Large Eddy Simulation of Homogeneous Rotating Turbulence

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Study of turbulent flows in rotating reference frames has proven to be one of the more challenging areas of turbulence research. The large number of theoretical, experimental, and computational studies performed over the years have demonstrated that the effect of solid-body rotation on turbulent flows is subtle and remains exceedingly difficult to predict (e.g., Greenspan 1968, Bardina et al. 1985, Jacquin et al. 1990, Mansour et al. 1991). Because of the complexities associated with non-homogeneous turbulence, it is worthwhile to examine the effect of steady system rotation on the evolution of an initially isotropic turbulent flow. The assumption of statistical homogeneity considerably simplifies analysis and computation; calculation of homogeneous turbulence is further motivated since it possesses the essential physics found in more complex rotating flows.

The principal objectives of the present study have therefore been to increase our fundamental understanding of turbulent flows in rotating reference frames through an examination of the asymptotic state of homogeneous rotating turbulence; particularly as to the existence of an asymptotic state which is self similar. Knowledge of an asymptotic similarity state permits prediction of the ultimate statistical evolution of the flow without requiring detailed knowledge of the complex, and not well understood, non-linear transfer processes. Aside from examination of possible similarity states in rotating turbulence, of further interest in this study has been an examination of the degree to which solid-body rotation induces a two-dimensional state in an initially isotropic flow.

Large-eddy simulation (LES) is ideally suited for examination of the long-time evolution of homogeneous rotating turbulence since it circumvents the Reynolds number restriction of direct numerical simulation. The drawback is of course that it requires use of a model to parameterize subgrid-scale stresses. In this study LES of homogeneous rotating turbulence was performed using the pseudo-spectral technique developed by Rogallo (1981) for incompressible turbulence together with the spectral eddy viscosity model of Chollet & Lesieur (1981) (modified for rotating flows) for parameterization of the subgrid scales.

Preliminary computations performed on cubic computational domains demonstrated an adverse impact of the domain on flow evolution. Numerical experiments showed that it was necessary to use a domain which was at least four times larger along the rotation axis than in the other directions in order that simulation results were not adversely impacted by the finite computational domain. The resolution of the simulations performed in this study was 96 × 96 × 384 corresponding to a maximum physical wavenumber of 72. Energy

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spectra possessing low wavenumber components proportional to both $k^2$ and $k^4$ were used as in initial conditions. For all simulations the wavenumber corresponding to the peak in the spectrum, $k_p$, was 51. The value of $k_p$ was made as large as possible in an effort to attain an asymptotic similarity state before the integral scales of the flow became large enough to be influenced by the domain size. For each initial spectrum type (i.e., $k^2$ or $k^4$) simulations using two non-zero rotation rates were performed.

Evolution of the power-law exponents of the kinetic energy decay are shown in Figure 1 (i.e., the instantaneous logarithmic time derivative) for both initial spectrum types and each rotation rate used in the simulations. For no system rotation the power-law exponents are in good agreement with theoretical predictions as well as previous LES computations (Chasnov 1993). For both initial spectrum shapes it is evident that rotation reduces the decay rate of turbulence kinetic energy consistent with the well known effect of inhibition of non-linear energy transfer caused by solid-body rotation. More importantly, the results in Figure 1 show for both initial spectrum shapes that, following development from an initial transient, the decay rate of kinetic energy is independent of rotation rate. Further, it may be shown that the power law exponents measured in the simulations are in good agreement with theoretical predictions based on simple dimensional analysis. Finally, the increase in the power-law exponent at later times in the simulation (e.g., for the higher rotation rates) shows the influence of the domain on the evolution of the flow.

![Figure 1. Evolution of the power-law exponent of turbulence kinetic energy in homogeneous rotating turbulence (the time axis has been non-dimensionalized by the eddy turnover time in the initial field).](image_url)

- $-$ $k^2$ spectrum, $\Omega = 0.0$; $-$ $k^4$ spectrum, $\Omega = 0.0$; $-$ $k^2$ spectrum, $\Omega = 0.5$; $-$ $k^2$ spectrum, $\Omega = 1.0$; $-$ $k^4$ spectrum, $\Omega = 0.5$; $-$ $k^4$ spectrum, $\Omega = 1.0$. 

2
Integral lengthscales defined in the direction along the rotation axis and orthogonal to the rotation vector are shown in Figure 2 for the initial spectrum with low wavenumber part proportional to $k^2$. The large rate of growth of the lengthscales along the rotation direction is apparent. It is also interesting to note that the integral scales defined perpendicular to the rotation axis are independent of rotation rate. The rapid growth of integral scales along the rotation axis provides one indication of a two-dimensionalization of the flow. Other indicators (not shown here) such as energy spectra as a function of both polar angle and wavenumber have shown a marked concentration of spectral energy in the equatorial plane and have provided the most conclusive evidence to date that there exists an asymptotic self-similar state of homogeneous rotating turbulence which is strongly two-dimensional. It is also found that though two-dimensionalization as demonstrated through concentration of spectral energy in the equatorial plane is significant, commonly used descriptors of two-dimensional turbulence such as development of the Reynolds stress anisotropy tensor show little departure from isotropic values. These and other results will be included in the full paper.

![Figure 2](image)

**Figure 2.** Evolution of the integral lengthscales in homogeneous rotating turbulence for an initial spectrum with low wavenumber part proportional to $k^2$ (the time axis has been non-dimensionalized by the eddy turnover time in the initial field). The integral scale measured in planes orthogonal to the rotation axis is the "horizontal" lengthscales while "vertical" refers to the integral scale measured along the rotation axis. ——, $\Omega = 0$; ——, horizontal lengthscales ($\Omega = 0.5$); ——, horizontal lengthscales ($\Omega = 1.0$); ——, vertical lengthscales ($\Omega = 0.5$); ——, vertical lengthscales ($\Omega = 1.0$).

**Addenda**

Note that there is no restriction on the presentation and publication of the paper described in this abstract.
References


