identical to the GS model, except that to predict localization accuracy for fixed stimulus durations, the serial attention stage of the GSA model is restricted to examining a fixed number of elements. If the target has not been found within the restricted presentation time, the model is forced to guess. The GSA model consists of two stages: a noisy parallel-processing stage and a noise-
free serial-attention stage. In the parallel-
processing stage, each element in the
display elicits a noisy response, its
activation. We assume that each element's
activation can be described by a Gaussian
probability distribution, and that the target
and distractor distributions have equal
variances. A target elicits, on average, a
larger activation than a distractor (Fig. 2).
The target-distractor discriminability is \( d' \),
a measure of the distance between the target
and distractor distributions. The results of
this parallel processing stage are then sent
to the noise-free serial attention stage, that
first orders the supra-threshold elements
(those with activations greater than the
threshold, \( \lambda \)) according to their activation.
Then, visual attention serially processes the
supra-threshold elements, beginning with
the element with the highest activation and
continuing in decreasing order of activation
(Fig. 1). Processing each element with
visual attention requires a fixed amount of
time and processing continues until all
supra-threshold elements have been
processed or the target is found, or until the
display presentation ends. If visual
attention processes the target, the model
always correctly identifies the location of
the target (even if its activation from the
initial parallel processing stage was by
chance lower than that of a distractor). If
the target was not processed during the
display presentation, the GSA model is
forced to guess. If all supra-threshold
elements have not been processed, the
model chooses the remaining supra-
threshold element with the highest
activation. Otherwise (if all supra-threshold
elements have been processed), it chooses
randomly among all remaining sub-
threshold elements. Errors can occur when
there are either more supra-threshold
elements than can be processed serially
prior to the display being terminated, or
when the activation of the target is sub-
threshold.

**Simplification of the GSA model**

To facilitate fitting the analytic GSA model
to human performance data, we have
simplified the original model (ref. 11) by
reducing the number of free parameters.
First, because in most target-localization
search tasks, the observer is searching for an
*a priori* known target and not an odd-man
out, we assume that the responses of the
parallel processing stage are entirely
determined by the similarity of an element
to the known target (top-down activation).
Thus, we assume that the contribution of the
bottom-up processing is negligible; either
the output associated with an element does
not depend on a weighted average of the
difference of filter outputs for that element
and its neighbors (no lateral inhibition), or if
it does, this effect does not vary across set-
size conditions (constant lateral inhibition).
Therefore the GSA model is applicable to
search displays that contain widely spaced
elements or that maintain a constant inter-
element distance for all set sizes. Second,
we assume that the activation threshold and
the maximum number of elements serially
processed are constant across trials. All
other aspects of the model are identical to
those of the Guided Search model 2.0 (ref.
11).

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1The equal-variance assumption is included to reduce
the number of free parameters in the GSA model, but
can be relaxed by adding a parameter specifying the
ratio of the variance of the target distribution to that of
the distractor.

2In the both the GS and GSA models and in other
models with a serial attention mechanism, visual
attention is assumed to be a homunculus that can
determine without error the identity of an attended
element.

3The GSA model could be extended to include effects
of lateral inhibition if an appropriate model of how
target detectability varies as a function of element
density is specified. Alternatively, lateral interactions
between the activity elicited by each element can be
avoided if the elements are far apart such that the
distance between elements is large relative to their
size.
Analytic implementation of the GSA model

We have developed analytic expressions for the performance accuracy (% correct) of the GSA model. Because both the target and distractor distributions are equal variance Gaussians, without loss of generality, we can rescale these distributions so that: 1) both target and distractor distributions have unit variance, 2) the mean distractor activation is zero, 3) the mean target activation is $d'$. Thus, the probability of a distractor producing an activation $x$ is:

$$d(x) = \frac{1}{\sqrt{2\pi}} \exp(-x^2/2)$$

(1)

The probability of the target producing an activation $x$ is:

$$t(x) = \frac{1}{\sqrt{2\pi}} \exp(-(x - d')^2/2)$$

(2)

We denote the cumulative probability of a distractor producing an activation less than $x$ by

$$D(< x) = \int_{-\infty}^{x} d(x)dx,$$

(3)

the probability of a distractor producing an activation greater than $x$ by

$$D(> x) = \int_{x}^{\infty} d(x)dx,$$

(4)

the probability of the target producing an activation less than $x$ by

$$T(< x) = \int_{-\infty}^{x} t(x)dx,$$

(5)

and the probability of the target producing an activation greater than $x$ by

$$T(> x) = \int_{x}^{\infty} t(x)dx.$$  

(6)

We compute the probability of the GSA model correctly locating the target in two separate regimes: 1) when the target activation is supra-threshold (i.e., larger than $\lambda$), 2) when the target activation is sub-threshold. Because, in many experiments, the display duration is fixed (usually brief: 50-300 ms) and examining each element requires a fixed amount of processing time, there is time to serially examine at most $k$ elements, which may be less than the total number of supra-threshold elements. Thus, we assume that on each trial, at most $k$ elements are serially examined, and that $k$ is less than or equal to $N$ (if $k\geq N$, there is time to search all the elements). If the target is examined, the model always correctly chooses it.

Otherwise, the model chooses the element with the next highest activation threshold, or if none of the remaining elements is supra-threshold, it randomly guesses among the sub-threshold elements.

In the first regime, when the target activation is supra-threshold, there are two ways the model can correctly identify the target: 1) if the target ranks among the $k$ highest activations, it is guaranteed to be processed and thus is always correctly located, 2) if the target has the $k+1$ highest activation, it is next in line at the end of the trial, so the model correctly chooses it.

\footnote{Instead of choosing the $k+1^{st}$ element, a variation of the model could guess among all unprocessed elements. This latter version of the model produces slightly worse performance for a given set of...}
Otherwise, it incorrectly chooses the distractor, which is next in line. Thus, for supra-threshold targets, the target is always correctly chosen if it is among the k+1 highest activation, and is never correctly chosen if it is not. To explicitly compute the probabilities, it is necessary to consider all possible permutations of distractor labeling, which is performed in the factorial terms of the subsequent equations.

The probability of a correct response given that the target is supra-threshold and is among the k+1 highest activations (Pc,) is the sum from j = 1 to k+1 of the probability of the target being supra-threshold and being the jth highest activation. Each term is the product of the probability of the target taking a value x, the probability of exactly j-1 distractors taking a value larger than x, the probability of exactly N-j distractors taking a value less than x, times a binomial coefficient describing the number of possible distractor permutations:

\[
P_{c_1} = \sum_{j=1}^{k+1} \binom{k}{j-1} \int_{\lambda} \left[ T(x) [D(x)]^{j-1} [D(x)]^{N-j} \right] dx
\]

Therefore, percent correct localization for the case in which the target activation is supra-threshold can be calculated as the sum of the probabilities of the target activation ranking among the k+1 highest activations (Pc1).

In the second regime, the target activation is sub-threshold, and is sometimes correctly guessed from all the unprocessed elements (Pc2). If the number of supra-threshold distractors, j, is greater than k, the model processes k distractors and then chooses the distractor next in line, and so is never correct. If the number of supra-threshold distractors, j, is less than or equal to k, the model first processes all j supra-threshold distractors and then randomly chooses one of the N-j sub-threshold elements. Therefore, it is correct only if this random choice is the target, which occurs with a probability of 1/(N-j). The probability of a correct guess (Pc2) is then the sum over j equal to k of 1/(N-j) times the product of the probability of the target being subthreshold, the probability of exactly j distractors being supra-threshold, and a binomial coefficient describing all possible distractor permutations:

\[
P_{c_2} = \sum_{j=0}^{k} \binom{k}{j} \frac{1}{N-j} \cdot T(<\lambda) [D(\lambda)]^j [D(<\lambda)]^{N-j}
\]

Because the two possibilities described by Eq. 7 and 8 are mutually exclusive, the total percent correct in the localization of the target for the GSA model is the sum of these two independent probabilities:

\[
P_c = P_{c_1} + P_{c_2}
\]

**Results**

We investigated the model's performance (Pc) as a function of its three free parameters: the activation threshold (\(\lambda\)), the maximum number of elements that can be processed serially within the presentation time (k), and the discriminability between the target and distractors (d').

**Effect of the activation threshold**

The activation threshold is a primary cause of performance errors for the GSA model. Figure 3 shows the probability distributions of the number of supra-threshold distractors for a display with 6 distractors (N=7) for four different values of the activation threshold (\(\lambda = -10, 0, 1, 2\)). The probabilities of the target (d'=1) exceeding the threshold are 0.98, 0.84, 0.5 and 0.17, respectively, for these activation thresholds. When there is no activation threshold (or it is very negative) then the activations of all 6 distractors are always supra-threshold. As

parameters than the version presented here. However, the two models have virtually identical sensitivity to variations in the parameters.
the threshold is increased, the expected number of distractors exceeding threshold decreases and for high thresholds the shape of the distribution also changes (Fig. 3). Similar decreases in the probability of the target's activation exceeding threshold also occur. Thus, an increase in the activation will generally decrease performance accuracy for two reasons. First, it decreases the probability that the target is supra-threshold and thus that it is examined by the serial processor, which forces the model to guess more frequently. Second, it decreases the number of distractors that are examined by the serial processor and discarded from the guessing subset.

For those set-size conditions in which the number of elements in the display is small so that all (or all but one) of the elements can be processed serially (N ≤ k + 1), the activation threshold is the only source of performance errors. If there were no activation threshold, for N≤k the model would serially process every element in the display and would always correctly identify the target (Pc =100). Similarly if N=k+1, the model would serially examine N-1 elements, then correctly guess the remaining element (if it hadn't already found the target) again producing perfect performance. Figure 4 shows the decrease in performance caused by increasing the activation threshold for a fixed k = 4 and d' = 1.0. As the activation threshold is increased, there is an increasing probability that the target activation is sub-threshold, and that the model incorrectly chooses a distractor. If the target is sub-threshold, the model first examines the supra-threshold distractors. If the number of supra-threshold distractors is greater than k, then the model incorrectly chooses the k+1th distractor. If the number of number of supra-threshold distractors is less than k, then the model is forced to guess randomly among the remaining elements. For high thresholds, the guessing is less accurate because there are a larger number of sub-threshold distractors. Therefore, a high threshold lowers performance, because it causes the model to guess more frequently and less accurately.

Effect of the maximum number of elements serially processed (k)

The time limit imposed on the serial allocation of visual attention is the second source of errors for the GSA model. If an element is processed in x ms and the processing is temporally serial, then in a presentation time τ, the model can only process k = τ/x elements. Therefore, if there are N > k+1 elements, the model will be unable to process all of the elements necessary to make a perfect decision. There will be a non-zero probability that the target is not processed by serial attention. In these cases the model incorrectly chooses the k+1th element. Alternately, when there are N < k+1 elements, the serial processor will be able to process all N-1 elements necessary for a perfect decision unless the target itself is sub-threshold (i.e. no errors will be generated due to the serial processing time). Figure 5 shows the performance accuracy as a function of set size for four different values of the maximum number of serially processed elements (k = 2, 4, 6, and 8). Increasing k changes the set size at which performance degrades due to the serial processing (inflection point in the curve) and the rate at which performance degrades (downward trend of the curve). Figure 5A shows that changes in k can greatly affect performance for a low activation threshold (λ = 0). Figure 5B shows the effect of varying k is much less dramatic for a larger activation threshold (λ = 1). Thus, when λ is high, most errors are generated by λ itself, and changes in k do not affect performance as much. This is because λ (together with N) determines the average number of supra-threshold elements. A high λ limits the effective number of elements available to the serial processor. If the effective (as opposed to actual) number of elements is
large, then performance is limited largely by k. If it is smaller than k, then performance is mostly limited by \( \lambda \).

**Effect of target-distractor discriminability (d')**

In the GSA model, target-distractor discriminability is determined by the distance between the target and distractor distributions (Fig. 2). Decreasing the physical difference between the target and the distractors decreases the difference between the mean activation of the target (d') and the mean activation of the distractor (always zero). As the target-distractor discriminability is reduced, the probability that the target does not rank among the k highest activations increases, thereby increasing errors. In addition, the probability that the target activation does not exceed threshold also increases thereby generating even more errors. As a result, decreasing target-distractor discriminability will reduce the GSA model performance.

Figure 6 shows the model's performance accuracy for four levels of target-distractor discriminability (d' = 2.0, 1.5, 1.0, 0.5) as a function of set size for k=4. Figure 6A shows accuracy for \( \lambda = 0 \) and Figure 6B for \( \lambda = 2 \). These results show that d' is an important factor that influences performance in two ways. First, d' determines the overall level of performance, lower d' values produce less accurate performance for all conditions examined (downward shift). Second, lower d' values increase the observed set-size effects. In particular, for high thresholds (\( \lambda = 2 \)), as shown in Figure 6B, performance initially decreases rapidly as a function of set size, then decreases much more slowly. The set size for which this change is observed depends on the value of d'; high d' values produce a rapid decrease in performance only for small set sizes, while low d' values produce rapid decreases in performance over a larger range of set sizes.

**Monte Carlo Simulations**

To verify that the analytic expressions above (Eqs. 1-9) accurately describe the GSA model, we compared the results from these expressions with predictions from standard Monte Carlo simulations of the sequence of probabilistic events described in Fig. 1. Performance predictions for the GSA analytic expressions were in good agreement with results from the brute force Monte-Carlo simulations of the model across the range of parameter settings tested.

**Conclusions**

We have developed an extension of the Guided Search model, the GSA model, which uses explicit analytic expressions to compute accuracy in a target-localization task. Our implementation allows the performance accuracy of the Guided Search model to be directly compared to that of human observers, the SDT model, or any other model of target-localization accuracy. We explicitly investigated the effect of varying three model parameters (activation threshold, maximum number of elements serially processed within the presentation time, and the target-distractor discriminability) on localization accuracy as a function set size. The GSA model will facilitate the direct and quantitative comparison of the ability of the Guided Search and Signal Detection Theory models to explain human search performance in target-localization tasks.
References


Figure 1. Schematic of the Guided Search Accuracy model. The parallel processing stage generates a noisy response for each element (its activation). If this activation is above a threshold ($\lambda$), it is passed to the serial processing stage. Attention sequentially examines the $k$ highest activations (only those that are supra-threshold) in descending order and always correctly identifies the target if was examined. If it runs out of time or supra-threshold elements, it guesses. If it runs out of time before running out of supra-threshold elements, it picks the unexamined element with the highest activation. If it runs out of supra-threshold elements, it randomly picks one of the subthreshold elements.
Figure 2. Probability distributions of the responses for the target and a distractor. The response distribution of a target whose $d'$ is 1 is illustrated.

Figure 3. Probability distributions of the expected number of supra-threshold elements for different values of activation threshold for a fixed target-distractor discriminability ($d'$ = 1.0) and set size ($N$ = 7).
Figure 4. Accuracy (Pc) as a function of set size for different values of activation threshold (\(\lambda\)) for a fixed maximum number of serially processed elements (\(k = 4\)), and a fixed target-distractor discriminability (\(d' = 1.0\)).
Figure 5. Accuracy (Pc) as a function of set size for different values of the maximum number of serially processed elements (k = 2, 4, 6, and 8), for two different activation thresholds, and a fixed target-distractor discriminability (d' = 1.0). A) λ = 0, B) λ = 1.
Figure 6. Accuracy (Pc) as a function of set size for different values of target-distractor discriminability ($d' = 0.5$, 1.0, 1.5 and 2.0), for two values of activation threshold and a fixed maximum number of serially processed elements ($k = 4$). A) $\lambda = 0$, B) $\lambda = 2$. 
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**Technological Memorandum**

Current models of human visual search have extended the traditional serial/parallel search dichotomy. Two successful models for predicting human visual search are the Guided Search model and the Signal Detection Theory model. Although these models are inherently different, it has been difficult to compare them because the Guided Search model is designed to predict response time, while Signal Detection Theory models are designed to predict performance accuracy. Moreover, current implementations of the Guided Search model require the use of Monte-Carlo simulations, a method that makes fitting the model’s performance quantitatively to human data more computationally time consuming. We have extended the Guided Search model to predict human accuracy in target-localization search tasks. We have also developed analytic expressions that simplify simulation of the model to the evaluation of a small set of equations using only three free parameters. This new implementation and extension of the Guided Search model will enable direct quantitative comparisons with human performance in target-localization search experiments and with the predictions of Signal Detection Theory and other search accuracy models.