

Simultaneous Aerodynamic and Structural Design Optimization (SASDO) for a 3-D Wing

Clyde R. Gumbert

NASA Langley Research Center
Hampton, Virginia

Gene J.-W. Hou

Old Dominion University
Norfolk, Virginia

Perry A. Newman

NASA Langley Research Center
Hampton, Virginia

preprint of the paper presented at the
AIAA 15th Computational Fluid Dynamics Conference

June 11-14, 2001

Anaheim, CA

Simultaneous Aerodynamic and Structural Design Optimization (SASDO) for a 3-D Wing

Clyde R. Gumbert*

NASA Langley Research Center, Hampton, VA 23681

Gene J. -W. Hou†

Old Dominion University, Norfolk, VA 23529-0247

Perry A. Newman‡

NASA Langley Research Center, Hampton, VA 23681

The formulation and implementation of an optimization method called **Simultaneous Aerodynamic and Structural Design Optimization (SASDO)** is shown as an extension of the **Simultaneous Aerodynamic Analysis and Design Optimization (SAADO)** method. It is extended by the inclusion of structure element sizing parameters as design variables and Finite Element Method (FEM) analysis responses as constraints. The method aims to reduce the computational expense incurred in performing shape and sizing optimization using state-of-the-art Computational Fluid Dynamics (CFD) flow analysis, FEM structural analysis and sensitivity analysis tools. SASDO is applied to a simple, isolated, 3-D wing in inviscid flow. Results show that the method finds the same local optimum as a conventional optimization method with some reduction in the computational cost and without significant modifications to the analysis tools.

Nomenclature

b	wing semispan	ΔQ_1	change in flow solver field variables due to better analysis convergence
C_D	drag coefficient	ΔQ_2	change in flow solver field variables due to design changes
C_L	lift coefficient	R	aerodynamic state equation residuals at each CFD mesh point
C_m	pitching moment coefficient	$ R/R_0 $	norm of the residual ratio, current/initial
c_r	wing root chord	s	surface area
c_t	wing tip chord	S	semispan wing planform area
F	design objective function	u	structural deflections (state variables)
g	design constraints	Δu_1	change in deflections due to better analysis convergence
K	stiffness matrix	Δu_2	change in deflections due to design changes
L	aerodynamic loads	W	wing weight
M_∞	free-stream Mach number	X	CFD volume mesh coordinates
n	unit normal vector	x_{LE}	location of wing root leading edge
p	local aerodynamic pressure	$\frac{x}{c}$	chordwise location normalized by local wing section chord
P	compliance, the work done by the aerodynamic load to deflect the structure	x_t	longitudinal location of wing tip trailing edge
q_∞	free-stream dynamic pressure	z_r	root section maximum camber
Q	flow-field variables (state variables) at each CFD mesh point	α	free-stream angle-of-attack
		β	design variables
		Γ	structural element size factor
		γ	line search parameter
		Δ	operator which indicates a change in a variable

*Research Engineer, Multidisciplinary Optimization Branch, M/S 159, c.r.gumbert@larc.nasa.gov

†Professor, Department of Mechanical Engineering, AIAA member, ghou@lions.odu.edu

‡Senior Research Scientist, Multidisciplinary Optimization Branch, M/S 159, p.a.newman@larc.nasa.gov

Copyright © 2001 by the American Institute of Aeronautics and Astronautics, Inc. No copyright is asserted in the United States under Title 17, U.S. Code. The U.S. Government has a royalty-free license to exercise all rights under the copyright claimed herein for Governmental Purposes. All other rights are reserved by the copyright owner.

$\varepsilon, \varepsilon'$	convergence tolerances for function and gradients
λ	adjoint variable for Q
μ	adjoint variable for u
τ	twist angle at wing tip, positive for leading edge up

Subscripts

d	deflected shape
j	jig (undeflected) shape
s	wing surface mesh
v	wing volume mesh

Superscripts

*	designates updated value
'	gradient with respect to design variables

Introduction

Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) is a procedure that incorporates design improvement within the iteratively solved (nonlinear) aerodynamic analysis so as to achieve fully converged flow solutions only near an optimal design. When SAADO is applied to a flexible wing rather than a rigid wing, the linear Finite Element Method (FEM) solution is iteratively coupled with the nonlinear Computational Fluid Dynamics (CFD) solution. When design variables that control structural element size are included, it is renamed Simultaneous Aerodynamic and Structural Design Optimization (SASDO). Overall computational efficiency is achieved because the many expensive iterative (nonlinear) solutions for non-optimal design parameters are not converged (i.e., obtained) at each optimization step. One can obtain the design in the equivalent of a few (rather than many) multiples of the computational time for a single, fully converged coupled aero-structural analysis. SAADO and similar procedures for simultaneous analysis and design (SAND) developed by others are noted and discussed by Newman et al.¹ These SAND procedures appear best suited for applications where the discipline analyses involved in the design are nonlinear and solved iteratively. Generally, convergence of these discipline analyses (i.e., state equations) is viewed as an equality constraint in an optimization problem. From this latter point of view, the SASDO method proceeds through infeasible regions of the design space which includes not only the design variables β , but also the state variables Q and u . A further advantage of SASDO is the efficient utilization of existing discipline analysis codes (without internal changes), augmented with sensitivity or gradient information, and yet effectively coupled more tightly than is done in conventional gradient-based optimization procedures, referred to as nested analysis and design (NAND) procedures.¹ A recent overview of aerodynamic shape

optimization² discusses both NAND and SAND procedures in the context of current steady aerodynamic optimization research.

For single-discipline design problems, the distinction between NAND and SAND procedures is fairly clear and readily seen. With respect to discipline feasibility (i.e., convergence of the generally nonlinear, iteratively solved state equations), these procedures can be viewed as accomplishing design by using only very well converged discipline solutions (NAND), or as converging a sequence of discipline solutions from poorly to well as the design progresses (SAND). However, the problem formulation and solution algorithms may differ considerably. About twenty SAND references are quoted by Newman et al.¹ and Newman et al.;² these references discuss a variety of formulations, algorithms, and results for single-discipline problems (mostly CFD applications) in the sense of SAND as defined above. For multidisciplinary design optimization problems, the distinction between NAND and SAND is somewhat blurred because there are feasibility considerations with respect to all the individual discipline state equations, as well as with respect to the multidisciplinary system compatibility and constraints. A number of the papers in Ref. 3 discuss MDO formulations and algorithms that are called SAND-like; however, not all of these latter MDO procedures appear to agree with the sense of SAND defined above and used herein; one that does is Ref. 4.

The computational feasibility of SAADO for quasi 1-D nozzle shape design based on the Euler equation CFD approximation was demonstrated by Hou et al.⁵ and Mani.⁶ Application of SAADO for turbulent transonic airfoil shape design based on a 2-D thin-layer Navier-Stokes CFD approximation was demonstrated and reported in a later paper by Hou et al.⁷ Both of these application results are summarized and briefly discussed in Ref. 1. The application of SAADO for rigid 3-D wing design based on the Euler CFD approximation was presented in Ref. 8. These SAADO procedures utilized quasi-analytical sensitivity derivatives obtained from hand-differentiated code for the initial quasi 1-D application^{5,6} and from automatically differentiated code for both the 2-D airfoil application⁷ and the 3-D rigid wing application.⁸ Different optimization techniques have also been used in these SAADO procedures.

The extension to multidisciplinary analysis with shape design variables only was presented in Ref. 9. Our initial results from SASDO are given in this paper. The analysis problem, the objective function, and the constraints are the same as those used in Ref. 9. That is, changes in design variables are sought to produce improvement in the lift-to-drag ratio of a simple wing subject to both aerodynamic and structural solution-dependent constraints. These constraints are the difference between the lift and weight, the pitch-

ing moment coefficient, and the compliance, a function representing work done by the aerodynamic load to deflect the structure. There are also geometric constraints.

The flexible wing studied here is formulated as a static aeroelastic problem. Similar problems have been used as examples in Refs. 10–15 to study various solution strategies for multidisciplinary analysis and optimization. In particular, Arian¹² analyzed the Hessian matrix of the system equations to derive the mathematical conditions under which the aeroelastic optimization problem can be solved in a “loosely” coupled manner. The multidisciplinary research of Walsh et al.^{13,14} emphasized engineering aspects of integrating high fidelity disciplinary analysis software and distributed computing over a network of heterogeneous computers. The aeroelastic analysis results of Reuther et al.¹⁵ were verified with experimental data.

Only a limited amount of literature related to aeroelastic problems has elaborated on the coupled sensitivity analysis. Kapania, Eldred and Barthelemy;¹⁶ Arslan and Carlson;¹⁷ and Giunta and Sobieszcanski-Sobieski¹⁸ derived global sensitivity equations (GSEs); some matrix coefficients in these GSEs were evaluated by finite differencing. Giunta¹⁹ later introduced modal coordinates to approximate the elastic displacement vector in order to reduce the size of the GSE. Newman, Whitfield, and Anderson²⁰ used the complex variable approach to obtain the aeroelastic sensitivity derivatives, whereas Reuther et al.¹⁵ employed the adjoint variable approach to derive the aeroelastic sensitivity equations. A mathematical study of the coupled nonlinear, incompressible aeroelastic analysis and sensitivity analysis problems has been given by Ghattas and Li.²¹ Recent results on aeroelastic sensitivity analysis and optimization can be found in Refs. 22–24. Particularly, Maute et al.²³ and Hou and Satyanarayana²⁴ explicitly formulated the deflection update and the load transfer between the separate flow and structures solvers as part of the coupled sensitivity equations. In the present study, the coupled sensitivity equations are constructed by differentiating the aeroelastic state equations and solving them by a Generalized Gauss-Seidel (GGS) method.¹² The present SASDO concept is very similar to that of Ghattas and others, Refs. 4, 21, 25, 26, but differs in the implementation details as described later.

Problem Description

To evaluate the efficacy of the SASDO procedure for a problem involving multidisciplinary analysis, it is applied herein to a simple, isolated, flexible wing. The wing shape consisted of a trapezoidal planform with a rounded tip. It was parameterized by fifteen variables; five described the planform, and five each described the root and tip section shapes. A schematic of the wing and its associated shape parameters is shown in

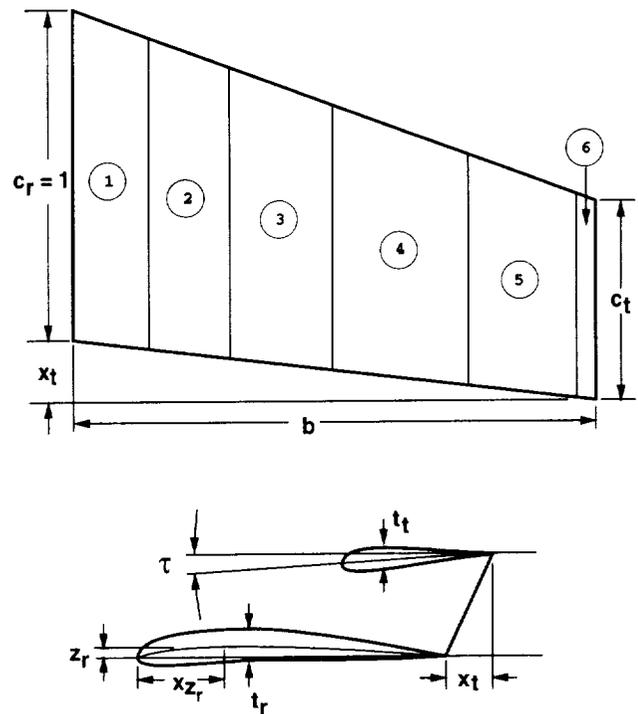


Fig. 1 Description of semispan wing parameterization.

Fig. 1. The baseline wing section varied linearly from an NACA 0012 at the root to an NACA 0008 at the tip. The wing structure consists of a skin, ribs, and spars. The ribs and spars consist of shear webs and trusses. Six spanwise zones of the structural model are defined as depicted in Fig. 1. The relative sizes of the skin and web thicknesses and the truss cross section areas are fixed within each zone. Each zone is assigned a parameter Γ which multiplies all the thicknesses and areas of the structural elements in that zone. The specific parameters selected as design variables in the sample optimization problems are identified in the section entitled Results. The objective function to be minimized was the negative of the lift-to-drag ratio, $-L/D$. Both coupled solution-dependent and geometric constraints were imposed.

The solution-dependent constraints were

- lower limit on the difference between the total lift and the structural weight, $C_L * S * q_\infty - W$
- upper limit on compliance, $P = \oint pu \cdot nds$
- upper limit on pitching moment, C_m , in lieu of a trim constraint

The purely geometric constraints were

- minimum leading edge radius, in lieu of a manufacturing requirement
- side constraints (bounds) on the active design variables

SASDO Procedure

Formulation

The flexible SASDO approach formulates the design optimization problem as follows:

$$\min_{\beta, Q, u} F(Q, X_{dv}(\beta, u), u, \beta) \quad (1)$$

subject to

$$g_i(Q, X_{dv}(\beta, u), u, \beta) \leq 0; \quad i = 1, 2, \dots, m \quad (2)$$

where the flow field Q and the structural deflection u are solutions of the coupled flow equation

$$R(Q, X_{dv}(\beta, u), \beta) = 0 \quad (3)$$

and the finite element structural equation

$$K(X_j(\beta), \beta)u = L(Q, X_{dv}(\beta, u)) \quad (4)$$

The deflected volume mesh, X_{dv} , is determined by the deflected surface mesh, X_{ds} , as $X_{dv} = X_{dv}(X_{ds})$. The deflected surface mesh is a result of the jig shape augmented by the elastic deflection, u , as $X_{ds} = X_j(\beta) + u$. The two disciplines are coupled through the deflection, u , and the load, L .

Recall that Q , R , and X_{dv} are very large vectors. This formulation treats the state variables, Q and u , as part of the set of independent design variables, and considers the state equations to be constraints. Because satisfaction of the equality constraints, Eqs. (3) and (4), is required only at the final optimum solution, the coupled steady-state aero-structural field equations are not converged at every design-optimization iteration. The easing of that restriction is expected to significantly reduce the excessively large computational cost incurred in the conventional approach. However, this advantage would likely be offset by the very large increase in the number of design variables and equality constraint functions, unless some remedial procedure is adopted.

Approximations

The SASDO method begins with a linearized design optimization problem solved for the most favorable change in the design variables, $\Delta\beta$, as well as for the changes in the state variables, ΔQ and Δu ; that is,

$$\begin{aligned} \min_{\Delta\beta, \Delta Q, \Delta u} & F(Q, X_{dv}(\beta, u), u, \beta) \\ & + \frac{\partial F}{\partial Q} \Delta Q + \left(\frac{\partial F}{\partial u} + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u \\ & + \left(\frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial F}{\partial \beta} \right) \Delta\beta \end{aligned} \quad (5)$$

subject to inequality constraints

$$\begin{aligned} g_i(Q, X_{dv}(\beta, u), u, \beta) & + \left(\frac{\partial g_i}{\partial u} + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u \\ & + \frac{\partial g_i}{\partial Q} \Delta Q + \left(\frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial g_i}{\partial \beta} \right) \Delta\beta \leq 0; \end{aligned} \quad (6)$$

$i = 1, 2, \dots, m$

and equality constraints

$$\begin{aligned} R(Q, X_{dv}(\beta, u), \beta) & + \frac{\partial R}{\partial Q} \Delta Q + \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u \\ & + \left(\frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial R}{\partial \beta} \right) \Delta\beta = 0 \end{aligned} \quad (7)$$

and

$$\begin{aligned} K(X_j(\beta), \beta)u & - L(Q, X_{dv}(\beta, u)) - \frac{\partial L}{\partial Q} \Delta Q \\ & + \left(K - \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u + \left(\frac{\partial K}{\partial \beta} u \right) \Delta\beta \\ & + \left(\frac{\partial K}{\partial X_j} u - \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) X'_j \Delta\beta = 0 \end{aligned} \quad (8)$$

Note that Eqs. (5) through (8) are linearized approximations of Eqs. (1) through (4), respectively.

In this formulation, neither the residual of the nonlinear aerodynamic field equations, $R(Q, X, \beta)$, nor that of the structures equation, $Ku - L$, is required to be zero (reach target) until the final optimum design is achieved. The linearized problem of Eqs. (5) through (8) is difficult to solve directly because of the number of design variables and equality constraint equations.

Direct differentiation method

One way to overcome this difficulty is by the direct differentiation method. In this method ΔQ , Δu , and Eqs. (7) and (8) are removed altogether from the linearized problem by direct substitution. This is achieved by expressing ΔQ and Δu as functions of $\Delta\beta$.

$$\begin{aligned} \Delta Q & = \Delta Q_1 + \Delta Q_2 \Delta\beta \\ \Delta u & = \Delta u_1 + \Delta u_2 \Delta\beta \end{aligned} \quad (9)$$

where vectors ΔQ_1 and Δu_1 are corrections in the aeroelastic solution due to the improvement of coupled aeroelastic analysis, while matrices ΔQ_2 and Δu_2 are corrections due to changes in the design variables. These vectors and matrices are solutions of the following coupled sets of equations, obtained from Eqs. (7) and (8):

$$\begin{aligned} \frac{\partial R}{\partial Q} \Delta Q_1 & = -R - \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1 \end{aligned} \quad (10)$$

$$K \Delta u_1 = \frac{\partial L}{\partial Q} \Delta Q_1 + \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \Delta u_1$$

where, for the linear FEM, $Ku - L = 0$ at every iteration, and

$$\frac{\partial R}{\partial Q} \Delta Q_2 + \frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) + \frac{\partial R}{\partial \beta} = 0$$

$$\begin{aligned} K \Delta u_2 & = \frac{\partial L}{\partial Q} \Delta Q_2 + \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) \\ & - \left(\frac{\partial K}{\partial X_j} X'_j + \frac{\partial K}{\partial \beta} \right) u \end{aligned} \quad (11)$$

Note that the number of columns of matrices ΔQ_2 and Δu_2 is equal to the number of design variables, β . Thus the computational cost of Eq. (11) is directly proportional to the number of design variables.

A new linearized problem with $\Delta\beta$ as the only design variables can be obtained by substituting Eq. (9) into

Eqs. (5) and (6) for ΔQ and Δu .

$$\begin{aligned} \min_{\Delta\beta} \quad & F(Q, X, u, \beta) + \frac{\partial F}{\partial Q} \Delta Q_1 \\ & + \left(\frac{\partial F}{\partial u} + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u_1 \\ & + \left\{ \frac{\partial F}{\partial Q} \Delta Q_2 + \frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j \right. \\ & \left. + \left(\frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} + \frac{\partial F}{\partial u} \right) \Delta u_2 + \frac{\partial F}{\partial \beta} \right\} \Delta\beta \end{aligned} \quad (12)$$

subject to

$$\begin{aligned} g_i(Q, X, u, \beta) + \frac{\partial g_i}{\partial Q} \Delta Q_1 \\ + \left(\frac{\partial g_i}{\partial u} + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) \Delta u_1 \\ + \left\{ \frac{\partial g_i}{\partial Q} \Delta Q_2 + \frac{\partial g_i}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} (X'_j + \Delta u_2) \right. \\ \left. + \frac{\partial g_i}{\partial u} \Delta u_2 + \frac{\partial g_i}{\partial \beta} \right\} \Delta\beta \leq 0; \quad i = 1, 2, \dots, m \end{aligned} \quad (13)$$

The appearance of ΔQ_1 and Δu_1 in the formulation indicates the difference between the SASDO (SAND) method and the conventional (NAND) aerodynamic optimization method. The ΔQ_1 and Δu_1 not only constitute a change in Q and u , but also play an important role in defining the objective function of Eq. (12) and the constraint violation of Eq. (13). We can directly solve Eqs. 10 for ΔQ_1 and Δu_1 , and, in fact, in previous SAADO applications that is how ΔQ_1 was determined. However, since ΔQ_1 and Δu_1 , as shown in Eq. (10), represent a single Newton's iteration on the coupled flow and structures equations, it is possible and less computationally expensive to approximate their influence on the solution Q and u by several Newton's iterations of the coupled aeroelastic equations. That is, ΔQ_1 and Δu_1 are not determined explicitly, but rather the first three terms of Eqs. 12 and 13 are viewed as updated values of F and g_i . Note that the terms in parentheses in Eqs. (12) and (13) are approximated gradients of the objective and constraint functions. Once established, this linearized problem can be solved using any mathematical programming technique for design changes, $\Delta\beta$. Results presented in this paper are computed using this direct differentiation approach.

Adjoint method

An alternative way to remove ΔQ and Δu from the linearized problems, Eqs. (5) and (6), is the adjoint variable method. The adjoint variables, λ and μ , can be introduced as the solutions of

$$\begin{aligned} \left(\frac{\partial R}{\partial Q} \right)^T \lambda &= \left(\frac{\partial L}{\partial Q} \right)^T \mu - \left(\frac{\partial F}{\partial Q} \right)^T \\ K \mu &= \left(\frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right)^T \mu \\ &- \left(\frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right)^T \lambda - \left(\frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right)^T - \frac{\partial F}{\partial u} \end{aligned} \quad (14)$$

so as to rewrite the expression of the objective function in Eq. (5) in terms of λ , μ , and $\Delta\beta$ as

$$\begin{aligned} F(Q, X_{dv}(\beta, u), u, \beta) + \lambda^T R + \mu^T (Ku - L) \\ + \left\{ \left(\frac{\partial F}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial F}{\partial \beta} \right) \right. \\ + \lambda^T \left(\frac{\partial R}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} X'_j + \frac{\partial R}{\partial \beta} \right) \\ \left. + \mu^T \left(\frac{\partial K}{\partial \beta} u + \left(\frac{\partial K}{\partial X_j} u - \frac{\partial L}{\partial X_{dv}} \frac{\partial X_{dv}}{\partial X_{ds}} \right) X'_j \right) \right\} \Delta\beta \end{aligned} \quad (15)$$

Note that the terms in the brace represent the gradient of the objective function. The terms $\lambda^T R$ and $\mu^T (Ku - L)$ indicate the effect on the design optimization formulation due to errors in the aeroelastic analysis. Furthermore, in the linear sense, the adjoint variables and the solution errors can be related by the following equations:

$$\frac{\partial F}{\partial \lambda} = R^T \quad (16)$$

and

$$\frac{\partial F}{\partial \mu} = (Ku - L)^T \quad (17)$$

These equations have been mentioned, for example, by Pierce and Giles²⁷ and Venditti and Darmofal²⁸ for aerodynamic problems.

In the typical optimization problem there are many design variables. When one can also pose the optimization problem such that there are only a few output quantities for objectives and constraints or, in the extreme, combine the constraints and objective function into a single cost function, the adjoint approach to sensitivity analysis has the advantage that the adjoint solutions are independent of the number of design variables.²⁹ However, when the disciplines are loosely coupled, this approach is impractical since the coupled sensitivity analyses would require an adjoint for each disciplinary output being transferred, i.e., the discretized loads and deflections. In a tightly or implicitly coupled multidisciplinary analysis the adjoint approach may prove practical since this system is analogous to a single discipline.

Line Search

A one-dimensional search on the step size parameter γ is then performed in order to find the updated values of $\Delta\beta$, ΔX , ΔQ , and Δu . Given the search direction $\Delta\beta$ determined by either the direct differentiation method (Eqs. 12 and 13) or the adjoint method equivalent, this line search functions to adjust its magnitude so as to simultaneously ensure better results for both design and analysis (converged solutions). The step size parameter γ plays the role of a relaxation factor in the standard Newton's iteration. The search procedure employed solves a nonlinear optimization problem of the form

$$\min_{\gamma} F(Q^*, X^*, u^*, \beta^*) \quad (18)$$

subject to

$$g_i(Q^*, X^*, u^*, \beta^*) \leq 0; \quad i = 1, 2, \dots, m \quad (19)$$

$$R(Q^*, X^*, \beta^*) = 0 \quad (20)$$

and

$$K(X^*, \beta^*)u^* = L(Q^*, X^*) \quad (21)$$

where step size γ is the only design variable. Again it is noted for emphasis that the equality constraints, Eqs. (20) and (21), are not required to be zero (reach target) until the final optimum design; violations of these equality constraints must simply be progressively reduced until the SASDO procedure converges.

The updated Q^* and u^* can be viewed as $Q^* = Q + \Delta Q$ and $u^* = u + \Delta u$, where ΔQ and Δu satisfy the first order approximations to Eqs. (20) and (21). That is, ΔQ and Δu are the solutions of Eqs. (7) and (8) where, in Eq. (9), $\Delta \beta$ is replaced by $\Delta \beta^* = \gamma \Delta \beta$. Consequently, $Q^* = Q + \Delta Q_1 + \gamma \Delta Q_2 \Delta \beta$ and $u^* = u + \Delta u_1 + \gamma \Delta u_2 \Delta \beta$ are readily available once γ is found. The $\Delta(\cdot)_1$ terms appearing in the above SASDO formulation are due to better convergence of the coupled analysis, whereas the $\Delta(\cdot)_2$ terms are due to changes in the design variables. In fact, ΔQ_2 and Δu_2 approach the flow field and deflection sensitivities, Q' and u' , as the solution becomes better converged.

Implementation

The following pseudocode shows algorithmically how the method was implemented.

```

set initial analysis convergence tolerance,  $\varepsilon$ 
set initial solution vectors,  $Q$  and  $u$ ,
set initial design variables,  $\beta$ 
do until converged
  1. solve coupled aeroelastic analysis, Eqs. (3) & (4),
    partially converged to  $\varepsilon$ 
  2. compute F and g
  3. solve coupled aeroelastic sensitivity analysis,
    Eq. (11), partially converged to  $\varepsilon'$ 
  4. compute  $\Delta \beta$  terms of Eqs. (5) & (6)
  5. solve optimization problem Eqs. (5) & (6) for  $\Delta \beta$ 
  6. solve Eqs. (18) through (21) for line search parameter,  $\gamma$ 
  7. update  $\beta$ ,  $u$ , and  $Q$ 
  8. tighten analysis convergence tolerance,  $\varepsilon = \varepsilon * \mathcal{Z}$ ,
     $0 < \mathcal{Z} < 1$ 
enddo
  
```

This pseudocode is similar to that used in the Biros and Ghattas²⁵ SAND approach. Specifically, both approaches use a Sequential Quadratic Programming (SQP) method to solve the design equations (step 5) and an approximate factorization method to solve the system equations (step 1). Step 3 above uses an incremental iterative method with approximate factorization to solve for derivatives in direct mode rather than as a solution of the adjoint equation of

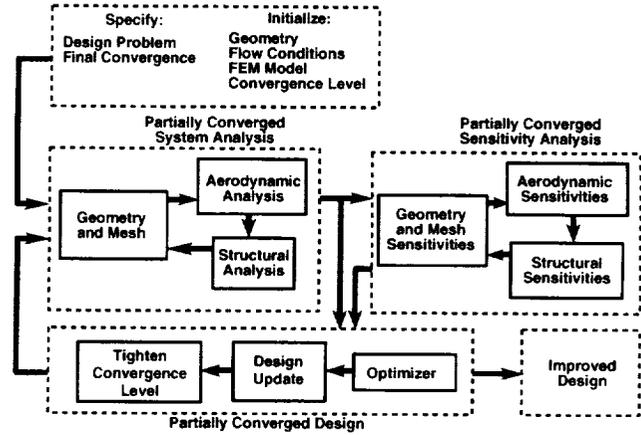


Fig. 2 Diagram of SASDO procedure.

Biros and Ghattas.²⁵ In addition, the line search step (step 6) and the convergence tightening step (step 8) were not included in the Biros and Ghattas method. A schematic of the present SASDO procedure is shown in Fig. 2. The dashed box, labeled “Partially Converged System Analysis,” depicts the coupled analysis iteration loop, Steps 1 and 2 of the pseudocode; that labeled “Partially Converged Sensitivity Analysis” depicts the coupled derivative iteration loop, Step 3; that labeled “Partially Converged Design” depicts the design steps, Steps 5–8 of the pseudocode. Specific computational tools and methods used to perform the tasks depicted by the solid boxes in Fig. 2 are identified in the next section.

Computational Tools and Models

Major computations in this SASDO procedure are performed using a collection of existing codes. These codes are executed by a separate driver code and scripts that implement the SASDO procedure as just discussed. Each code runs independently, some simultaneously, on different processors, and the required I/O transfers between them, also directed by the driver, are accomplished via data files.

The aerodynamic flow analysis code used for this study is a version of the CFL3D code.³⁰ Only Euler analyses are performed for this work, although the code is capable of solving the Navier-Stokes equations with any of several turbulence models. The gradient version of this code, which was used for aerodynamic sensitivity analysis, was generated by an unconventional application³¹ of the automatic differentiation code ADIFOR^{32,33} to produce a relatively efficient, direct mode, gradient analysis code, CFL3D.ADH.³⁴ It should be pointed out that the ADIFOR process produces a discretized derivative code consistent with the discretized function analysis code. The addition of a stopping criterion based on the norm of the residual of the field equations was the only modification of the CFL3D.ADH code made to accommodate the SASDO procedure.

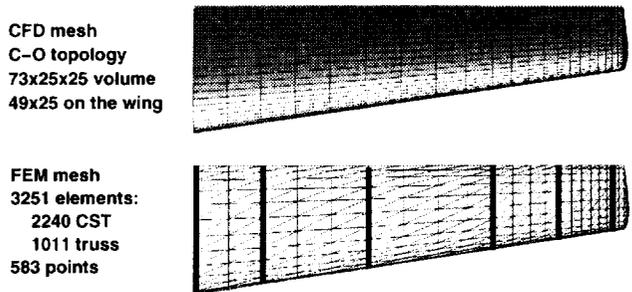


Fig. 3 Computational meshes for wing analysis.

The surface geometry was generated based on the parameters described in a previous section by a code utilizing the Rapid Aircraft Parameterization Input Design (RAPID) technique developed by Smith, et al.³⁵ This code was preprocessed with ADIFOR to generate a code capable of producing sensitivity derivatives as well.

The CFD volume mesh needed by the flow analysis code was generated using a version of the CSCMDO³⁶ grid generation code. The associated grid sensitivity derivatives needed by the flow sensitivity analysis were generated with an automatically differentiated version of CSCMDO.³⁷ In addition to the parameterized surface mesh and accompanying gradients, CSCMDO requires a baseline volume mesh of similar shape and identical topology. The 45,000 grid point baseline volume mesh of C-O topology used in the present flexible wing examples was obtained with the GridgenTM code. The wing surface portion of the mesh is shown in Fig. 3. This mesh is admittedly particularly coarse by current CFD analysis standards.

The structural analysis code³⁸ used to compute the deflection of the elastic wing was a generic finite element code. The flexible structure for the wing shown in Fig. 3 was discretized by 583 nodes; there were 2,141 constant-strain triangle (CST) elements and 1,110 truss elements. Zone boundaries for the design variables controlling element size are also shown in Fig. 3. Because the elastic deformation was assumed to be small, linear elasticity was deemed to be appropriate. The structural sensitivity equations were derived based upon the direct differentiation method. We note that the sensitivity of the aerodynamic forces appears as a term on the right-hand side (RHS) of the deflection sensitivity equations. The derivative of the stiffness matrix in these sensitivity equations was also generated³⁹ by using the ADIFOR^{32,33} technique. We note that the coefficient matrix of the structural sensitivity equations was identical to that of the structural equations. Consequently, these structural sensitivity equations were solved efficiently by backward substitution with different RHSs for each sensitivity.

At the wing surface, i.e., the interface where aerodynamic load and structural deflection information is transferred, it was assumed that surface nodes of the

FEM structural model were a subset of the CFD aerodynamic surface mesh points (see Fig. 3) for the present SASDO application. This lack of generality allowed for simplifications in the data transfers and, although an important issue, it was not deemed crucial for these initial SASDO demonstrations. Future applications to more complex configurations should allow for transfer of conserved information between arbitrary meshes as required by the individual disciplines. A recent review of such data transfer techniques and a specific proposed one are given in Ref. 40.

Conventional (NAND) and SASDO (SAND) procedures were implemented using the SQP method of the DOT⁴¹ optimization software. All computations were executed on an SGI Origin 2000TM workstation with 250MHz R10000TM processors. The CFD sensitivity calculations were partitioned and run on several processors to reduce required memory and elapsed optimization time. This partitioning, however, results in additional accumulated computation time due to the nature of ADIFOR-generated sensitivity analysis code.

Results

The optimization results shown in this work are for design problems involving only four or eight design variables out of the 21 available wing parameters. The results shown by these authors in an earlier work⁹ used design variables that directly affected either the aerodynamic analysis alone or both the aerodynamic analysis and the structural analysis. In this work, additional design variables are chosen that directly affect only the structural analysis. The flow conditions for the wing optimizations were $M_\infty = 0.8$ and $\alpha = 1^\circ$.

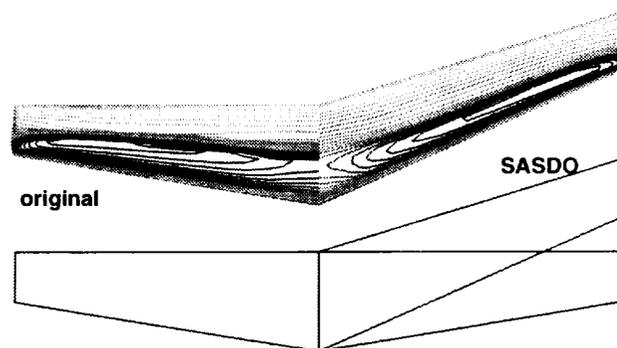


Fig. 4 Comparison of planform shapes and surface pressure contours for 4-design-variable cases, $M_\infty = 0.8$, $\alpha = 1^\circ$.

Four-Design-Variable Problems

Table 1 and Fig. 4 show results of several optimization problems involving four design variables: the tip chord c_t , the tip setback x_t , and the structural element size factor for the two most inboard zones, Γ_1 and Γ_2 . Two of the cases, designated Conv_C and SASDO_C in Table 1, represent direct comparisons of SASDO and the conventional method for consistent accuracy

Table 1 Summary of Four-Design-Variable Cases.

	baseline	Optimizations			
		SASDO _A	Conv _B	SASDO _C	Conv _C
c_t	1.	1.07778	1.07281	1.07827	1.07697
x_t	1.	1.93259	1.91880	1.93052	1.92614
Γ_1	1.	1.06631	1.08008	1.09637	1.09982
Γ_2	1.	0.69510	0.69187	0.68664	0.68691
F	-7.149	-10.187	-10.181	-10.186	-10.183
g_1	-.0302	-.000760	-.00177	-.0000629	-.000915
g_2	-.8882	-0.880	-0.876	-0.880	-0.879
g_3	-.2647	-.000639	-.000159	-.000149	+.000108
ε	9.6e-7	9.9e-5	8.9e-7	6.6e-6	4.0e-6
\sum state	1	1.1	4.8	2.0	3.9
\sum grad	—	27.1	63.1	26.9	40.4

of function and gradient analyses. The other two cases show effects of changes in the accuracy of the function and gradient analyses. The resulting designs are essentially identical for all four cases. Figure 4 shows a comparison of the wing planform and the surface pressure coefficient results for the baseline design and the design designated SASDO_C in Table 1. The shock wave has been weakened somewhat in the optimized cases from that on the original wing, as one would expect. As one can see qualitatively in Fig. 4 and numerically from the values of the objective function F , the constraints g_i , and the final design variables in Table 1, the final designs are very similar for the four problems. The relative computational costs of

have been separated in Fig. 5. The total time for performing this optimization problem was reduced by 36 percent using the SASDO method. The analysis alone was reduced by 55 percent, but the gradient evaluation was the dominant cost.

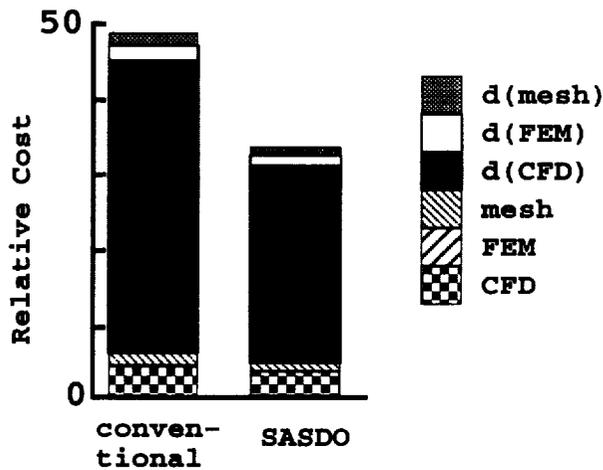


Fig. 5 Comparison of computation cost of four-design-variable optimization problem using the conventional and SASDO methods.

the optimizations are shown in Table 1 and Fig. 5. The accumulated function and gradient analysis times (denoted as Σ state and Σ grad) are shown separately. They have been normalized by the cost of the baseline coupled function analysis. The components of the analyses were lumped together in Table 1 but they

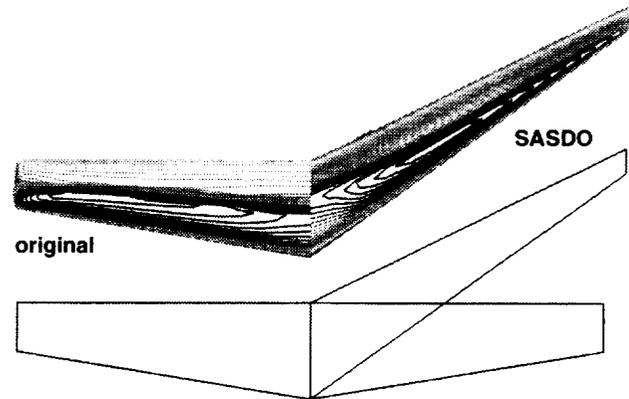


Fig. 6 Comparison of planform shapes and surface pressure contours for 8-design-variable cases, $M_\infty = 0.8$, $\alpha = 1^\circ$.

Eight-Design-Variable Problems

Table 2 and Fig. 6 show results of three optimization problems involving eight design variables: the same set used in the four-design-variable cases with the inclusion of the span b , the root section max camber z_r , and the structural element size factor for two more zones, Γ_3 and Γ_4 . Two of the cases, designated Conv_B and SASDO_B in Table 2, represent direct comparisons of SASDO and the conventional method for consistent accuracy of the function and gradient analyses. Figure 6 shows a comparison of wing planform and surface pressure coefficient results for the baseline design and the design designated SASDO_B in Table 2. The relative computational costs of the optimizations are shown in Table 2 and Fig. 7. The total time for performing this optimization problem was reduced by

Table 2 Summary of Eight-Design-Variable Cases.

	baseline	Optimizations		
		Conv _A	SASDO _B	Conv _B
c_t	1.	0.76574	0.75929	0.75794
x_t	1.	2.60000	2.60000	2.60000
b	1.	1.05992	1.07543	1.07269
z_r	1.	1.11511	1.11173	1.11175
Γ_1	1.	2.42444	2.55263	2.44498
Γ_2	1.	2.09537	2.09924	2.09394
Γ_3	1.	1.03635	1.01883	1.05778
Γ_4	1.	0.65137	0.81046	0.76612
F	-7.149	-21.255	-21.235	-21.244
g_1	-.0302	-.7536	-.7255	-.7316
g_2	-.8882	-.00336	-.00671	-.000241
g_3	-.2647	-.000420	-.000962	-.000134
ϵ	9.6e-7	6.8e-7	1.8e-5	1.6e-5
\sum state	1	13.5	4.4	11.3
\sum grad	-	146.7	115.9	148.6

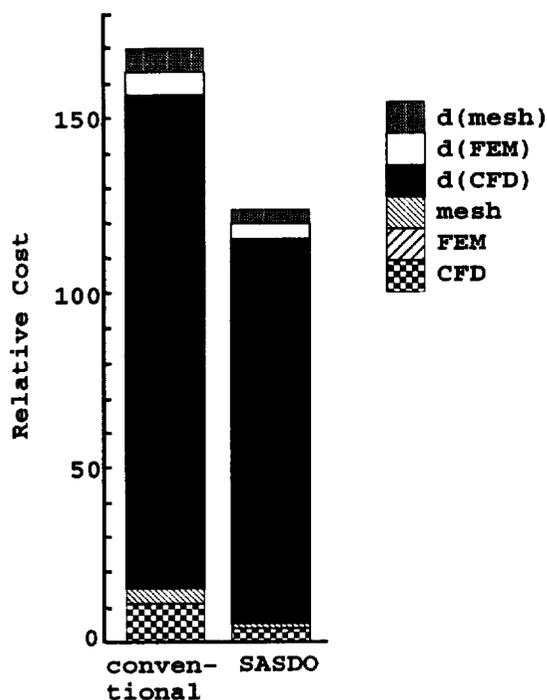


Fig. 7 Comparison of computation cost of eight-design-variable optimization problem using the conventional and SASDO methods.

26 percent using the SASDO method. The analysis alone was reduced by 60 percent, but, as with the four-design-variable problem, the gradient evaluation was the dominant cost.

Further Discussion

The relative costs, based on CPU timing ratios, for the SASDO (SAND) procedures applied to these present small 3-D aerodynamic/structural design opti-

mization problems are about seven-tenths of the costs of the corresponding conventional (NAND) procedures. This range is very similar to that reported for 2-D nonlinear aerodynamic shape design optimization in Refs. 1 and 4, even though many of the computational details differ. The results given in Ref. 1 were for a turbulent transonic flow with shock waves computed using a Navier-Stokes code; a direct differentiation approach (using ADIFOR) was used for the sensitivity analysis. The results reported in Ref. 4 were for a compressible flow without shock waves computed using a nonlinear potential flow code; an adjoint approach was used for the sensitivity analysis. Since these two optimization problems were also not the same, no timing comparison between these adjoint and direct differentiation solution approaches would be meaningful. As indicated earlier, an expected speed-up for using an adjoint approach instead of the direct differentiation approach was estimated in Ref. 1. Ghattas and Bark²⁶ recently reported 2-D and 3-D results for optimal control of steady incompressible Navier-Stokes flow that demonstrate an order-of-magnitude reduction of CPU time for a SAND approach versus a NAND approach. These results were obtained using reduced Hessian SQP methods that avoid converging the flow equations at each optimization iteration. The relationship of these methods with respect to other optimization techniques is also discussed in Ref. 26. The "Control Theory" approach of Jameson²⁹ and several other SAND-like methods for simultaneous analysis and design, which were summarized and discussed by Ta'asan,⁴² have been applied to aerodynamic shape design problems at several fidelities of CFD approximation. These techniques have obtained an aerodynamic design in the equivalent of several analysis CPU times for some sample problems.

Concluding Remarks

This study has introduced an implementation of the SASDO technique for a simple, isolated wing. Initial results indicate that SASDO

1. is feasible under dual simultaneity (i.e. simultaneity not only with respect to analysis and design optimization, but also simultaneity with respect to flexible wing aero-structural interaction)
2. finds the same local minimum as a conventional technique
3. is computationally more efficient than a conventional gradient-based optimization technique
4. requires few modifications to the analysis and sensitivity analysis codes involved
5. is effective at reducing the function analysis cost, but the gradient analysis time is the dominant cost

Acknowledgment

The second author, G. J.-W. Hou, was supported in this work by NASA through several Tasks under contract NAS1-19858 and NASA P.O. No. L-9291 with the Old Dominion University Research Foundation.

References

- ¹Newman, P. A., Hou, G. J.-W., and Taylor, III, A. C., "Observations Regarding Use of Advanced CFD Analysis, Sensitivity Analysis, and Design Codes in MDO," *Multidisciplinary Design Optimization: State of the Art*, edited by N. M. Alexandrov and M. Y. Hussaini. SIAM Proceedings Series, Philadelphia, 1997, pp. 263-279, also ICASE Report 96-16, NASA CR 198293, (available electronically at www.icas.edu).
- ²Newman, III, J. C., Taylor, III, A. C., Barnwell, R. W., Newman, P. A., and Hou, G. J.-W., "Overview of Sensitivity Analysis and Shape Optimization for Complex Aerodynamic Configurations," *Journal of Aircraft*, Vol. 36, No. 1, 1999, pp. 87-96.
- ³Alexandrov, N. M. and Hussaini, M. Y., editors, *Multidisciplinary Design Optimization: State of the Art*, SIAM Proceedings Series, Philadelphia, 1997.
- ⁴Ghaffas, O. and Orozco, C. E., "A Parallel Reduced Hessian SQP Method for Shape Optimization," *Multidisciplinary Design Optimization: State of the Art*, edited by N. M. Alexandrov and M. Y. Hussaini, SIAM Proceedings Series, Philadelphia, 1997, pp. 133-152.
- ⁵Hou, G. J.-W., Taylor, III, A. C., Mani, S. V., and Newman, P. A., "Simultaneous Aerodynamic Analysis and Design Optimization," *Abstracts from 2nd U.S. National Congress on Computational Mechanics*, Aug. 1993, pp. 130.
- ⁶Mani, S. V., *Simultaneous Aerodynamic Analysis and Design Optimization*, Master's thesis, Old Dominion University, Norfolk, VA, Dec. 1993.
- ⁷Hou, G. J.-W., Korivi, V. M., Taylor, III, A. C., Maraju, V., and Newman, P. A., "Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) of a Turbulent Transonic Airfoil Using a Navier-Stokes Code With Automatic Differentiation (ADIFOR)," *Proceedings of the Computational Aerodynamics Workshop 95*, edited by W. J. Feiereisen and A. K. Lacer, NASA CD CP-20010, Jan. 1996, pp. 82-85.
- ⁸Gumbert, C. R., Hou, G. J.-W., and Newman, P. A., "Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) of a 3-D Rigid Wing," *Proceedings of 14th AIAA Computational Fluid Dynamics Conference*, Norfolk, June 1999, pp. 402-418, also AIAA Paper 99-3296.
- ⁹Gumbert, C. R., Hou, G. J.-W., and Newman, P. A., "Simultaneous Aerodynamic Analysis and Design Optimization (SAADO) of a 3-D Flexible Wing," AIAA Paper 2001-1107, Jan. 2001.
- ¹⁰Haftka, R. T., Sobieszczanski-Sobieski, J., and Padula, S. L., "On Options for Interdisciplinary Analysis and Design Optimization," *Structural Optimization*, Vol. 4, 1992, pp. 65-74.
- ¹¹Shubin, G. R., "Application of Alternative Multidisciplinary Optimization Formulations to a Model Problem for Static Aeroelasticity," *Journal of Computational Physics*, Vol. 118, No. 1, 1995, pp. 73-85.
- ¹²Arian, E., "Analysis of the Hessian for Aeroelastic Optimization," ICASE Report 95-84, Dec. 1995.
- ¹³Walsh, J. L., Townsend, J. C., Salas, A. O., Samareh, J. A., Mukhopadhyay, V., and Barthelemy, J. F., "Multidisciplinary High-Fidelity Analysis and Optimization of Aerospace Vehicles, Part 1: Formulation," AIAA Paper 2000-0418, Jan. 2000.
- ¹⁴Walsh, J. L., Weston, R. P., Samareh, J. A., Mason, B. H., Green, L. L., and Biedron, R. T., "Multidisciplinary High-Fidelity Analysis and Optimization of Aerospace Vehicles, Part 2: Preliminary Results," AIAA Paper 2000-0419, Jan. 2000.
- ¹⁵Reuther, J. J., Alonso, J. J., Martins, J. R., and Smith, S. C., "A Coupled Aero-Structural Optimization Method for Complete Aircraft Configurations," AIAA Paper 99-0187, Jan. 1999.
- ¹⁶Kapania, R. K., Eldred, L. B., and Barthelemy, J. -F. M., "Sensitivity Analysis of a Wing Aeroelastic Response," *Journal of Aircraft*, Vol. 30, No. 4, 1993, pp. 496-504, also AIAA Paper 91-1103.
- ¹⁷Arslan, A. E. and Carlson, L. A., "Integrated Determination of Sensitivity Derivatives for an Aeroelastic Transonic Wing," *Proceedings for the 5th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Panama City Beach, FL, Sept. 1994, pp. 1286-1300, also AIAA Paper 94-4400 CP.
- ¹⁸Giunta, A. A. and Sobieszczanski-Sobieski, J., "Progress Toward Using Sensitivity Derivatives in a High-Fidelity Aeroelastic Analysis of a Supersonic Transport," *Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, St. Louis, MO, Sept. 1998, pp. 441-453, also AIAA Paper 98-4763 CP.
- ¹⁹Giunta, A. A., "Sensitivity Analysis for Coupled Aero-Structural Systems," NASA TM-1999-209367, Aug. 1999.
- ²⁰Newman, III, J. C., Whitfield, D. L., and Anderson, K. W., "A Step-Size Independent Approach for Multidisciplinary Sensitivity Analysis and Design Optimization," *Proceedings of the 17th Applied Aerodynamics Conference*, Norfolk, VA, June 1999, pp. 12-22, also AIAA Paper 99-3101.
- ²¹Ghaffas, O. and Li, X., "Domain Decomposition Methods for Sensitivity Analysis of a Nonlinear Aeroelastic Problem," *International Journal of Computational Fluid Dynamics*, Vol. 11, 1998, pp. 113-130.
- ²²Møller, H. and Lund, E., "Shape Sensitivity Analysis of Strongly Coupled Fluid-Structure Interaction Problems," *Proceedings of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, Sept. 2000, also AIAA Paper 2000-4823.
- ²³Maute, K., Nikbay, M., and Farhat, C., "Analytical Based Sensitivity Analysis and Optimization of Nonlinear Aeroelastic Systems," *Proceedings of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, Sept. 2000, also AIAA Paper 2000-4825.
- ²⁴Hou, G. and Satyanarayana, A., "Analytical Sensitivity Analysis of a Static Aeroelastic Wing," *Proceedings of 8th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, Sept. 2000, also AIAA Paper 2000-4824.
- ²⁵Biros, G. and Ghaffas, O., "Parallel Newton- Krylov Algorithms For PDE-Constrained Optimization," *Proceedings of Scientific Computing 99*, Portland, OR, 1999.
- ²⁶Ghaffas, O. and Bark, J.-H., "Optimal Control of Two- and Three-Dimensional Navier-Stokes Flows," *Journal of Computational Physics*, Vol. 136, No. 2, 1997, pp. 231-244.
- ²⁷Pierce, N. A. and Giles, M. B., "Adjoint Recovery of Superconvergent Functionals From PDE Approximations," *SIAM Review*, Vol. 42, No. 2, 2000, pp. 247-264.
- ²⁸Venditti, D. A. and Darmofal, D. L., "Multilevel Error Estimation and Grid Adaptive Strategy for Improving the Accuracy of Integral Outputs," *Proceedings of the AIAA 15th Computational Fluid Dynamics Conference*, Norfolk, June 1999, also AIAA Paper 1999-3292.
- ²⁹Jameson, A. and Vassberg, J. C., "Computational Fluid Dynamics for Aerodynamic Design: Its Current and Future Impact," AIAA Paper 2001-0538, Jan. 2001.
- ³⁰Rumsey, C., Biedron, R., and Thomas, J., "CFL3D: Its History and Some Recent Applications," NASA TM-112861, May 1997.
- ³¹Sherman, L., Taylor, III, A., Green, L., Newman, P., Hou, G., and Korivi, M., "First- and Second- Order Aerodynamic Sensitivity Derivatives via Automatic Differentiation with Incre-

mental Iterative Methods," *Journal of Computational Physics*, Vol. 129, No. 2, 1996, pp. 307-336.

³²Bischof, C. H., Carle, A., Corliss, G. F., Griewank, A., and Hovland, P., "ADIFOR: Generating Derivative Codes from Fortran Programs," *Scientific Programming*, Vol. 1, No. 1, 1992, pp. 1-29.

³³Bischof, C. and Griewank, A., "ADIFOR: A Fortran System for Portable Automatic Differentiation," *Proceedings of the 4th AIAA/USAF/NASA/OAI Symposium on Multidisciplinary Analysis and Optimization*, Cleveland, OH, Sept. 1992, pp. 433-441, also AIAA Paper 92-4744 CP.

³⁴Taylor, III, A. C., Olosio, A., and Newman, III, J.C., "CFL3D.ADH (Version 2.0): An Efficient, Accurate, General-Purpose Code for Flow Shape-Sensitivity Analysis," AIAA Paper 97-2204, June 1997.

³⁵Smith, R. E., Bloor, M. I. G., Wilson, M. J., and Thomas, A. T., "Rapid Airplane Parametric Input Design (RAPID)," *Proceedings of the 12th AIAA Computational Fluid Dynamics Conference*, San Diego, June 1995, pp. 452-462, also AIAA Paper 95-1687.

³⁶Jones, W. T. and Samareh-Abolhassani, J., "A Grid Generation System for Multidisciplinary Design Optimization," *Proceedings of the 12th AIAA Computational Fluid Dynamics Conference*, San Diego, June 1995, pp. 474-482, also AIAA Paper 95-1689.

³⁷Bischof, C., Jones, W. T., Samareh-Abolhassani, J., and Mauer, A., "Experiences with the Application of the ADIC Automatic Differentiation Tool to the CSCMDO 3-D Volume Grid Generation Code," AIAA Paper 96-0716, Jan. 1996.

³⁸Nguyen, D. T., "Finite Element Software for Multidisciplinary Design Optimization, Final Report," Tech. rep., Old Dominion University Research Foundation, Contract NAS1-19858, Task 69, NASA Langley Research Center, Hampton, VA, Nov. 1995.

³⁹Hou, G., Arunkumar, S., and Tiwari, N. S., "First- and Second-Order Sensitivity Analysis of Finite Element Equations via Automatic Differentiation," *Proceedings of the 7th AIAA/USAF/NASA/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, St. Louis, MO, Sept. 1998, pp. 454-464, also AIAA Paper 98-4764.

⁴⁰Samareh, J. A. and Bhatia, K. G., "A Unified Approach to Modeling Multidisciplinary Interactions," *Proceedings of the 8th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization*, Long Beach, Sept. 2000, also AIAA Paper 2000-4704.

⁴¹Anon., *DOT Users Manual: Version 5.0*, Vanderplaats Research & Development, Inc., Colorado Springs, May 1999.

⁴²Ta'asan, S., "Trends in Aerodynamic Design and Optimization: A Mathematical Viewpoint," *Proceedings of the 12th AIAA Computational Fluid Dynamics Conference*, San Diego, June 1995, pp. 961-970, also AIAA Paper 95-0001.



