Effects of Space Environment on Flow and Concentration During Directional Solidification

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ABSTRACT

A study of directional solidification of a weak binary alloy (specifically, Bi – 1 at% Sn) based on the fixed grid single domain approach is being undertaken. The enthalpy method is used to solve for the temperature field over the computational domain including both the solid and liquid phases; latent heat evolution is treated with the aid of an effective specific heat coefficient. A source term accounting for the release of solute into the liquid during solidification has been incorporated into the solute transport equation. The vorticity-stream function formulation is used to describe thermo-solutal convection in the liquid region.

In this paper we numerically investigate the effects of g-jitter on directional solidification. A background gravity of 1 μg has been assumed, and new results for the effects of periodic disturbances over a range of amplitudes and frequencies on solute field and segregation have been presented.

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1. INTRODUCTION

The investigation of solidification and melting processes in low gravity conditions is of great practical importance for crystal growth techniques. The quality of single crystals grown from the melt strongly depends on growth morphology and macro-segregation caused by convective heat and mass transfer effects. A low gravity environment produces conditions in which convection is decreased to a level at which crystal growth is largely diffusion controlled. Residual accelerations in orbiting space vehicles are of the order of one to several thousand μg (where 1 μg = 9.81×10^{-6} ms^{-2}). To achieve diffusion dominated conditions, much effort has been expended in recent years in performing crystal growth experiments in the microgravity environment of a spacecraft in earth orbit. Such effects as constitutional and kinetic supercooling, and the influence of convection on solute distribution in the melt, have been investigated under microgravity.

However, gravity in an orbiting space vehicle may not be steady in either magnitude or direction. Perturbations to the anticipated steady microgravity environment may arise from, for example, crew actions, the operation of machinery and thruster rocket firings. Such perturbations are known as g-jitter. 1,2,13-16

The effects of gravity perturbations on composition distribution in the Bridgman crystal growth configuration have been investigated numerically in a number of papers. Alexander, Ouazzani and Rosenberger 1 investigated the effects of steady and impulse residual acceleration on dopant distribution in Bridgman-Stockbarger crystal growth with different gravity vector orientations. It was found that lateral non-uniformity in composition is very sensitive to the orientation of the steady component of the residual gravity vector.

Transient and periodic accelerations have been considered in Alexander, Amiroudine, Ouazzani, and Rosenberger 2. It was found that the largest compositional nonuniformities occur for disturbances with amplitudes above 10^{-6}g and frequencies below 10^{-2} Hz. At higher frequencies, larger acceleration amplitudes are required to obtain significant nonuniformities. Numerical results for the effect of g-jitter on the average interface concentration during Bridgman crystal growth in space are presented in Garandet, Corre, Favier and Alexander 3. In all of these works a pseudo-steady-state model was adopted with the constraint that the interface is planar. This is a simplification of the true unsteady solidification process.

In actual growth situations the solid-liquid interface can be non-planar due to thermal and mass transfer conditions and also due to morphological instability factors. Pseudo-steady-state models neglect transient effects such as changes in the interface velocity, temperature and concentration with time due to changes in the length of melt caused by the finite length of the ampoule. Investigation of solute redistribution during the initial transients becomes crucial for an alloy with a low partition coefficient solidifying at low rates because the length of the initial transient is very long, thus steady state is difficult to reach.

In this work we investigate effects of periodic gravity perturbations on segregation and solute distribution during transient directional solidification of Bi-1 at % Sn alloy in a Bridgman furnace. A background gravity of 1 μg, which corresponds to a typical spacecraft environment, is considered. The influence of solute on liquid density is included. The interface is assumed to be at the melting temperature of pure Bi. The diffusion coefficient for Sn in Bi is assumed to be constant. The general boundary conditions used are similar to NASA's MEPHISTO-2 and -4 Shuttle Flight experiments.
2. MATHEMATICAL FORMULATION

We consider a Bridgman furnace in which a moving temperature profile consisting of a cold zone \((T_c)\), a nominally adiabatic zone and a hot zone \((T_h)\) is imposed on the boundary of the ampoule. This boundary temperature profile is translated with a constant pulling velocity, as a result of the furnace movement, causing the solid/liquid interface to move along the ampoule. The material in the ampoule is thus divided into two regions: solid and liquid.

Although the ampoule is three-dimensional, a two-dimensional model is used. This simplification is valid because, under the microgravity conditions being considered, convection is very weak and the solidification process remains largely diffusion-controlled and the flow that does arise is predominantly 2-D in nature. Newtonian and laminar flow is assumed in the liquid phase, and the Boussinesq approximation has been used, in which the liquid density is assumed to be constant except in the buoyancy term of the equation of motion.

The governing time dependent equations describing mass, momentum, heat and solute transport in a vorticity-stream function formulation are:

\[
\rho \left( \frac{\partial \zeta}{\partial t} + \nabla (\bar{v} \cdot \zeta) \right) = \nabla \rho \times \bar{g} \| \bar{g} \| + \mu \nabla^2 \zeta \tag{1}
\]

\[
\nabla \cdot \bar{v} = -\zeta \tag{2}
\]

\[
\rho c_p \left( \frac{\partial T}{\partial t} + \nabla (\bar{v} \cdot T) \right) = \lambda \nabla^2 T \tag{3}
\]

\[
\frac{\partial C}{\partial t} + \nabla (\bar{v} \cdot C) = D \nabla^2 C \tag{4}
\]

where \(t, \rho, \mu, c_p, \lambda\) and \(D\) are respectively the time, density, viscosity, specific heat and thermal conductivity of the alloy and the diffusivity of the solute; \(\zeta, \psi, T, \bar{v}\) and \(C\) are respectively the vorticity, stream function, temperature, velocity vector and solute concentration; \(\bar{g}\) is the magnitude of the gravitational acceleration, and \(\hat{g}\) is the unit vector in the direction of gravity. The density in the buoyancy term of equation (1) is assumed to be a linear function of temperature and solute concentration:

\[
\rho = \rho_k \left[ 1 - \beta_r (T - T_k) + \beta_c (C - C_k) \right] \tag{5}
\]

where \(\beta_r\) and \(\beta_c\) are the thermal and solutal expansion coefficients,

\[
\beta_r = -\frac{1}{\rho_k} \frac{\partial \rho}{\partial T} \tag{6}
\]

and

\[
\beta_c = \frac{1}{\rho_k} \frac{\partial \rho}{\partial C} \tag{7}
\]

\(\rho, T, \rho_R, T_R\) and \(C_R\) the are the reference density, temperature and concentration.
The gravitational acceleration is taken as:

\[ g(t) = g_0 + A \sin(2\pi \omega t) \]  

(8)

where \( A \) is the amplitude of the acceleration, \( \omega \) is the frequency and \( g_0 \) is the steady component of the acceleration.

**Enthalpy method**

To model the process of directional solidification we have chosen the enthalpy method\(^4\) which avoids explicit tracking of the solid/liquid interface.

Latent heat evolution during phase change is incorporated in the energy equation using the following definition of enthalpy. For each phase \( \phi \), enthalpy is defined as

\[ h = \int_0^t c_{p\phi} \, dT + f_\ell L, \]  

(9)

where \( L \) is latent heat and \( f_\ell \) is the local liquid volume fraction.

For isothermal phase change the liquid fraction is determined by the melting temperature \( T_m \):

\[
\begin{align*}
  &\text{for } T > T_m \quad f_\ell = 1 \\
  &\text{for } T < T_m \quad f_\ell = 0.
\end{align*}
\]

(10)

With the assumption that specific heat \( c_{p\phi} \) is constant in each phase, (9) can be written as

\[ h = c_{p\phi} T + f_\ell L = h_{\text{sens}} + f_\ell L. \]  

(11)

Here \( h_{\text{sens}} \) is the sensible heat, and the subscripts \( l \) and \( s \) refer to the liquid and solid phases.

Using the apparent heat capacity method\(^5\), an effective specific heat can be defined by

\[ C^*(T) = \frac{\partial h}{\partial T} = c_{p\phi} + L \frac{\partial f_\ell}{\partial T}. \]  

(12)

Using (12), the energy equation (3) can be written:

\[ \rho [C^*(T) \frac{\partial T}{\partial \ell} + c_{p\phi} \nabla (\ell T)] = \lambda \nabla^2 T. \]  

(13)

To solve equation (13), an effective heat capacity coefficient \( \frac{\partial f_\ell}{\partial T} \) has to be calculated. We define

\[ \frac{\partial f_\ell}{\partial T} = \frac{\partial f_\ell / \partial \ell}{\partial \ell / \partial T} = \frac{(f_\ell)_l T_n}{T_n} = \frac{(f_\ell)_l T_n + (f_\ell)_s T_s}{T_n + T_s} \]  

(14)

where the subscripts \( n \) (denoting the normal direction), \( x \) and \( y \) denote differentiation.
Since isothermal phase change is under consideration, the liquid fraction undergoes a step change when the interface crosses a grid line. This abrupt change in the liquid fraction, defined by the step function (10), can cause serious numerical instabilities. To overcome this problem, a control volume was defined around each grid point, in which the liquid fraction could be estimated. Phase change was considered to take place over one control volume, in which the step function (10) is replaced by a linear approximation:

\[
\begin{align*}
\text{for } T_u > T_* + \Delta T & \quad f_i = 1 \\
\text{for } T_* - \Delta T \leq T_u \leq T_* + \Delta T & \quad f_i = \frac{T_u - T_* + \Delta T}{2\Delta T} \\
\text{for } T_u < T_* - \Delta T & \quad f_i = 0
\end{align*}
\]

(15)

where $2\Delta T$ is a temperature interval chosen to represent the range over which phase change occurs in the $(i, j)$ control volume.

Based on the calculated values of liquid fraction at each mesh point the computational domain was subdivided into sub-regions of solid and liquid phases.

In the solid, the vorticity, stream function and velocities were set to zero. In the liquid, they are calculated from the stream function defined as:

\[\vec{V} = \nabla \times \vec{\psi}\]

(16)

**Solute transport with phase change**

The release of solute into the liquid during solidification can be described by considering an average concentration in an arbitrary control volume which is undergoing phase change\(^4\). This control volume can be treated as partially solidified with an average concentration defined as:

\[C = f_s C_s + f_l C_l\]

(17)

where $f_s = 1 - f_i$ is the local solid volume fraction. Since diffusion in the solid is neglected, the concentration in the solid remains constant over time. Noting that $C_s = kC_l$ we can thus write:

\[\frac{\partial C_l}{\partial t} = -\frac{\partial}{\partial x} (1-k)C_l + (1-f_s) \frac{\partial C_l}{\partial x}\]

(18)

When (18) is used in the solute transport equation (4), we obtain the solute conservation equation in the form:

\[\frac{\partial C_l}{\partial t} + \nabla \times (\vec{r} \times C_l) = D \nabla^2 C_l + S_s\]

(19)

in which

\[S_s = \frac{\partial}{\partial t} (1-k)C_l + f_s \frac{\partial C_l}{\partial t}\]

(20)
The formulation for solute transport during phase change described by equations (19) and (20) allows for the solution for liquid concentration only and hence bypasses the concentration discontinuity at the interface.

3. NUMERICAL METHOD

An algorithm entitled SOLCON 6, which incorporates the closely coupled solution of the transport equations in the vorticity-stream function formulation, was used. To ensure stability of the computational process, all source terms and non-linear coefficients depending on liquid fraction are linearized based on the value of liquid fraction obtained from the previous iteration.

The vorticity, stream function and energy equations were discretized using central differences and solved by a modified ADI scheme with internal iterations. Interface boundary conditions for vorticity and stream function were applied at those mesh points in the solid sub-region which are adjacent to the liquid. For the calculation of vorticity boundary conditions, the definition of vorticity was used: 
\[ \zeta = \nabla \times \mathbf{v} \]. The boundary condition \( \psi = 0 \) was used for the stream function. The concentration equation (19) was discretized and solved using a control volume approach. This ensures mass balance during phase change in the partially solidified control volume. A second order upwind scheme (SOU) was used for the convection fluxes with central differences for the diffusion terms.

To account for the fact that the computed concentration is a cell average value, an exponential extrapolation procedure based on the liquid fraction has been introduced to find the values of the concentration at the solid/liquid interface. The liquid side interface solute concentration \( C_I \) can then be used to determine the concentration in the solid as it forms.
4. CODE VALIDATION

To validate the code, computations were performed for directional solidification of gallium-doped germanium crystal grown by the Bridgman-Stockbarger technique as described by Alexander et al.\textsuperscript{2}. Comparisons were made for the case of sinusoidal acceleration with an amplitude of $10^{-3}g$ and a frequency of $10^{-1}$ Hz oriented parallel to the solid-liquid interface.

In Alexander et al.\textsuperscript{2} a pseudo-steady model was adopted with the assumption that the ampoule translation rate and the growth rate are equal. In their model the solid-liquid interface is located at a fixed distance from the top of computational domain, which is completely occupied by the melt. The aspect ratio of the computational domain was equal to 1. On the other-hand the model used in the current study considers transient effects and hence the gradual decrease in the length of melting zone. Both solid and liquid phases are included in the computational domain (see Figure 1).

The temperature profile is translated with a constant pulling velocity along the boundary causing the interface movement inside the domain. The boundary temperature profile, size of the computational domain and physical properties of the alloy were chosen to approximate the idealized model of the Bridgman-Stockbarger system in Alexander et al.\textsuperscript{2}. $T_c$ was equal to 1131K and $T_h$ was 1331K. The length of the computational domain was taken to be 21 mm, the height was 10 mm and the adiabatic zone was 5 mm. A uniform square 51 x 106 mesh was used.

A steady solution for the temperature and flow was used as the initial condition for the transient growth. This steady solution was obtained by keeping the boundary temperature profile stationary with the solid-liquid interface located at 7 mm from the left wall of the ampoule. The initial solute concentration in the liquid was uniform at 1 at%. Solidification was first started with a constant gravity level of $1_g$ and when 3 mm of material was solidified, a sinusoidal acceleration with an amplitude of $10^{-3}g$ and frequency of $10^{-1}$ Hz (oriented parallel to the solid-liquid interface) was imposed for another 2 mm of solidification. The translation velocity was 6.5 $\mu$m/s.

Figure 2 shows the maximum vertical velocity as a function of time computed using the present model. The velocity field is in phase with the residual acceleration. This result is in excellent agreement with Figure 7b from Alexander et al.\textsuperscript{2}.

Table 1. Comparison of the two models.

<table>
<thead>
<tr>
<th></th>
<th>$U_{max}$ (mm/s)</th>
<th>$V_{max}$ (mm/s)</th>
<th>$\gamma_s$ - $\gamma_0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alexander et al.\textsuperscript{2}</td>
<td>0.1140</td>
<td>0.1325</td>
<td>19.14</td>
</tr>
<tr>
<td>Present work</td>
<td>0.1175</td>
<td>0.1338</td>
<td>19.80</td>
</tr>
<tr>
<td>% difference</td>
<td>3.07</td>
<td>0.98</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Table 1 shows a quantitative comparison of the results obtained from the two models. $U_{max}$ and $V_{max}$ are the maximum velocity components along and across the ampoule respectively (during one period), and $\gamma_s$ is the radial segregation at the interface in the liquid defined by

$$\gamma_s = \frac{(C_{max} - C_{min})}{C_{max}} \times 100\%$$

(21)

where the three values of the concentration are taken in the liquid at the interface and $\gamma_0$ is segregation.
Figure 1. Model used in the present work for comparison with Alexander et al.²

Figure 2. Maximum vertical velocity for a sinusoidal acceleration with an amplitude of $10^{-3}\,g$ and frequency of $10^{-1}\,Hz$. 
before the g-jitter starts. In the case of Alexander et al.,\textsuperscript{2} $\gamma_0$ was equal to zero, in the present calculations the initial segregation was 4.0%.

The maximum change in segregation during g-jitter was equal to 19.14\% (Alexander et al.,\textsuperscript{2} and 19.8\% (our calculations). This was reached after 230 seconds of solidification.

It is obvious that computed results are in very good agreement despite the difference in the physical and mathematical models.

**Solidification of Bi-Sn Alloy**

Simulations were performed for directional solidification of Bi-1 at\% Sn alloy in a Bridgman furnace. Property values are taken from Timchenko et al.\textsuperscript{6} and shown in the Table 2 below.

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>$\rho$</td>
<td>10,070 kg/m$^3$</td>
</tr>
<tr>
<td>Specific heat</td>
<td>$C_p$</td>
<td>144.87 J/kg K</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\lambda$</td>
<td>12.4 W/m K</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>$\alpha$</td>
<td>8.500x10$^{-6}$ m$^2$/s</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>$\nu$</td>
<td>1.837x10$^{-7}$ m$^2$/s</td>
</tr>
<tr>
<td>Diffusion coefficient for Sn in Bi</td>
<td>$D$</td>
<td>2.7x10$^{-9}$ m$^2$/s</td>
</tr>
<tr>
<td>Thermal expansion coefficient</td>
<td>$\beta_T$</td>
<td>1.25x10$^{-4}$ K$^{-1}$</td>
</tr>
<tr>
<td>Solutal expansion coefficient</td>
<td>$\beta_C$</td>
<td>-0.305 (kg Sn/kg alloy)$^{-1}$</td>
</tr>
<tr>
<td>Temperature gradient in adiabatic zone</td>
<td>$dT/dx$</td>
<td>200 K/cm</td>
</tr>
<tr>
<td>Partition coefficient</td>
<td>$k$</td>
<td>0.029 at%/at%</td>
</tr>
</tbody>
</table>

The domain studied has a height of 6 mm and a length of 42 mm. The boundary temperature profile imposed on the outer surface of the liquid boundary consisted of a cold zone ($T_c = 50$ °C), linear temperature profile with a gradient 20 K/mm (for a length of 32.5 mm) and a hot zone ($T_h = 700$ °C). That is, conduction in the ampoule wall was not considered. The computational domain initially contains only liquid with a uniform solute concentration $C_0$ of 1 at\% and uniform temperature of 700 °C. At the left boundary an initial temperature of 272 °C was imposed. The pulling velocity, the rate of translation of the boundary temperature distribution, was 3.34 $\mu$m/s, and solidification occurred from left to right as time progressed. These boundary conditions are very similar to those present during the MEPHISTO-2 and -4 Space Flight experiments.\textsuperscript{11,12}

**Mesh validation**

To ensure the accuracy of the solution a mesh validation was performed for 500 seconds of solidification. The gravity vector was 1 $\mu$g, acting in a direction normal to the axis of the ampoule.

Three different mesh sizes were used, with the number of mesh points equal to 31 x 211, 31 x 421 and 61 x 421. The time step used with the 31 x 211 mesh was 0.1 s, and for the 31 x 421 and 61 x 421 meshes it was 0.01 s. The difference between those results is listed in Table 3. The length of the solute boundary layer is about 2 mm (estimated from the diffusion based analytical solution as $2D/R$ and previous numerical solutions\textsuperscript{6}) thus the coarsest mesh provides about 10 mesh points within the solute
boundary layer.

From Table 3, it can be concluded that a 31 x 211 mesh can be used in the calculations.

<table>
<thead>
<tr>
<th></th>
<th>31 x 211</th>
<th>31 x 421</th>
<th>61 x 421</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_{\text{max}} ) mm/s</td>
<td>2.286 x 10^{-4} (0.7%)</td>
<td>2.286 x 10^{-4} (0.7%)</td>
<td>2.302 x 10^{-4} (-)</td>
</tr>
<tr>
<td>( V_{\text{max}} ) mm/s</td>
<td>1.395 x 10^{-4} (0.35%)</td>
<td>1.400 x 10^{-4} (0.0%)</td>
<td>1.400 x 10^{-4} (-)</td>
</tr>
<tr>
<td>( C_{\text{max}} ) at%</td>
<td>3.2997 (0.4%)</td>
<td>3.2871 (0.0%)</td>
<td>3.2868 (-)</td>
</tr>
</tbody>
</table>
5. RESULTS AND DISCUSSION

The effects of periodic disturbances on the compositional profile and segregation at the interface have been investigated. The amplitudes were varied from $10^{-5}g$ to $10^{-2}g$ for a range of frequencies from $10^{-2}$ to 1 Hz.

At first, computations were performed for solidification with a constant gravity level of 1 $\mu$g. After 1500 seconds, sinusoidal accelerations oriented parallel to the solid-liquid interface were imposed and computations with g-jitter were performed for a further 500 seconds.

For comparison purposes, computations with a steady gravity level of 1 $\mu$g were performed. The results of these computations are summarised in Table 4. It can be seen that for a given acceleration magnitude, larger segregation at the interface occurs for smaller frequencies or longer periods of disturbances. The same effect was reported in Alexander et al. Disturbances with amplitudes of $10^{-5}g$ produce very little effect on the segregation or the compositional profile. Disturbances with amplitudes of $10^{-4}g$ cause increases in the segregation from 2.7 to 4.9% (1.8% for steady 1 $\mu$g gravity) when the frequencies of the disturbances become lower than 0.1 Hz. Larger effects on segregation were observed for disturbances with an amplitude of $10^{-3}g$.

Figure 3 shows the vertical velocity at a reference point as a function of time, for frequencies of the disturbances of (a) 0.01 Hz, (b) 0.1 Hz and (c) 1 Hz with an amplitude of $10^{-3}g$. The reference point is moving with the interface and is always located 2 mm in front of the interface at the mid-height of the domain. The development of the velocity field occurs in phase with the gravitational acceleration. The maximum velocity exhibits a transient before reaching steady oscillations. This transient includes number of periods which increases with increasing frequency as can be seen clearly in Figure 3(c). As the frequency is reduced the maximum velocity increases.

**Velocity Field**

Consider Movie 1, Movie 2 and Movie 3, they show that the configuration of the flow field depends on frequency of the disturbance. The velocity field in these three movies is quite different, especially during the reversal period. For the small frequency of disturbance 0.01 Hz, the reversal period is very small (2-3 seconds), as seen in Movie 1 when the contour colour of stream function changes from red to blue or blue to red. During the reversal period of the 0.1 Hz case, a weak recirculation occurs in front of the interface, as seen in Movie 2 which is not present in the 0.01 Hz case. For 1.0 Hz of disturbance, the flow configuration is much different from the previous two cases. At high frequency of disturbance, the flow does not reverse. As seen in Movie 3, the flow is periodically changing from strong clockwise flow to weak clockwise flow. It is also observed that there is a weak recirculation cell in front of the interface.

The velocity field needs time to respond to the acceleration, therefore, a major factor on the velocity field is frequency or the period of the disturbance. For the small frequencies of the disturbance, the velocity field has enough time to develop corresponding to the changing acceleration field. As a result, the maximum velocity in this case is close to that with the steady gravitational field at the peak of the disturbance. In contrast, for high frequencies of the disturbance, the flow field does not have enough time (in one cycle) to develop corresponding to the oscillating part of the acceleration. Therefore, the maximum velocity is close to that with the steady gravitational field at the back ground level, as seen in Figure 4.

**Concentration Field**

It has been shown from Table 4 that the low frequencies and large amplitudes of the disturbance do have an effect on the segregation of the solute. Therefore, the following movies will demonstrate how the solutal field is responding to the disturbance.

Movie 4 shows the effects of the disturbance of 0.01Hz and $10^{-2}g$. Due to a very strong amplitude and low frequency of the disturbance, strong convection is observed. The flow is strong enough to
<table>
<thead>
<tr>
<th>Amplitude (m/s²)</th>
<th>Frequency (Hz)</th>
<th>Maximum velocity (mm/s)</th>
<th>Maximum segregation (%)</th>
<th>Segregation after 500s solidification (with g-jitter) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$U_{max}$</td>
<td>$V_{max}$</td>
<td></td>
</tr>
<tr>
<td>10^{-2}g</td>
<td>0.01</td>
<td>2.50</td>
<td>0.89</td>
<td>188.7</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>1.26</td>
<td>0.78</td>
<td>192.4</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.75</td>
<td>0.32</td>
<td>39.9</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>0.19</td>
<td>9.66x10^{-2}</td>
<td>20.2</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.78x10^{-2}</td>
<td>4.81x10^{-2}</td>
<td>10.9</td>
</tr>
<tr>
<td>10^{-3}g</td>
<td>0.01</td>
<td>0.22</td>
<td>0.11</td>
<td>38.2</td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0.12</td>
<td>6.75x10^{-2}</td>
<td>11.3</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>7.49x10^{-2}</td>
<td>4.13x10^{-2}</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>1.90x10^{-2}</td>
<td>9.78x10^{-3}</td>
<td>3.1</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>9.95x10^{-3}</td>
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*Table 4* Velocity and radial interface segregation results in the liquid after 2000s of growth at 3.34 μm/s, with 1500s of growth at constant 1μg, the next 500s of growth with g-jitter added. The direction $U$ is along the length of the sample, $V$ is parallel to the interface.
Figure 3. Vertical velocity at the reference point for an amplitude of $10^{-3} \text{g}$ and frequencies of 
(a) $10^{-2} \text{ Hz}$, (b) $10^{-1} \text{ Hz}$ and (c) $1 \text{ Hz}$. 
Figure 3. Vertical velocity at the reference point for an amplitude of $10^{-3}g$ and frequencies of (a) $10^{-2}$ Hz, (b) $10^{-1}$ Hz and (c) 1 Hz.
Figure 4 The relation between maximum axial velocity and frequency of the disturbance.

Background gravity (1µg)

Peak of the gravity (11µg)
convectively carry the solute away from the interface. As a result, the concentration near the interface is decreased after one cycle of the disturbance. Because the acceleration changes slowly, the velocity field has enough time to change with it; and since flow velocities are high the transport of solute is dominated by the flow. As seen in the animation, oscillations in velocity and concentration are almost in phase. For this case longitudinal segregation is expected to be high due to solute transport to the far field liquid.

**Movie 5** shows the effects of the disturbance of 0.05Hz and 10^{-2}g. In this case, the solute profile established during steady 1 μg conditions is significantly modified when g-jitter starts with the decrease of concentration in the mid-height. As a result, the highest amount of radial segregation at the interface was observed in this case (see **Table 4**). Far field mixing of solute appears to be less in this case as compared to **Movie 4**, thus longitudinal segregation is expected to be less.

**Movie 6** shows the effects of the disturbance of 0.1Hz and 10^{-2}g. In the previous 2 cases, the flow was swift enough to sweep the solute across the interface with higher concentrations resulting in the corners where velocities are restricted. For this case, the moderate flow only has enough time and strength to sweep the solute about half way across the interface before reversing; thus resulting in the highest concentrations being located near the centreline and a reversal in radial segregation as compared to **Movies 4** and **5**, as seen in the animation. In this case, a small time delay between concentration field and velocity field is observed.

**Movie 7** shows the effects of the disturbance of 1.0Hz and 10^{-2}g. From the animation, it can be seen that even though the amplitude is high its effect on the solute field is relatively small due to the higher frequency. Though the solute field is still significantly corrupted from the pure diffusion case as seen in the radial segregation. (**Table 4**).

**Movie 8** shows the effects of the disturbance of 0.01Hz and 10^{-3}g. In this case, the frequency is the same as that in **Movie 4**. However, the amplitude of the disturbance is smaller (10^{-3}g). Therefore, the convection is not strong enough to mix the solute much with the far field liquid (at this growth rate). However, the velocity field is still strong enough to induce radial solute variations, as seen in the animation and in **Table 4**.

**Movie 9** shows the effects of the disturbance of 0.1Hz and 10^{-3}g. From the animation, little if any effect on solute due to g-jitter can be seen; however, **Table 3** does show radial segregation caused by the g-jitter.

**Average Concentration at the interface**

**Figure 5** shows the solute concentration distribution at the mid-height of the sample at the start of g-jitter and after a further 500 seconds of solidification for the oscillation frequencies of 0.01, 0.1 and 1 Hz, with an amplitude of 10^{-2}g and also for a steady gravitational acceleration.

At the time g-jitter starts, 5 mm of the sample had been solidified, creating a solute rich boundary layer in front of the interface. The peak value of concentration in the liquid at the interface caused by solute rejection into the liquid reached almost 7 at%. The solute concentrations in the liquid at the interface decay exponentially with the distance away from the interface to a value of C₀.

After another 500 seconds of solidification the maximum concentration at the mid-height of the ampoule reached 8.33 at% for the steady gravitational acceleration, 8.43 at% and 8.38 at% for frequencies of 0.1 and 1Hz, and 7.26 at% for 0.01 Hz. The largest difference in the maximum value (12.8%) occurs at the lowest frequency of 0.01 Hz. In this case convection has developed, resulting in the redistribution of solute in the melt and hence at the interface. The difference between the low 0.01 Hz frequency and the other cases can clearly be observed in **Figure 6** which shows the history of the average concentration at the interface during 500 seconds of solidification. Even though longitudinal segregation in the 0.1 Hz case is nearly the same as the pure diffusion case, **Table 4** and **Figure 7** show significant radial segregation at the 10^{-3}g, 0.1 Hz condition; thus this case is not resulting in diffusion controlled growth.
Figure 5. Distribution of solute concentration at the mid-height of the ampoule after 500 seconds of solidification with a g-jitter amplitude of $10^{-3}g$.

Figure 6. Average liquid interface concentration during growth with g-jitter of $10^{-3}g$ at frequencies of $10^{-2}$, $10^{-1}$ and 1 Hz. Starting time is after 1500s of growth without g-jitter.
Figure 7. Radial segregation in the liquid at the interface during growth with g-jitter of amplitude of $10^{-3}$ g and frequencies of $10^{-2}$, $10^{-1}$ and 1 Hz; showing growth at 1 Hz to be diffusion controlled and the solute profiles for the $10^{-2}$ and $10^{-1}$ Hz cases to be modified by convection.
The resulting radial segregation in the liquid at the interface can be seen in Figure 7. The segregation at the interface for the 0.1 and 0.01 Hz cases fluctuates with a frequency corresponding to that of the disturbances. The oscillations are real, however the different magnitude of the peaks in the case of the 0.1 Hz frequency and their periodic change are not real but rather an artefact of the fixed grid finite volume formulation which is used. In this formulation all the computed values are cell averaged values which are changing while the interface passes through one partially solidified cell.

Finally, disturbances with an amplitude of $10^{-2}g$ are considered. The time history of the average liquid concentration at the interface during 500 seconds of solidification with g-jitter (after 1500 seconds of solidification at constant $1 \mu g$) is shown in Figure 8. At 1 Hz the average interface concentration follows closely that expected during the initial transient for diffusion controlled growth (see Figure 6); thus indicating that far field mixing of the solute being produced at the interface is negligible. Growth for this case ($10^{-2}g, 1 Hz$) is not, however, dominated by diffusion, since the solute profiles near the interface are corrupted from the diffusion case as shown in the radial segregation results, Table 4. For the 0.1 Hz ($10^{-2}g$) case the departure of the average interface concentration from the diffusion case indicates significant mixing of solute from the neighbourhood of the interface with the far field liquid; radial segregation is also high (Table 4) with the maximum radial segregation occurring between (at $10^{-2}g$) 0.1 and 0.01 Hz. The decrease in radial segregation after the frequency is lowered from 0.05 to 0.01 Hz (at $10^{-2}g$) and interface concentration approaching $C_0$ (Figure 8) indicates that the solute is becoming well mixed with the far field liquid for the 0.01 Hz, $10^{-2}g$ case.

At 0.1 Hz, $10^{-2}g$ the flow at first carries low concentration solute from the bulk of the liquid closer to the interface, some mixing occurs in the liquid and as a result the average concentration at the interface decreases while the interface is moving through the liquid. However, once the flow starts to oscillate backwards and forward, and additional solidification occurs with solute rejection and mixing with the far field liquid, the average concentration levels out and is expected to rise as solidification continues. The radial segregation in this case reaches 39.9% after 500 seconds of g-jitter. At 0.01 Hz the average concentration drops quickly due to bulk mixing of solute. At 60 seconds the flow reverses and brings back high concentration solute causing an increase in the average concentration at the interface. As the magnitude of the velocity increases in the opposite direction, further mixing occurs in the liquid and the average concentration drops to about 1.5%.

A non-dimensional quantity of interest in this analysis is the solutal Peclet number, defined as $Pe_s = \frac{V_f \delta}{D}$, where $V_f$ is the predominant flow velocity in the solute boundary layer, $\delta$ is the boundary layer thickness equal to $2D/R$, and $R$ is the growth rate. In general, growth is considered to be diffusion dominated if $Pe_s < 1$. If we consider the vertical velocity maximum, $V_{max}$, to be characteristic of the flow inside the solute boundary layer, it can be seen from Table 4 that no signs of convection (radial segregation) are present for $V_{max}$ values less than about $1 \times 10^{-3}$ mm/s and that diffusion conditions are clearly departed from at $V_{max}$ values greater than about $1 \times 10^{-2}$ mm/s; these velocities correspond to $Pe_s$ of between about 0.6 to 6.0 respectively, which is in excellent agreement with the stated general application of this non dimensional number. The Rayleigh numbers ($\text{Ra} = \frac{g \beta \rho_s h^4}{\kappa}$) assuming constant $10^{-5}g$ and $10^{-4}g$, and thermally driven flow for these $V_{max}$ values are 1.09 and 10.9 respectively. By scaling $V_{max}$ to the Rayleigh number the consequences of changing the g level, furnace temperature gradient, ampoule diameter (h), and sample material can be estimated. With $Pe_s$ the consequences of changing the growth velocity can also be estimated.

It is also noteworthy that the effects of solute on the fluid flow are not readily seen in these simulations because only about 6.7 mm of sample are solidified; thus simulations take place in the early stage of the initial transient. The result is that the solute boundary layer is not built up very high. With further solidification, additional solute would be rejected and possibly the creation of a solutally driven flow.
cell (rather than the thermally driven cells found in this work). Previous work has shown the development of solutally driven cells.\(^6\)

**Temperature field and interface shape**

Figure 8. Average liquid concentration at the interface during growth with g-jitter of \(10^{-2}\)g at frequencies of \(10^{-2}, 10^{-1}\) and 1 Hz; showing no solute mixing with the far field liquid at 1 Hz, partial mixing at \(10^{-1}\) Hz, and nearly complete mixing of the solute rejected at the interface with the far field liquid at \(10^{-2}\) Hz.

Figure 8 shows the average liquid concentration at the interface during growth with g-jitter of \(10^{-2}\)g at frequencies of \(10^{-2}, 10^{-1}\) and 1 Hz; showing no solute mixing with the far field liquid at 1 Hz, partial mixing at \(10^{-1}\) Hz, and nearly complete mixing of the solute rejected at the interface with the far field liquid at \(10^{-2}\) Hz.

Temperature field and interface shape

Figure 9 and Figure 10 show the interface shape with time for the disturbance of 0.01 Hz with \(10^{-2}\)g and \(10^{-3}\)g respectively. It can be concluded that the interface shape is not flat when there is a strong convection. As seen in Figure 9, the interface shape develops corresponding to the flow inside the domain. However, for \(10^{-3}\)g amplitude of disturbance, it is shown that (while assuming constant interface temperature) the flow field is not strong enough to change the shape of the interface (see Figure 10).

When considering Figures 9 and 10 it should be kept in mind that the S/L interface temperature has been assumed constant. Thus the effects of solutal gradients at the interface on the interface temperature have not been included. As seen in Table 4, the concentration in the liquid across the interface can vary by a factor of 2 or more. If, for example, we assume a concentration variation from 5 at% to 10 at% at the interface, these locations would vary in temperature by about \(12\,\text{°C} = (10-5\,\text{at%}) \times 2.32\,\text{°C/at%}\). With an axial temperature gradient of \(20\,\text{°C/mm}\), these locations would vary axially by about 0.6 mm, which indicates much more interface curvature due to solute dependent interface temperature than that caused by the flow modifying the temperature field.
Figure 9. Isothermal interface shape for the disturbance of $10^2$g and 0.01 Hz from 400 - 500 sec (time interval 10 sec). Note that the horizontal and vertical scales are not the same; the curvature of the interface is exaggerated.
Figure 10. Isothermal interface shape for the disturbance of $10^{-3}$ g and 0.01 Hz from 400 - 500 sec (time interval 10 sec). This simulation does not include the influence of radial segregation and effect of solute on the temperature (and thus the shape) of the interface.
6. CONCLUSIONS

We have investigated numerically effects of sinusoidal disturbances with amplitudes from $10^{-5}g$ to $10^{-2}g$ and frequencies from $10^{-2}$ to 1 Hz on the solute redistribution and segregation at the interface during directional solidification of Bi-1at% Sn alloy.

It was found that for large frequencies a higher amplitude of the gravitational acceleration is required to produce an effect on the segregation. For example, disturbances with frequencies from 0.5 to 1 Hz and amplitudes less than $10^{-2}g$ produce very little effect on the segregation. For this frequency range an amplitude of $10^{-2}g$ resulted in 20.2% and 10.9% segregation compared with a segregation of 1.8% for the steady 1 $\mu$g case. For frequencies from 0.05 Hz to 0.1 Hz, an amplitude of $10^{-3}g$ results in the segregation changing to 11.3% and 8.9% respectively. The largest effect on the segregation was produced by disturbances with a frequency of 0.01 Hz, where the maximum segregation was equal to 4.9% for an amplitude of $10^{-4}g$, 38.2% for an amplitude of $10^{-3}g$ and 188.7% for an amplitude of $10^{-2}g$. In the last case significant mixing of the solute in the cavity was observed. Diffusion dominated conditions were observed at solutal Peclet numbers below 0.6, convection was significant at $Pe_s > 6.0$. Rayleigh numbers below about 1.1 resulted in diffusion dominated growth at $R = 3.34 \mu m/s$, at $Ra > 11.0$ corruption of diffusion conditions was observed (at the growth rate examined). Under the conditions examined in this work, the shape of the interface can not be accurately simulated if the influence of solute on interface melting temperature is omitted.
REFERENCES

A study of directional solidification of a weak binary alloy (specifically, Bi - 1 at% Sn) based on the fixed grid single domain approach is being undertaken. The enthalpy method is used to solve for the temperature field over the computational domain including both the solid and liquid phases; latent heat evolution is treated with the aid of an effective specific heat coefficient. A source term accounting for the release of solute into the liquid during solidification has been incorporated into the solute transport equation. The vorticity-stream function formulation is used to describe thermosolutal convection in the liquid region. In this paper we numerically investigate the effects of g-jitter on directional solidification. A background gravity of 1 μg has been assumed, and new results for the effects of periodic disturbances over a range of amplitudes and frequencies on solute field and segregation have been presented.