The Effect of Doppler Frequency Shift, Frequency Offset of the Local Oscillators, and Phase Noise on the Performance of Coherent OFDM Receivers

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March 2001
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The Effect of Doppler Frequency Shift, Frequency Offset of the Local Oscillators, and Phase Noise on The Performance of Coherent OFDM Receivers

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Abstract
This paper first shows that the Doppler frequency shift affects the frequencies of the RF carrier, subcarriers, envelope, and symbol timing by the same percentage in an OFDM signal or any other modulated signals. Then the SNR degradation of an OFDM system due to Doppler frequency shift, frequency offset of the local oscillators and phase noise is analyzed. Expressions are given and values for 4-, 16-, 64-, and 256-QAM OFDM systems are calculated and plotted. The calculations show that the Doppler shift of the D3 project is about 305 kHz, and the degradation due to it is about 0.01 to 0.04 dB, which is negligible. The degradation due to frequency offset and phase noise of local oscillators will be the main source of degradation. To keep the SNR degradation under 0.1 dB, the relative frequency offset due to local oscillators must be below 0.01 for the 16QAM-OFDM. This translates to an offset of 1.55MHz (0.01×155 MHz) or a stability of 77.5ppm (0.01×155 MHz/20GHz) for the D3 project. To keep the SNR degradation under 0.1dB, the relative linewidth (β) due to phase noise of the local oscillators must be below 0.0004 for the 16QAM-OFDM. This translates to a linewidth of 0.062 MHz (0.0004 × 155MHz) of the 20 GHz RF carrier. For a degradation of 1dB, β=0.04, and the linewidth can be relaxed to 6.2 MHz.

I. Introduction

An orthogonal frequency division multiplexing (OFDM) signal can be written as

\[ v(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i) \]  

where \( A_i, \omega_i, \) and \( \phi_i \) are the amplitude, angular frequency, and phase of the \( i \)th subcarrier. \( N_s \) is the number of subcarriers. In order for the subcarriers to be orthogonal to each other, \( f_i = \omega_i / 2\pi \) must be integer multiples of \( 1/2T \) [1, p. 90, 101][2], where \( T \) is the symbol period of the data, and \( f_i \) are spaced in frequency by \( \Delta f = 1/T \) (if all \( \phi_i \) are the same, i.e.,
coherent, the spacing can be multiples of 1/2T, and the minimum spacing is 1/2T). Usually \( f_i \) are chosen as integer multiples of the symbol rate \( R_s = 1/T \).

If the signal is amplitude shift keyed (ASK), \( A_i \) is determined by the data, and \( \phi_i \) is a random phase uniformly distributed in \([-\pi, \pi]\). If the signal is phase shift keyed (PSK), \( A_i \) is constant, and \( \phi_i \) is determined by the data. Therefore the above is a general form of the OFDM signal in the baseband. It is called the baseband OFDM signal because this signal is usually modulated on a higher frequency carrier before transmission. However, this signal can also be transmitted directly, without further frequency conversion (e.g., the system depicted in [3]).

The signal \( v(t) \) is modulated on a RF carrier with a frequency \( f_c \) for transmission:

\[
s(t) = 2v(t)\cos \omega_c t
\]

\[
= 2 \sum_{i=0}^{N-1} A_i (\cos(\omega_i t + \phi_i)\cos \omega_c t
\]

\[
= \sum_{i=0}^{N-1} A_i \left\{ \cos[(\omega_c + \omega_i) t + \phi_i] + \cos[(\omega_c + \omega_i) t - \phi_i] \right\}
\]

(2)

where \( \omega_c = 2\pi f_c \). The phase of the carrier has been assumed to be zero without loss of generality. The amplitude of the carrier is set to 2 to simplify the analysis later. A single sideband transmission is enough to convey the information imbedded in \( A_i \) and \( \phi_i \). Assume upper sideband is used, the transmitted signal is

\[
s(t) = \sum_{i=0}^{N-1} A_i \cos[(\omega_c + \omega_i) t + \phi_i]
\]

(3)

If the lower sideband is used, the transmitted signal is

\[
s(t) = \sum_{i=0}^{N-1} A_i \cos[(\omega_c - \omega_i) t + \phi_i]
\]

(4)

II. Doppler Frequency Shift

Doppler shift \( f_D \) is proportional to the frequency \( f \) of the electromagnetic wave in propagation, and is given by

\[
f_D = \frac{v_r f}{c} \cos \alpha
\]

(5)

where \( v_r \) is the relative speed between the transmitter and the receiver, \( f \) is the carrier frequency, \( c \) is the speed of light \((3 \times 10^8 \text{m/s})\), and \( \alpha \in [0, \pi] \) is the angle of the velocity vector. The maximum \( f_D \) happens when \( \alpha = 0 \).

\[
\max(f_D) = \frac{v f}{c}
\]

(6)
If the speed is in unit of kilometers per hour, (5) can be changed to

\[ f_o = \frac{v_f}{3.6 \times 3 \times 10^8} \cos \alpha \]  

Assuming \( \alpha = 0^\circ \), the values of \( f_D \) for 20GHz carrier and 30GHz carrier and various speeds are listed in Table 1. Those are the maximum values of \( f_D \) among the values for different angles. From the table we can see that the Doppler shift is about 2 kHz to 20 kHz for \( f = 20 \) GHz and 3 kHz to 30 kHz for \( f = 30 \) GHz, respectively, in the speed range of 100 km/h to 1000 km/h. The relative Doppler shift \( \xi = f_D / f \) is, therefore, about \( 10^{-7} \) to \( 10^{-6} \), which is very small.

<table>
<thead>
<tr>
<th>V (km/h)</th>
<th>( f_D ) (Hz) ( f = 20 ) GHz</th>
<th>( f_D ) (Hz) ( f = 30 ) GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>( 1.852 \times 10^3 )</td>
<td>( 2.778 \times 10^3 )</td>
</tr>
<tr>
<td>200</td>
<td>( 3.704 \times 10^3 )</td>
<td>( 5.556 \times 10^3 )</td>
</tr>
<tr>
<td>300</td>
<td>( 5.556 \times 10^3 )</td>
<td>( 8.333 \times 10^3 )</td>
</tr>
<tr>
<td>400</td>
<td>( 7.407 \times 10^3 )</td>
<td>( 1.111 \times 10^4 )</td>
</tr>
<tr>
<td>500</td>
<td>( 9.259 \times 10^3 )</td>
<td>( 1.389 \times 10^4 )</td>
</tr>
<tr>
<td>600</td>
<td>( 1.111 \times 10^4 )</td>
<td>( 1.667 \times 10^4 )</td>
</tr>
<tr>
<td>700</td>
<td>( 1.296 \times 10^4 )</td>
<td>( 1.944 \times 10^4 )</td>
</tr>
<tr>
<td>800</td>
<td>( 1.481 \times 10^4 )</td>
<td>( 2.222 \times 10^4 )</td>
</tr>
<tr>
<td>900</td>
<td>( 1.667 \times 10^4 )</td>
<td>( 2.500 \times 10^4 )</td>
</tr>
<tr>
<td>1000</td>
<td>( 1.852 \times 10^4 )</td>
<td>( 2.778 \times 10^4 )</td>
</tr>
</tbody>
</table>

Table 1: Doppler shifts for various speeds

### III. Doppler Shift Affects the Frequencies of the RF Carrier, Subcarriers, Envelope, and Symbol Timing by the Same Percentage

Consider the OFDM signal as the sum of various frequency components; each component's Doppler shift is given by (5). This implies that the Doppler shift affects each component with the same percentage in the OFDM signal. This is to say

\[( f_c \pm f_i ) \xrightarrow{\text{Doppler Shift}} (1 + \xi)( f_c \pm f_i )\]

where \( \xi \) (positive or negative) is the percentage of change. Obviously

\[ \xi = \frac{f_D}{f} = \frac{v_f}{c} \cos \alpha \]  

which is determined by the relative velocity of the transmitter and the receiver.

Since

\[(1 + \xi)(f_c \pm f_i) = (1 + \xi)f_c \pm (1 + \xi)f_i\]

we can consider

\[ f_c \xrightarrow{\text{Doppler Shift}} (1 + \xi)f_c \]

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and
\[ f_i \xrightarrow{\text{Doppler Shift}} (1 + \xi_i) f_i \]
That is, the Doppler shift affects the carrier frequency \( f_c \) and the subcarrier frequencies \( f_i \) by the same percentage \( \xi \). The Doppler shifts are
\[ f_{D,c} = \xi f_c = \frac{vf_c}{c} \cos \alpha \]  
(9)
and
\[ f_{D,i} = \xi f_i = \frac{vf_i}{c} \cos \alpha \]  
(10)
Thus the OFDM signal with Doppler shift is
\[ s(t) = \sum_{i=0}^{N-1} A_i \cos[(1+\xi)(\omega_c + \omega_i)t + \phi_i] \]
\[ = \sum_{i=0}^{N-1} \left[ A_i \cos[(1+\xi)\omega_i t + \phi_i] \cos[(1+\xi)\omega_i t] \right. \]
\[ - A_i \sin[(1+\xi)\omega_i t + \phi_i] \sin[(1+\xi)\omega_i t] \]  
(11)
Consider \( A_i \cos[(1+\xi)\omega_i t + \phi_i] \) as the envelope of the carrier \( \cos[(1+\xi)\omega_i t] \) and similarly for the second term, this expression shows that the Doppler shift affects the carrier frequency and the envelope frequency by the same percentage. This conclusion can be extended beyond OFDM signal to any modulated signal.

The clock timing is derived from the symbol rate \( R_s = 1/T \) in the modulated signal. Since each subcarrier frequency is an integer multiple of \( R_s \), when the subcarrier frequency changes by a percentage \( \xi \) due to Doppler shift, \( R_s \) changes by the same percentage \( \xi \). This conclusion also applies to any modulated signal.

In conclusion, Doppler shift affects the frequencies of the RF carrier, subcarriers, envelope, and symbol timing by the same percentage \( \xi \).

The fact that all subcarrier frequencies change by the same percentage destroys the orthogonality between subcarriers. This is because the separations between subcarriers are no longer \( mR_s \), \( (m \text{ integer}) \). Instead, they become \( (1 + \xi)mR_s \). The OFDM system performance degradation due to frequency offset caused by Doppler shift and other sources will be discussed in Section V.

The Doppler shift rate \( r_D \), which is the rate of changes in Doppler shift due to the changes in relative velocity, can be derived as follows. Rewrite (5) as
\[ f_D(t) = \frac{vf(t)f}{c} \cos\alpha(t) \]
where the Doppler shift and the velocity \((v(t)\text{ and } \alpha(t))\) are functions of time \( t \). Taking the derivative we have...
\[ r_D(t) = f_D(t) = \frac{f}{c} \left[ v_r(t) \cos \alpha(t) - v_r(t) \alpha'(t) \sin \alpha(t) \right] \]

\[ = \frac{f}{c} \left[ a_r(t) \cos \alpha(t) - v_r(t) \alpha'(t) \sin \alpha(t) \right] \] (12)

where \( a_r \) is the relative acceleration rate.

From (5) we know that the maximum \( f_D \) happens when \( \alpha = 0 \). To determine at what angle maximum \( r_D \) happens, we fix the acceleration rate \( a_r(t) \) and rewrite (12) as

\[ r_D(t) = \frac{f}{c} \left[ a_r(t) \cos \alpha(t) - v_r(t) \alpha'(t) \sin \alpha(t) \right] \]

The maximum of this can be found by letting its derivative be zero, and is at \( \alpha(t) = \alpha = 0 \).

\[ \max(r_D) = \frac{a_r f}{c} \] (13)

IV. Doppler Shifts of the OFDM Signal for the D^3 Project

In the proposed D^3 project, the OFDM modem developed in this branch will be put on the space shuttle and at a ground station. There is only a down link with a carrier frequency of 19.035 GHz. The maximum relative speed is 4811.00 m/s and the maximum relative acceleration is 168.00 m/s². In Table 2, the maximum Doppler shifts are calculated using (6) and the Doppler shift rates are calculated using (13).

<table>
<thead>
<tr>
<th>Sym.</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_r )</td>
<td>maximum relative velocity (seen by ground terminal)</td>
<td>4811.00</td>
<td>m/s</td>
<td>Dean Schrage's analysis</td>
</tr>
<tr>
<td>( a_r )</td>
<td>maximum relative acceleration (seen by ground terminal)</td>
<td>168.00</td>
<td>m/s²</td>
<td>Dean Schrage's analysis</td>
</tr>
<tr>
<td>( c )</td>
<td>speed of light</td>
<td>3.00E+08</td>
<td>m/s</td>
<td>standard</td>
</tr>
<tr>
<td>( f_c )</td>
<td>source frequency (carrier)</td>
<td>19.035</td>
<td>GHz</td>
<td>project spec.</td>
</tr>
<tr>
<td>( f_{Dc} )</td>
<td>maximum carrier Doppler shift</td>
<td>305.26</td>
<td>kHz</td>
<td>calculated</td>
</tr>
<tr>
<td>( r_{Dc} )</td>
<td>maximum carrier Doppler shift rate</td>
<td>10.66</td>
<td>kHz</td>
<td>calculated</td>
</tr>
<tr>
<td>( f_d )</td>
<td>source frequency (data clock)</td>
<td>155.52</td>
<td>MHz</td>
<td>project spec.</td>
</tr>
<tr>
<td>( f_{Dd} )</td>
<td>maximum data clock Doppler shift</td>
<td>2.49</td>
<td>kHz</td>
<td>calculated</td>
</tr>
<tr>
<td>( r_{Dd} )</td>
<td>maximum data clock Doppler shift rate</td>
<td>0.09</td>
<td>kHz</td>
<td>calculated</td>
</tr>
<tr>
<td>( \pm ) frequency variation required</td>
<td>16.04</td>
<td>ppm</td>
<td>calculated</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Worst case Doppler shifts for D^3 Project

From the table we can see that the maximum Doppler shift is about 300 kHz in the carrier frequency and 2.5 kHz in the clock frequency. In both cases the relative shift is only

\[ \frac{f_{Dc}}{f_c} = \frac{f_{Dd}}{f_d} = \frac{v_r}{c} = 16.04 \times 10^{-6} \]
The symbol rate of the 622 Mbps 16QAM-OFDM modem is about 60 Mbps per channel when coding is considered. Thus the channel separation is also 60 MHz, the relative Doppler shift (relative to the channel separation) in carrier is

\[ \varepsilon = \frac{305.26 \times 10^3}{60 \times 10^6} = 5 \times 10^{-3} \] (14)

This is the number that determines the performance degradation in the coherent OFDM demodulator.

V. Performance Degradation of Coherent OFDM Modem Due to Doppler Frequency Shift, Frequency Offset of Local Oscillators, and Phase Noise

Performance degradation of coherent OFDM modem due to channel frequency offset, including Doppler frequency shift and frequency offset of local oscillators in the receiver, is analyzed in [4] and [5]. In both papers, the OFDM signals are assumed to be generated by the inverse discrete Fourier transform (IDFT) and demodulated by the discrete Fourier transform (DFT). Also, channel phase noise is included in the channel models in both papers. But there are some different assumptions in the two papers. We will point out them in the following description of the major results of these papers.

In [4], the channel is modeled by a complex transfer function \( H_k \) for the \( k \)th subcarrier and a frequency offset \( \varepsilon \) for all subcarriers, where \( \varepsilon \) is relative to the data symbol rate \( R_s = 1/T \) (AWGN is also present). And \( \varepsilon \) includes the frequency offsets due to Doppler shift and the local oscillator (LO) frequency error. Recall that we have demonstrated that each subcarrier suffers from the same percentage but different values of Doppler shift. This model is therefore not accurate. However, when the Doppler shift in the carrier \( (f_{DC}) \) is not tracked and the offset in the LO \( (\Delta f_{LO}) \) is not compensated for, this model is approximately correct. In this case the total offset for each subcarrier after down conversion is

\[ \Delta f_i = f_{DC} + f_{Di} + \Delta f_{LO} \]

where \( f_{DC} \) and \( f_{Di} \) are given by (9) and (10), respectively. It is clear that \( f_{DC} >> f_{Di} \), since \( f_c >> f_i \). Thus

\[ \Delta f_i = f_{DC} + \Delta f_{LO} = \Delta f \]

That is, the offset is approximately independent of subcarriers. The relative offset

\[ \varepsilon = \frac{\Delta f}{R_s} \]

is thus approximately independent of subcarriers.

Based on this model, the signal-to-noise ratio (SNR) at the output of the DFT at the OFDM receiver is found as

\[ SNR \geq \frac{E_c}{N_0} \left( \frac{\sin \pi \varepsilon}{\pi \varepsilon} \right)^2 \frac{1}{1 + 0.5947 \left( \frac{E_c}{N_0} \right) (\sin \pi \varepsilon)^2}, \quad |\varepsilon| < 0.5 \] (15)
At BER = 10^{-6} the SNR is 10.5, 14.4, 18.8, and 23.5 dB for QPSK, 16-QAM, 64-QAM, and 256-QAM respectively. The degradations for these SNRs are shown in Figures 1 and 2. Figure 1 shows how SNRs degrade as relative offset increases. Figure 2 shows the direct degradations versus the relative offset.

![SNR versus frequency offset ε](image)

**Figure 1: SNR versus frequency offset ε**

Return to the D³ project. If the offset includes only the Doppler shift, the maximum relative offset $\varepsilon = 5 \times 10^{-3}$. 16-QAM is the modulation used in the 622-Mbps D³ OFDM system. It is seen from Figure 2 (d) that for $\varepsilon = 5 \times 10^{-3}$ the degradation is below 0.04 dB. Therefore, the SNR degradation, hence the BER performance degradation, caused by the Doppler shift is negligible in D³ OFDM system. This happens because the symbol rate (60 Mbps) is very high. If the symbol rate is lower, say, 10 Mbps, then the relative offset is higher: $\varepsilon = 0.03$. Consequently, the SNR degradation of 16-QAM would be around 1 dB for SNR$_{\text{e}}$ = 14.4 dB. Further lowering the symbol rate will cause the degradation even bigger.

Similar results are reported in [5], where the channel is modeled by a time-varying phase $\theta(t)$ caused by either a carrier offset between the receiver and transmitter carrier, or the phase noise of these carriers. In the first case, $\theta(t)$ is deterministic and equals $2\pi \Delta F t + \theta_0$, where $\Delta F$ is the carrier offset. In the latter case, $\theta(t)$ is modeled as a Wiener process for which $E[\theta(t)] = 0$ and $E[\theta(t+\tau) - \theta(t)]^2 = 4\pi \beta |\tau|$, where $\theta$ denotes the one-sided 3 dB linewidth of the Lorentzian power density spectrum of the free-running carrier generator.
Figure 2: SNR degradation versus relative frequency offset $\epsilon$

Based on this model, the degradation $D$ (dB) of SNR at the input of the decision device is found as follows. For a fixed total symbol rate $R$, with $R = N/T = NR_s$, where $N$ is the number of subcarriers, for OFDM, and $R = 1/T = R_s$ for single channel (SC), the degradation is

- Frequency offset case:

$$D(dB) = \begin{cases} 
\frac{10}{\ln 10} \left( \frac{\pi N \Delta F}{R} \right)^2 \frac{E_s}{N_0} & \text{OFDM} \\
\frac{10}{\ln 10} \left( \frac{\pi \Delta F}{R_s} \right)^2 & \text{SC}
\end{cases}$$ (16)
For OFDM, the degradation is proportional with $E_s/N_0$, but it is not the case for SC. For both OFDM and SC, the degradation is proportional with the square of the frequency offset. For OFDM, the degradation is also proportional with the $E_s/N_0$, and with the square of the number of subcarriers.

- Phase noise case

$$D(dB) \equiv \begin{cases} \frac{10}{\ln 10} \frac{11}{60} \left( \frac{4\pi N \beta}{R} \right) \frac{E_s}{N_0} & \text{OFDM} \\ \frac{10}{\ln 10} \frac{1}{60} \left( \frac{4\pi \beta}{R} \right) \frac{E_s}{N_0} & \text{SC} \end{cases}$$

(17)

For both OFDM and SC, the degradation is proportional with $E_s/N_0$, and the linewidth $\beta$. For OFDM, the degradation is also proportional with the number of subcarriers.

From (16) and (17) it is seen that the degradation (in dB) of OFDM due to frequency offset or phase noise is $N^2E_s/N_0$ or $N$ times greater than that of SC, respectively.

Using the subcarrier spacing ($R_s$), the OFDM part of (16), (17) can be written as

$$D(dB) \equiv \frac{10}{\ln 10} \frac{11}{60} \left( \frac{4\pi}{R_s} \right)^2 \frac{E_s}{N_0}$$

OFDM

(18)

and

$$D(dB) \equiv \frac{10}{\ln 10} \frac{1}{60} \left( \frac{4\pi \beta}{R_s} \right) \frac{E_s}{N_0}$$

OFDM

(19)

respectively. In the expressions, $\Delta F/R_s$ and $\beta/R_s$ are relative frequency offset and linewidth versus the subcarrier spacing, respectively.

Some numerical results for (18) and (19) are plotted in Figure 3. For 16-QAM, which is the modulation used in the 622-Mbps D3 OFDM system, from the figure it can be seen that the degradation due to Doppler shift $\varepsilon = \Delta F/R_s = 5 \times 10^{-3}$ (log $\varepsilon = -2.3$) is about 0.01 dB (This is similar to the results in Figure 2(d)). However, if the relative phase noise linewidth is also $5 \times 10^{-3}$, then the degradation is much larger: about 1.0 dB. This means the degradation is more sensitive to the phase noise.
Figure 3: SNR degradation in dB versus the normalized frequency offset (a) and the normalized phase noise linewidth (b). The original SNRs are the values at which BER = 10^-6: 23.5 dB for 256-QAM, 18.8 dB for 64-QAM, 14.4 dB for 16-QAM, and 10.5 dB for QPSK.

VI. Conclusions and Discussions

The results of Pollet show that for a fixed total symbol rate, the SNR degradation of OFDM due to frequency offset is N^2 times more sensitive than SC and the SNR degradation due to phase noise is N times more sensitive than SC. The SNR expression of Moose (15) does not include the channel number N explicitly. However $\epsilon$ can be expressed as $\epsilon = \Delta F / R_s = N \Delta F / R$, where $R = NR_s$, then (15) will include $N^2$ in it and the SNR degradation increases with $N^2$ approximately.

The maximum Doppler shift of $D^3$ project is 304 kHz, or $5 \times 10^{-3}$ relative to 60Mbps symbol rate per channel or channel separation.

The SNR degradation for 16QAM-OFDM at SNR=14.4 dB (BER=10^-6) is about 0.01 dB (Pollet) or 0.04 dB (Moose) for the maximum Doppler shift. Since the SNR degradation due to the Doppler shift is negligible, the degradation due to frequency offset and phase noise of local oscillators will be the main source of degradation. To keep the SNR degradation under 0.1 dB, the relative frequency offset due to local oscillators must be below 0.01 for 16QAM-OFDM. This translates to an offset of 0.6 MHz (0.01x60MHz) or a stability of 30 ppm (0.01 x 60 MHz/20 GHz) for the $D^3$ project. To keep the SNR degradation under 0.1 dB, the relative linewidth ($\beta$) due to phase noise of the local oscillators must be below 0.0004 for 16QAM-OFDM. This translates to a line width of 0.024 MHz (0.0004 x 60 MHz) of the 20 GHz RF carrier. For a degradation of 1 dB, $\beta=0.04$, and the linewidth can be relaxed to 2.4 MHz.
References


# The Effect of Doppler Frequency Shift, Frequency Offset of the Local Oscillators, and Phase Noise on the Performance of Coherent OFDM Receivers

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## Abstract

This paper first shows that the Doppler frequency shift affects the frequencies of the RF carrier, subcarriers, envelope, and symbol timing by the same percentage in an OFDM signal or any other modulated signals. Then the SNR degradation of an OFDM system due to Doppler frequency shift, frequency offset of the local oscillators and phase noise is analyzed. Expressions are given and values for 4-, 16-, 64-, and 256-QAM OFDM systems are calculated and plotted. The calculations show that the Doppler shift of the D3 project is about 305 kHz, and the degradation due to it is about 0.01 to 0.04 dB, which is negligible. The degradation due to frequency offset and phase noise of local oscillators will be the main source of degradation. To keep the SNR degradation under 0.1 dB, the relative frequency offset due to local oscillators must be below 0.01 for the 16 QAM-OFDM. This translates to an offset of 1.55 MHz (0.01×155 MHz) or a stability of 77.5 ppm (0.01×155 MHz/20 GHz) for the D3 project. To keep the SNR degradation under 0.1 dB, the relative linewidth (\(\beta\)) due to phase noise of the local oscillators must be below 0.0004 for the 16 QAM-OFDM. This translates to a linewidth of 0.062 MHz (0.0004×155 MHz) of the 20 GHz RF carrier. For a degradation of 1 dB, \(\beta=0.04\), and the linewidth can be relaxed to 6.2 MHz.

## Subject Terms

Doppler

## Distribution/Availability Statement

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