Human Exploration Missions Study

Space Surveillance Telescope
Transfer to and Station at a Halo Orbit
At the Earth-Sun Libration Point L2

MSFC/Alpha Technology, Inc.

FINAL REPORT

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INTRODUCTION

This study was undertaken to determine mission profile and delta velocity requirements to place a telescope at the Earth-Sun libration point L2. The program "Integrated Mission Program" (IMP), was selected to be used in the investigation.

A description of IMP and its capabilities may be found in the Addenda. The Addenda also contains the libration halo equations, constants and other parameters. Comments regarding the chaotic nature of numerical integration near the libration points are also attached in the Addenda.

A basic two stage S/C with a simple mission profile was selected. This profile is shown immediately below.

SPACE SURVEILLANCE TELESCOPE MISSION PROFILE
TBD DAY TRANSFER WITH COURSE CORRECTION AT 2-3 DAYS

DEPART FROM NODE OF PARKING ORBIT (TLIB)
160 NM : INCLINATION 23.45 DEG

DROP 1ST STAGE

MIDCOURSE

HALO ORBIT INSERTION

DROP 2ND STAGE

The original study began with a space craft (S/C) weight of 6200 lbs. This was raised scaling by 10 to 62000 lbs. The two S/C are described in Table 3 and 6 respectively. The study results will be shown for both vehicles.

An important part of the study was to identify opportunities for departure and arrival. The initial study focused on a constant date of arrival. Trip times of 20, 30, 40, 60, 80, and 100 days were investigated to arrive on 25 Sep 2011. Later a constant trip time of 100 days to arrive at selected dates in September, October, November, and December was studied.

TRANSFER (TLIB)

In a classical Newtonian two body system, the optimum transfer from one circular orbit to another is a Hohmann transfer of 180 degrees. In an oblate system, an exact 180 degree transfer in inertial space cannot be accomplished by a single burn at the departure point, but must have a midcourse to account for the transfer plane's regression. Or a transfer can be made to the regressed apsides. Our system is a three body oblate system, with perturbations that also pose restrictions. Two techniques for surmounting these restrictions are discussed on the next page.
Another requirement was an algorithm to be used in selecting departure date, time, orbit, position etc to arrive after varying trip times. The departure and arrival points, Table 1, were selected using the following as an algorithm.

Select a transfer time and an arrival date. This chooses a departure date and time. The departure date/time is used as program time zero, and sets the coordinate system that will be used for the calculations.

With program zero time set, use the insertion event in IMP to insert a target vehicle at the arrival time at the L2 point. From the output, note the arrival geocentric Latitude and the inertial Longitude. Note also whether the arrival azimuth is ascending or descending. The departure point is set to the opposite side of the Earth in plane with the Earth-Sun.

Using IMP INSERTION, option 0, set the main S/C at the negative of the arrival Latitude, at the arrival inertial Longitude + 180 degrees, and at the departure altitude of 160 NM. Set the azimuth flag to compute the proper direction of the orbit velocity. Input an inclination of 23.45 degrees. Then if velocity is set to 0., IMP will set circular velocity and azimuth.

This yields an almost inplane departure with close to 180 degrees of transfer. A small coast can then set the transfer less than 179 degrees so that it can be solved. This coast can also be used for optimization. A second technique to get away from the 180 degree point is to offset the longitude by a few degrees. This accounts for the transfer plane's regression rate varying as the S/C moves from the oblate gravity field to a spherical field. The second method was not used other than a few selected test cases.

Table 1 assumes a constant arrival date 25 Sep 2011 is desired and shows departure Lat/Lon for several trip times.

<table>
<thead>
<tr>
<th>TRIP TIME DAY</th>
<th>DEPARTURE DATE</th>
<th>LAT</th>
<th>LON(G)</th>
<th>ARRIVAL DATE</th>
<th>LAT</th>
<th>LON(IN)</th>
<th>ASC</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>SEP 05</td>
<td>-.731</td>
<td>107.68</td>
<td>DSC</td>
<td>SEP 25</td>
<td>.731</td>
<td>-72.32</td>
</tr>
</tbody>
</table>
Table 2 assumes a constant trip time of 100 days to arrive at the dates shown.

<table>
<thead>
<tr>
<th>DATE</th>
<th>LAT</th>
<th>LON(G)</th>
<th>DATE</th>
<th>LAT</th>
<th>LON(IN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JUN 7</td>
<td>3.146</td>
<td>187.418</td>
<td>DSC</td>
<td>SEP 15</td>
<td>-3.146</td>
</tr>
<tr>
<td>JUN 17</td>
<td>-7.31</td>
<td>186.530</td>
<td>DSC</td>
<td>SEP 25</td>
<td>.731</td>
</tr>
<tr>
<td>JUL 07</td>
<td>-8.398</td>
<td>185.043</td>
<td>DSC</td>
<td>OCT 15</td>
<td>8.398</td>
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<tr>
<td>AUG 07</td>
<td>-18.399</td>
<td>184.691</td>
<td>DSC</td>
<td>NOV 15</td>
<td>18.399</td>
</tr>
<tr>
<td>SEP 07</td>
<td>-23.298</td>
<td>187.400</td>
<td>DSC</td>
<td>DEC 16</td>
<td>23.298</td>
</tr>
</tbody>
</table>

**TLIB TRANSFERS TO L-2**

A simple method of breaking the initial orbit and moving out of the Earth's major influence was desired. Two stages are used. It is planned to use a fixed size first stage for the TLIB burn, drop it and then after 2-3 days do a midcourse correction with the second stage. There are several techniques for the TLIB burn.

**HOHMANN**

Departure point is perigee. The target apogee altitude is near the L2 radius from the Earth and can be used for optimization. Velocity at departure is calculated from closed form two body equations then modified with BELL LAB's correction for an oblate Earth. The TLIB burn is in the initial departure orbit plane.

**INPLANE**

An inplane burn is made such that the S/C is in an orbit intersecting the departure orbit. At the intersection, the new orbit has a velocity and flight path angle as input. Normally 10908 M/S and 0 degrees. Again the begin event time is as given above and can be used for optimization.

**LAMBERT**

The halo insertion point is used as a target. The departure point is the initial position. A spherical Lambert problem is solved for the velocity needed at the initial point to arrive at the target point at the time desired. This solution is basically for two bodies and cannot be a 180 degree inertial transfer. The transfer may be modified for the presence of the sun using IMP'S algorithm "NXDN" which numerically solves for a new XDN. This is referred to as an oblate Lambert. It includes the oblate Earth gravity effects, and also the Sun's. Since a Lambert solution for 180 degrees does not exist, the begin event state must insure less than 180 degrees transfer. It still may be used for optimization.
OTHER THRUST EVENTS

MIDCOURSE
An oblate Lambert is solved from the midcourse point to the halo insertion point to arrive at the time desired. The midcourse transfer angle should be between 5 to 20 degrees. And in a posigrade direction. Midcourse should be accomplished while the Earth's gravity field still predominates. If delayed, things might get squirrely in the simulation.

INJECTION
Halo velocity calculated, and burn guided such that a minimum state error is obtained. On arrival this might require a big delta V. It is very affected by the length of time the sun's gravity has to change the approach trajectory. Injection is into a retrograde Halo at a point on the LOS from L2 to the Earth. A current initial halo x is 2000 KM.

NUMERICAL RESULTS

The Integrated Mission program described on page 11 was used to integrate the equations of motion. IMP uses double precision FORTRAN and early in the study a requirement for at least Quad precision was suspected. Since this was not available, means of getting better results were investigated.

See pages 13 to 19 for a description of the basic equations used in IMP's libration simulations. In particular note the definition of a libration point. At such a point, the sum of all forces acting on a body is zero. Additionally the attraction of gravity by either the sun or the earth is very small. This means that numerical roundoff and truncation can be a chaotic factor, and indeed, they were found so.

One method for easing this problem is a modified patch conic procedure for TLIB and midcourse. The transfers are integrated with the Sun's gravity omitted from the equation of motion. On reaching the halo insertion point, it is turned back on to calculate the Halo velocity needed. In the tables below, with Sun refers to results leaving the Sun in the equation of motion. A notation of OK means that the transfer did not exceed the DV capability of either stage. Most of the light vehicle transfers were in the not OK status. It was expected to be difficult to design a small system to put a telescope at L2.
## TABLE 3. GENERIC LIGHT VEHICLE DATA

<table>
<thead>
<tr>
<th>STAGE</th>
<th>THRUST (LB)</th>
<th>ISP (SEC)</th>
<th>FUEL (LB)</th>
<th>DRY (LB)</th>
<th>TOTAL (LB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1ST STAGE: BREAK ORBIT AND TRANSFER</td>
<td>24000</td>
<td>300</td>
<td>4100</td>
<td>825</td>
<td>4925</td>
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<tr>
<td>2ND STAGE: MIDCOURSE AND FINAL INJECTION</td>
<td>1000</td>
<td>285</td>
<td>220</td>
<td>325</td>
<td>545</td>
</tr>
<tr>
<td>PAYLOAD</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TELESCOPE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>700</td>
</tr>
<tr>
<td>CONTINGENCY</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>TOTAL</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>730</td>
</tr>
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</table>

**TOTAL INITIAL WEIGHT:** 6200 LB

Basic HOHMANN transfers to radii near L2 were used to size the first stage of both vehicles.

## TABLE 4. HOHMANN TRANSFERS LIGHT VEHICLE

<table>
<thead>
<tr>
<th>RADIUS (KM)</th>
<th>PERIOD (HRS)</th>
<th>DV (M/S)</th>
<th>FUEL (LB)</th>
<th>TRANSFER (DAYS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1500000.</td>
<td>1818.4</td>
<td>3174.7</td>
<td>4092.6</td>
<td>37.8</td>
</tr>
<tr>
<td>1600000.</td>
<td>2001.6</td>
<td>3176.2</td>
<td>4093.7</td>
<td>41.7</td>
</tr>
<tr>
<td>1700000.</td>
<td>2190.5</td>
<td>3177.5</td>
<td>4094.6</td>
<td>45.6 OK</td>
</tr>
<tr>
<td>1800000.</td>
<td>2385.1</td>
<td>3178.7</td>
<td>4095.5</td>
<td>49.7</td>
</tr>
<tr>
<td>1900000.</td>
<td>2585.0</td>
<td>3179.8</td>
<td>4096.2</td>
<td>53.8</td>
</tr>
</tbody>
</table>

## TABLE 5. HOHMANN TRANSFERS LIGHT VEHICLE

<table>
<thead>
<tr>
<th>TDAYS</th>
<th>TLIB</th>
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<th>INS</th>
<th>SUM</th>
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</thead>
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<tr>
<td>20</td>
<td>3178</td>
<td>98</td>
<td>607</td>
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<td>30</td>
<td>3178</td>
<td>74</td>
<td>378</td>
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<tr>
<td>40</td>
<td>3178</td>
<td>75</td>
<td>359</td>
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<td>60</td>
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<td>192</td>
<td>384</td>
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<td>612</td>
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<td>3178</td>
<td>398</td>
<td>715</td>
<td>4292</td>
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### TABLE 6. LIGHT VEHICLE OTHER TRANSFERS

<table>
<thead>
<tr>
<th>TDAYS</th>
<th>INPLANE 1ST FIXED</th>
<th>M/S</th>
<th>WITH SUN G</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TLIB MID INS</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td>3175 104 609</td>
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<td>3888</td>
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<td>40</td>
<td>3175 106 329</td>
<td></td>
<td>3610 OK</td>
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<td>60</td>
<td>3175 220 405</td>
<td></td>
<td>3800</td>
<td></td>
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<td>80</td>
<td>3175 366 574</td>
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<td>4115</td>
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<td>100</td>
<td>3175 399 701</td>
<td></td>
<td>4275</td>
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<table>
<thead>
<tr>
<th>TDAYS</th>
<th>INPLANE 1ST FIXED</th>
<th>M/S</th>
<th>WITHOUT SUN G</th>
<th>SUM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TLIB MID INS</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>3175 72 548</td>
<td></td>
<td>3795</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>3175 6 254</td>
<td></td>
<td>3435 OK</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>3175 16 330</td>
<td></td>
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<tr>
<td>80</td>
<td>3175 40 399</td>
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<td></td>
</tr>
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<td>100</td>
<td>3175 58 451</td>
<td></td>
<td>3685 OK</td>
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<table>
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<tr>
<th>TDAYS</th>
<th>LAMBERT SPH M/S</th>
<th>WITH SUN G</th>
<th>SUM</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>TLIB MID INS</td>
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<tr>
<td>20</td>
<td>3202 20 611</td>
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<tr>
<td>30</td>
<td>3237 40 369</td>
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<td>40</td>
<td>3291 81 320</td>
<td>3692</td>
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<td>60</td>
<td>3345 246 426</td>
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<td>3375 427 592</td>
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<td>100</td>
<td>3402 142 493</td>
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<table>
<thead>
<tr>
<th>TDAYS</th>
<th>LAMBERT SPH M/S</th>
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<tbody>
<tr>
<td></td>
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<td>3202 27 552</td>
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<td>30</td>
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<td>3290 27 254</td>
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<td>3334 27 330</td>
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<td>80</td>
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<tr>
<td>100</td>
<td>3401 26 453</td>
<td>3880</td>
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</tr>
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</table>
TABLE 7. GENERIC HEAVY VEHICLE DATA

1ST STAGE: BREAK ORBIT AND TRANSFER

<table>
<thead>
<tr>
<th>Thrust</th>
<th>ISP</th>
<th>Fuel</th>
<th>Dry</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>60000</td>
<td>300</td>
<td>41000</td>
<td>8250</td>
<td>49250</td>
</tr>
</tbody>
</table>

3185 m/s DV
.968 T/W
200 LB/S

2ND STAGE: MIDCOURSE AND FINAL INJECTION

<table>
<thead>
<tr>
<th>Thrust</th>
<th>ISP</th>
<th>Fuel</th>
<th>Dry</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>285</td>
<td>4250</td>
<td>1250</td>
<td>5500</td>
</tr>
</tbody>
</table>

1130 m/s DV
.784 T/W
35 LB/S

PAYLOAD

<table>
<thead>
<tr>
<th>Telescope</th>
<th>Contingency</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>7000 LB</td>
<td>250 LB</td>
<td>7250 LB</td>
</tr>
</tbody>
</table>

PF .117

TOTAL INITIAL WEIGHT

62000 LB

See Tables 1 and 2 for the departure and arrival positions that were used for the following transfers.

TABLE 8. HEAVY VEHICLE TRANSFERS FIXED ARRIVAL DATE

<table>
<thead>
<tr>
<th>TDays</th>
<th>TLIB</th>
<th>MID</th>
<th>INS</th>
<th>SUM</th>
<th>M/S</th>
<th>M/S</th>
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</thead>
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<tr>
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<td>80</td>
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<td>401</td>
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</table>

<table>
<thead>
<tr>
<th>TDays</th>
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<th>MID</th>
<th>INS</th>
<th>SUM</th>
<th>M/S</th>
<th>M/S</th>
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</thead>
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<td>-----</td>
<td>-----</td>
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</tr>
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<td>SEP 15</td>
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<td>465</td>
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</tbody>
</table>
HEAVY INPLANE 100 DAY 1=SUN 2=NO

SUM

0.09 0.10 0.11 0.12 0.13
ARR MO X 10**2
"IMP" IS A SIMULATION LANGUAGE THAT IS USED TO MODEL MOST PRESENT OR FUTURE MISSIONS ABOUT THE EARTH, MARS, MOON OR OTHER BODY. MISSIONS ARE USER CONTROLLED THROUGH SELECTION FROM A LARGE EVENT/MANEUVER MENU. MISSION PROFILES, TIMELINES, PROPELLANT REQUIREMENTS, FEASIBILITY AND PERTURBATION ANALYSIS MAY BE QUICKLY, ACCURATELY CALCULATED. ONE, TWO OR THREE SPACECRAFT MAY BE USED: A MAIN, A TARGET AND AN OBSERVER.

- A FEHLBERG 7/13 RUNGE-KUTTA INTEGRATOR WITH ERROR AND STEP SIZE CONTROL IS USED TO NUMERICALLY INTEGRATE THE EQUATIONS OF MOTION.

- OBLATE OR SPHERICAL GRAVITY CAN BE USED FOR THE CENTRAL BODY. ADDITIONAL EFFECTS OF SUN GRAVITY, SOLAR PRESSURE, OR MOON GRAVITY ARE AVAILABLE. WHEN ADDED, THE SUN OR MOON GRAVITY IS SPHERICAL.

- AERODYNAMIC LIFT AND DRAG WHILE IN THE EARTH OR MARTIAN ATMOSPHERE INCLUDED WHEN REQUESTED.

- INPUT/OUTPUT HAS BEEN SIMPLIFIED AND IS IN METRIC UNITS, WITH THE EXCEPTION OF THRUST AND WEIGHT WHICH ARE IN ENGLISH UNITS. INPUT IS READ FROM THE VDT KEYBOARD AND THE USER'S INPUT FILE. REAL TIME KEYBOARD INPUT HAS BEEN MINIMIZED.

- THE CODE EXECUTES EVENTS IN THE ORDER THEY ARE INPUT. AN EVENT IS ENTERED BY INVOKING ITS NAME, READING ITS OPTIONS, DATA, AND THEN EXECUTING. ON COMPLETION, THE NEXT EVENT IS STARTED.

- EVENTS/MANEUVERS MAY INVOLVE NONE, ONE, OR MULTI VELOCITY CHANGES. THRUST LEVEL, AND PROPELLANT USAGE ARE AS THE USER SELECTS. THE VELOCITY CHANGES MAY BE IMPULSIVE, OR OF FINITE DURATION WITH GUIDANCE CALCULATED INTERNALLY OR PRESET BY THE USER. ALGORITHMS FOR TWO POINT BOUNDARY VALUES PROBLEMS INVOLVING VELOCITY CHANGES AND GUIDANCE ARE AUTOMATICALLY INVOKED AS NEEDED.

- MAIN OUTPUT IS TO A USER NAMED PRINT FILE, TO A PLOT FILE, AND TO A DEBUG FILE. MAJOR MANEUVERS ALSO USE THEIR OWN PRENAMED FILES TO OUTPUT ADDITIONAL DATA IN TABULAR AS WELL AS PLOT FORM.

- THE CODE IS PROGRAMMED IN DOUBLE PRECISION FORTRAN AND WILL COMPILE WITHOUT ERRORS TO THE STANDARDS OF LAHEY (TM) FORTRAN 90.

THE PROGRAM WAS INITIALLY CODED FOR MARSHALL SPACE FLIGHT CENTER (MSFC), SE-AERO-G, HUNTSVILLE, AL. THE AUTHOR WAS EMPLOYED BY NORTHROP SERVICES, INC., HUNTSVILLE, AL. LATER IT WAS EXTENSIVELY MODIFIED WHEN HE WAS A MISSION ANALYST AT PRELIMINARY DESIGN, PD33, MSFC. SINCE RETIREMENT HE HAS CONTINUED TO UPDATE AND IMPROVE IMP.
The following is data used and or computed in IMP to model the Lagrangian libration point L2. The spherical gravity mode of IMP and an ideal 3 body simulation, ideal constants as below, were used to validate the equations of motion on page 15.

```
* * * * EARTH / SUN * * * *

YEAR 2011 DATA

IDEAL LIBRATION SYSTEM S/E
GM1 M**3/S**2 0.398601200000E+15
GM2 M**3/S**2 0.132718490000E+21
MEAN R M 149600000000.00
OMG SYS DEG/DAY 0.98561057
DEG/S 0.114075297544E-04
PERIOD DAYS 365.25582281
OMG M1 DEG/SEC 0.417807419642E-02

LIBRATION DATA SYS S/E POINT 2
YEAR MAX MIN DIF P/C
RSYS KM 152098953.00 147101055.15 4997897.85 3.29
OMG D/S 0.00001170 0.00001113 0.00000057 4.89
RLIB KM 1526613.13 1476449.37 50163.77 3.29
VLIB M/S 30338.524 29835.906 502.618 1.66
BC KM 456.81 441.80 15.01 3.29

PRESENT PARAMETERS AT LIB POINT
GRAVITY PARTIAL WRT R FORCE
EARTH -0.24767952E-12 0.18284634E-03 (M/S**2)
SUN -0.80924389E-13 0.60118772E-02 (M/S**2)

CENTRIFUGAL 0.61947235E-02 (M/S**2)

DISTANCES
SYSTEM 147103631.82 (KM)
LP TO EARTH 1476475.23 (KM)
BC TO SUN 441.80 (KM)
```
LIBRATION SIMULATIONS CAN BE CHAOTIC. CALCULATIONS, EVEN DOUBLE PRECISION, ARE NOT STABLE. ROUNDOFF AND TRUNCATION DEGRADE ACCURACY.

FOR S/E SYSTEM, OR E/M SYSTEM, IMP IS STABLE FOR A SIMULATION OF ABOUT ONE HALO ORBIT, S/E 180 DAYS, M/E 11 DAYS. USE LONGER TIMES WITH CAUTION.

LIBRATION SIMULATIONS MAY BE

1) AN IDEAL RESTRICTED THREE-BODY SOLUTION USING CIRCULAR ORBITS, SPHERICAL GRAVITY, IDEALIZED CONSTANTS, (MU, RADII, ETC) AND EITHER

   A) CLOSED FORM EQUATIONS OR,
   B) NUMERICAL INTEGRATION OF EQUATIONS OF MOTION

2) A REAL WORLD SIMULATION USING ELLIPTICAL ORBITS, OBLATE GRAVITY, EPHEMERIDES, CURRENT CONSTANTS, (MU, RADII, ETC) AND EITHER

   A) APPROXIMATIONS USING PERTURBATION ANALYSIS OF THE IDEAL AND ASSUMED CLOSED FORM EQUATIONS
   B) NUMERICAL INTEGRATION OF EQUATIONS OF MOTION

IT IS EVIDENT THAT THE ABOVE ARE ALL USEFUL TOOLS IN STUDYING LIBRATION. 1A IS USED FOR FAST PRELIMINARY RESULTS AND AT MOST IS ONLY ADVISORY. 1B IS USED TO TEST THE IDEAL EQUATIONS AND THE RESULTS OF ROUNDOFF AND TRUNCATION. 2A AGAIN GIVES FAST RESULTS AND IN GENERAL IS USED FOR PRELIMINARY PLANNING. 2B REQUIRES A LOT OF COMPUTER TIME AND IS NORMALLY USED LAST TO REFINE SOLUTIONS OBTAINED BY THE OTHER METHODS.

IMP USES NUMERICAL INTEGRATION OF THE EQUATIONS OF MOTION, AND CAN BE USED IN AN IDEAL (1B) OR PERTURBED (2B) MODES.
KEATON'S EQUATIONS AS SHOWN IN REFERENCE 4, WERE ADAPTED AND
USED IN IMP FOR LIBRATION STUDIES. REFERENCES 6, 7, AND 8 WERE
ALSO USED IN PREPARING THE SIMULATION. THESE ARE ALL EXAMPLES
OF THE CLOSED FORM IDEAL EQUATIONS.

BY DEFINITION, THE SUM OF FORCES AT A LIB POINT IS ZERO. EACH
SYSTEM HAS 5 PLACES WHERE THAT CAN OCCUR. THE FIRST THREE EARTH
SUN ARE ON THE EARTH SUN LINE. L1 ON THE SUNNY SIDE, L2 ON THE
DARK SIDE, AND L3 ON THE SIDE OF THE SUN AWAY FROM THE EARTH.
L4 IS AHEAD OF THE EARTH IN ITS ORBIT, AND L5 TRAILS THE EARTH
IN ITS ORBIT. L4(L5) WITH THE EARTH AND SUN FORMS AN EQUILATERAL
TRIANGLE. IMP HAS SIMULATIONS FOR L1, L2, L4 AND L5. L3 IS NOT
NORMALLY STUDIED.

IT IS KNOWN THAT THE LIB POINTS 1, AND 2 ARE UNSTABLE. HALO ORBITS
ABOUT L1 OR L2 CAN BE MADE STABLE IN THE RESTRICTED 3 BODY CASE.

L4 AND L5 ARE STABLE AT WHAT ARE TERMED THE TROJAN POINTS.
HALO ORBITS DO NOT EXIST ABOUT L4 AND L5.

HALO ORBITS ABOUT L1 AND L2 ARE USUALLY WHAT INVESTIGATORS STUDY.
WE INTEND STATIONING FOR A PERIOD OF TIME, IF PERTURBATIONS ARE
PRESENT, INSTABILITY MAY RESULT. IN FACT, TRUNCATION OR ROUND OFF
EVENTUALLY DISTURB THE SIMULATION OF THE SYSTEM. THEREFORE IN
TESTING, WE NEED TO EVALUATE THE IDEAL AS WELL AS THE REAL WORLD.

TESTS SHOW THAT IMP'S EQUATIONS, ARE ACCEPTABLE FOR AT LEAST
ONE IDEAL EARTH/SUN, MOON/EARTH OR MARS/SUN HALO ORBIT. THAT IS,
IN THE NON PERTURBED RESTRICTED 3 BODY MODE, THE PROGRAM WILL
SIMULATE HALO ORBITS ABOUT E/S, E/M OR M/S LIBRATION POINTS L1
AND L2. AS A MEASURE OF THE SIMULATIONS RELIABILITY, A HALO ORBIT
ABOUT THE EARTH/SUN L1 POINT, RESTRICTED 3 BODY SOLUTION, NEEDS
LESS THAN 1 M/S DELTA V TO ACCOUNT FOR ROUND OFF AND TRUNCATION.

IDEAL STATIONING AT L4 AND L5 IS ALSO MODELED IN IMP, AND 90
DAYS AT THE EARTH-SUN L5 POINT REQUIRES NO CORRECTIVE DELTA-V.

AS PART OF THE INVESTIGATION, PARAMETERS FOR THE IDEAL AND
PERTURBED SYSTEMS WERE CALCULATED. FOR THE EARTH/SUN SYSTEM
MODELED IN IMP, THE DATA OBTAINED WAS PREVIOUSLY SHOWN.
**PERTURBED RESULTS**

STUDIES OF WHAT HAPPENS WHEN THE GRAVITY IS NON KEPLERIAN, OR ECCENTRIC ORBITS ARE USED, OR A FOURTH BODY IS PRESENT YIELD SOME INTERESTING RESULTS.

HALO ORBITS ABOUT THE E/S LIBRATION POINTS L1 AND L2 ARE FAIRLY STABLE. THEY ARE AFFECTED BY THE EARTH'S ORBIT, AND THE PRESENCE OF THE MOON, BUT THEY REQUIRE VERY LITTLE ENERGY TO REMAIN ON STATION.

HALO ORBITS ABOUT THE M/E LIBRATION POINTS, ARE AFFECTED BY THE MOON'S UNUSUAL ORBIT. CONSIDER MOON EVECTION. WHENEVER THE MOON'S VELOCITY IS DIRECTED TOWARD THE SUN, IT SPEEDS UP. IT SLOWS DOWN WHEN MOVING AWAY FROM THE SUN. THIS EFFECT IS ALSO FELT BY THE 3RD BODY IN ITS HALO ORBIT. THE SUN'S PRESENCE HAS AN EFFECT ON M/E HALO ORBITS ABOUT A LIBRATION POINT. STATIONING IS HOWEVER NOT AS DIFFICULT AS I ONCE THOUGHT. THE SUN'S EVECTION EFFECT IS EFFECTIVELY CANCELLED TO A GREAT DEGREE AND I NOW CONCLUDE THAT:

* HALO ORBITS ABOUT THE M/E LIBRATION POINTS (L1 OR L2) CAN BE USED AS TRANSPORTATION NODES. DELTA-V FOR STATIONING IS APPROXIMATELY 7 TIMES THE EARTH SUN SYSTEM.
THE FOLLOWING EQUATIONS ARE MODELED IN IMP IDEAL LIBRATION NODE/HALO SYSTEMS (SEE REFERENCES 4, 5, 6, 7, AND 8)

B1 SMALLER BODY WITH GRAVITY GM1
B2 LARGER " " GM2
BC BARYCENTER OF SYSTEM

L1 LIBRATION POINT B2 >>>> L1 > B1 INSIDE
L2 LIBRATION POINT B2 >>>> B1 > L2 OUTSIDE

DISTANCES
RS B1 TO B2 SYSTEM
R1 L1 TO B1
R2 L1 TO B2
R3 B2 TO BC
R4 BC TO L1

OMEGA SYSTEM ROTATION RATE
OMEGA**2 = (GM1+GM2)/RS**3

CALCULATIONS
ALPHA = GM2/(GM1+GM2)
R3 = (ONE-ALPHA)*RS

ITERATE R1 UNTIL SUM OF F = VERY SMALL

R2 = F(LIB POINT, R1, RS)
R4 = R2-RS

F1= GM1/R1**2 GRAVITY BODY1
F2= GM2/R2**2 GRAVITY BODY2
F3= R4*OMEGA**2 CENTRIFUGAL FORCE AROUND BC

SIGN F1, F2, AND F3 ACCORDING TO LIB POINT DESIRED AND SUM

F1+F2+F3 = ZERO

THEN SET KEATON'S EQUATIONS REFERENCE 4
FSQ=(ONE-ALPHA)*(RS/R1)**3+ALPHA*(RS/R2)**3
F=SQRT(FSQ)
BSQ=ONE-FSQ/TWO+F*SQRT(2.25D0*FSQ-TWO)
B=SQRT(BSQ)
GAMH=(ONE+BSQ+TWO*FSQ)/(TWO*B)
BOMGS=B*OMEGA
FOMGS=F*OMEGA
Sper= 2*PI/OMEGA SYSTEM PERIOD
HPER=SPER/B HALO PERIOD
COORDINATE SYSTEMS


HX POINTS FROM B1 TO B2
HY IS TO LEFT VIEWED FROM TOP IN ROTATION PLANE
HZ COMPLETES A RIGHT HAND SYSTEM

T0 INSERTION TIME (ONLY AT BANG = ZERO, OR PI)
X0 INSERTION POSITION ALWAYS POSITIVE
Y0 ALWAYS AT ZERO
Z0 OUT OF PLANE COMPONENT AT INSERTION ALWAYS POSITIVE
PHASE0 INSERTION PHASE ZERO IF INSERT AT BANG = ZERO
PI IF INSERT AT BANG = PI

T = POSITION TIME

THALO = T - T0
BANG = BOMGS * THALO + PHASE0
FANG = FOMGS * THALO + PHASE0

HX = X0 * COS(BANG)
HY = -X0 * GAMH * SIN(BANG)
HZ = Z0 * COS(FANG)

HXD = -X0 * BOMGS * SIN(BANG)
HYD = -X0 * BOMGS * GAMH * COS(BANG)
HZD = -Z0 * FOMGS * SIN(FANG)

ANOMALY ANGLE IN HX, HY PLANE
"Z" ANGLE WRT TO HX, HY PLANE
HALO POSITION

HALO VELOCITY
**LIBRATION NOTES**

IT IS IMPORTANT THAT THE USER KNOW THE DIFFERENCE BETWEEN THE
RESTRICTED THREE BODY SYSTEM (IDEAL) AND THE REAL WORLD. IN
THE IDEAL SYSTEM, THERE ARE ONLY THREE BODIES PRESENT. THESE
ARE IN IMP'S SIMULATION, DESIGNATED B1, B2, AND B3. THE MASSES
ARE SUCH THAT

\[ B3 << B1 < B2. \]

B1 AND B2 ROTATE ABOUT A COMMON BARYCENTER WITH A CONSTANT RATE
AND AT A CONSTANT DISTANCE. THIS IMPLIES SPHERICAL GRAVITY FIELDS
FOR BOTH B1 AND B2, AND THAT B3, AT THE LIBRATION POINT, HAS A
MINIMAL INFLUENCE ON THE SYSTEM.

IN THE REAL WORLD, THERE ARE PERTURBATIONS.

IN THE SUN-EARTH SYSTEM, THE EARTH IS IN AN ELLIPTICAL ORBIT ABOUT
THE BARYCENTER. ALTHOUGH THE DISTANCE TO THE SUN IS SUCH THAT THE
SUN APPEARS TO HAVE A SPHERICAL GRAVITY FIELD, THE THIRD BODY IS
CLOSE ENOUGH TO THE EARTH SO THAT IT SEES AN OBLATE EARTH GRAVITY
FIELD. FURTHER, THERE IS A FOURTH BODY, THE MOON, CLOSE ENOUGH TO
EFFECT THE SYSTEM. HOWEVER, FOR B3 AT THE EARTH-SUN LIBRATION
POINTS, THE MOON IS FAR ENOUGH AWAY SO THAT ITS EFFECTS ARE MINIMAL.

FOR THE EARTH-MOON SYSTEM, THE SUN IS A FOURTH BODY THAT DOES
AFFECT THE SYSTEM, AND MUST BE ACCOUNTED FOR IN A SIMULATION.
ADDITIONALLY, THE MOON'S ORBIT IS 3 TIME MORE ECCENTRIC THAN
THE EARTH, AND THE MOON ORBITS THE EARTH ABOUT 13 TIMES A YEAR.
**REFERENCES**


2. ----, NATURAL ENVIRONMENT AND PHYSICAL STANDARDS FOR THE APOLLO PROGRAM AND THE APOLLO APPLICATIONS PROGRAM, MD-E 8020.008C, SE 015-001-1B, OFFICE OF MANNED SPACE FLIGHT, WASHINGTON, D.C.,


