Multidisciplinary System Reliability Analysis

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CHAPTER I

INTRODUCTION

The purpose of this study is to develop a methodology to estimate the reliability of engineering systems that encompass several disciplines. The methodology is implemented using the NESSUS probabilistic analysis code, which has mostly been applied exclusively in the discipline of structural engineering. In order to apply the NESSUS probabilistic structural analysis code to analyze a multi-disciplinary engineering system, the equivalences between system behavior models in different disciplines are investigated, and the effect of physical interaction among the failure modes is quantified in this study.

System reliability analysis is a method of estimating the effects of uncertainties in an engineering system on the probability of successful performance. Usually, an engineering system consists of multiple subsystems and components, which may require the knowledge of different disciplines of engineering. Such disciplines may include structural engineering, mechanical engineering, heat transfer theory, fluid mechanics, electrical engineering, etc. Such a system is called a multi-disciplinary engineering system. The reliability analysis of any engineering system usually begins with the identification and reliability computation of individual failure modes within the system. Then the reliability analysis of the overall system can be carried out.

Traditionally, reliability methods have primarily concentrated on failures in one
particular discipline, e.g. structural analysis, not on an overall system which consists of multiple disciplines. Furthermore, conventional methods of system reliability estimation usually only consider the statistical correlation between individual failure events, ignoring the fact that more often than not, those individual failure modes also have a physical correlation. This leads to inaccuracy in system reliability estimation.

The method presented in this report uniquely computes the failure probability interactions between different modes and overall system failure probability through the imposition of one failure mode on another field and reanalysis of the latter. This method is used to compute the probabilities of critical system failure events after accounting for the contributing non-critical failure modes in all different fields. However, it is not an easy task to estimate the reliability interactions between different failure modes. The success of such a method primarily depends on the availability of effective reliability tools. The software system NESSUS developed under the leadership of NASA Lewis Research Center is uniquely suited for this purpose. Currently, this code has been applied primarily to the structural engineering problems. In order to perform system reliability analysis including the interactive failure modes, this study uniquely develops behavior analogies between the structural model and heat transfer model, and between the structural model and fluid mechanics model. By doing so, the probability estimation of heat transfer and fluid mechanics failures can be pursued similarly to structural reliability analysis.

The objective of this research project is to develop a method, using system reliability theory, for the reliability estimation of multi-disciplinary engineering systems. The method is implemented on the software system NESSUS (Numerical Evalu-
ation of Stochastic Structures under Stress) developed by NASA Lewis Research Center. An example application to a three-discipline system involving mechanical stress-strain behavior, heat transfer and fluid mechanics is provided. In order to compute the individual failure mode probability of non-structural problems such as heat transfer and fluid mechanics within NESSUS, it is necessary to develop a new methodology for the analysis of heat transfer problems using the concept of equivalence between the computational quantities in structural analysis, such as stiffness, displacement vector, load vector, etc. and similar quantities such as conduction, temperature distribution and heat flux in heat transfer theory, flow velocity, pressure and flow factor in fluid mechanics. This is the first important contribution of this study.

The second important contribution is the method for the computation of the physical dependence of critical failure mode probabilities on non-critical failure modes in various disciplines. This involves the imposition of the non-critical modes and reanalysis of the system with appropriate discipline equivalences, for various levels of progressive damage. The combination of these two ideas - inter-disciplinary analogies and physical failure mode correlation - makes a reliability analysis program such as NESSUS very powerful for application to a variety of multi-disciplinary systems.

The concepts and methods discussed above are examined in detail in the next four chapters of this report. In Chapter II, the basic reliability analysis concepts for individual component-level and system-level events are reviewed, and their implementation in the NESSUS program is described. In Chapter III, the behavior analogies between the structural analysis model and heat transfer problem, and be-
tween the structural analysis model and fluid mechanics model are developed. The finite element numerical examples with NESSUS/FEM are demonstrated for this concept. Chapter IV consists of two major parts: in the first part, the failure probability analyses for individual events including structural, heat transfer and fluid flow failure modes are performed using NESSUS; in the second part, the system failure probability is studied. The effect of non-critical failure events of heat transfer and fluid mechanics upon a critical structural failure event is investigated, followed by system reliability analysis with the consideration of physically correlated component-level events. A numerical example of system reliability analysis of a multi-disciplinary system consisting of structural, heat transfer and fluid mechanical modes is demonstrated. The conclusions and recommendations of the study are summarized in Chapter V.
CHAPTER II

SYSTEM RELIABILITY ANALYSIS

Individual failure modes and effects

An engineering system consists of a number of functional components. Before the system-level analysis begins, the modes of failure for individual components should be specified. The analyses of the failure modes and effects can be carried out by starting at the component level and expanding upward to the whole system. A failure mode is the manner by which a failure is observed. All units in a system are designed to fulfill one or more functions. A failure is thus defined as non-fulfillment of one of these functions. Analytically, each failure mode has a corresponding limit state which separates the design space into "failure" and "safe" regions. The probability of failure, $P_f$, is denoted as

$$P_f = P[g \leq 0]$$

(1)

where $g$ is the value of the performance function $g(X)$. The limit-state is denoted by the equation $g(X) = 0$.

An exact solution of $P_f$ can be obtained by the integration of the multiple integral denoted as

$$P_f = \int_{g(X) \leq 0} f_X(x) dx$$

(2)
where $f(X)$ is the joint probability density function of the vector of uncertain variables $\bar{X}$.

In general, the solution of this multiple integral is too complicated to obtain. This is not only because the individual distributions are not always available but also because the integral is multi-dimensional for a realistic problem and is difficult to evaluate. Therefore, for practical purposes, efficient approximate analysis tools are needed.

Fig. 1 illustrates the concept of the first-order approximation to the limit state for an estimate of the failure probability.

The uncertain variables ($\bar{X}$) are all transformed to equivalent uncorrelated standard normal variables ($u$). The most probable point $MPP$ of the limit state is defined at the minimum distance $\beta$ from the origin to the limit state surface. Therefore, the first-order estimate of the failure probability is

$$P_f = \Phi(-\beta)$$

where $\Phi$ is the distribution function of a standard normal variables.

In the NESSUS computer code, this is referred to as the Fast Probability Integration ($FPI$) method. The limit state is constructed as:

$$g = Z(X) - Z_0 = 0$$
Figure 1: Failure Probability Estimation
where \( Z_0 \) is a real value of the random variable \( Z(X) \), which is a performance function or a response function, such as stress, displacement, temperature, etc.

\[
Z(X) = Z(X_1, X_2, \ldots, X_n)
\]

(5)

where \( X_i (i = 1, 2, \ldots, n) \) are the input random variables.

The NESSUS program searches for the \( MPP \) by computing the sensitivities of the limit state to the random variables using iterative perturbation (in NESSUS/PFEM), and using these sensitivities to obtain a mean value first order (MVFO) or second-order (MVSO) estimate of the failure probability (in NESSUS/FPI).

By using a first or second-order Taylor’s series expansion around the \( MPP, u^* \), the exact \( g(u) \)-function is replaced by the first-order polynomial, \( g_1(u) \),

\[
g_1(u) = a_0 + \sum_{i=1}^{n} a_i(u_i - u_i^*)
\]

(6)

or a second-order polynomial, \( g_2(u) \),

\[
g_2(u) = a_0 + \sum_{i=1}^{n} a_i(u_i - u_i^*) + \sum_{i=1}^{n} b_i(u_i - u_i^*)^2 + \sum_{i=1}^{n} \sum_{j=1}^{n-1} C_{ij}(u_i - u_i^*)(u_j - u_j^*)
\]

(7)

where the coefficients can be obtained by perturbation.

Once these functions are obtained, the \( MPP \) is found. The probability of failure can be computed easily using Equation 3. This is the mean value first order (MVFO) estimate of the failure probability. This is improved using the Advanced Mean Value (AMV) analysis. Point probability estimate is made using specific limits for \( Z_0 \), and the cumulative distribution function (CDF) is obtained by varying \( Z_0 \).
NESSUS/FEM employs innovative finite element technology and solution strategies. It provides a choice of algorithms for the solution of static and dynamic problems, both linear and nonlinear, together with an interactive perturbation analysis algorithm to evaluate the sensitivity of the response to small variations in one or more user-defined random parameters.

NESSUS/FPI (Fast Probability Integrator) is used to evaluate structural response cumulative distribution functions (CDF). There are two methods in the code, the first-order reliability method and the advanced first-order reliability method. In general, the structural performance or response functions (e.g., stresses, displacements, vibration frequencies) are implicitly defined and each function evaluation may require intensive computation. The AMVFO (Advanced Mean Value First Order) method reduces the computational burden and is the main probabilistic tool in NESSUS. NESSUS/PFEM automates the AMV procedure by integrating the FPI code and the FEM code.
System and component-level failure modes

After the individual reliability analysis is completed, one then proceeds to the system or subsystem level analysis. System failure may occur due to a combination of any of the individual component failure modes. Many physical systems that are composed of multiple components can be classified as series-connected or parallel-connected systems, or combinations of series and parallel conditions. Description of these simple system structures is as follows.

• Series System

A system that is functioning if and only if all of its \( n \) components are functioning is called a series system structure. Fig. 2 illustrates such a system.

If \( E_i \) denotes the failure mode \( i \), then the failure of a series system is the event

\[
E_f = E_1 \cup E_2 \cup \ldots \cup E_n
\]  

(8)

Then the failure probability of the system is

\[
P_f = P(E_f) \quad \text{(9)}
\]

If each failure mode \( E_i \) is represented by a limit state \( g(X) = 0 \) in basic variable space, the failure probability can be obtained by the integration denoted as

\[
P_f = \int_{g(X) = 0} \ldots \int f_x(x)dx \quad \text{(10)}
\]
Figure 3: Basic reliability problem in two dimensions
where $X$ represents the vector of all the basic random variable (loads, material properties, etc.) and $\Omega$ is the domain in $X$ defining failure of the system.

This is defined in terms of the various failure modes as $g_i(X) \leq 0$. In two-dimensional $X$ space, expression (10) is defined in Fig. 3.

- Parallel System

A system that is functioning if at least one of its $n$ components is functioning is called a parallel system structure. A parallel structure of order $n$ is illustrated in Fig. 4.
In this case, the system failure event can be written

\[ E_f = E_1 \cap E_2 \cap ... \cap E_n \]  

Combined Series-Parallel System Structure

This refers to systems which are a combination of series and parallel structures. Fig. 5 shows an example of such systems.

The failure event of this system is written, for example, as

\[ E_f = [E_1 \cap (E_2 \cup E_3)] \cup E_4 \]  

It should be noted that not all engineering systems can be represented simply as described above. Practical systems may be more complex and need more effort to model.
System reliability computation

In NESSUS, two methods have been implemented for system reliability computation [4]: (1) probabilistic fault tree analysis combined with importance sampling (Torng et al, 1992) and (2) a structural reanalysis procedure to accurately estimate the failure regions for various critical failure modes affected by progressive damage (Mahadevan et al, 1992).

Consider an engineering system subject to a sequence of loads (duty cycles) and which may fail in any one (or more) of a number of possible failure modes under any one load in the loading sequence. The total probability of the system failure may then be expressed in terms of the individual mode failure probabilities as

\[ P_f = P(E_1) \cup P(E_2 \cap S_1) \cup P(E_3 \cap S_2 \cap S_1) \cup P(E_4 \cap S_3 \cap S_2 \cap S_1) \cup ... \]  

where \( E_i \) denotes the "failure of the system due to failure in \( i \)th mode and \( S_i \) denotes the complementary "survival event of the \( i \)th mode.

Since \( P(E_2 \cap S_1) = P(E_2) - P(E_2 \cap E_1) \), ..., Eq. 13 may be written also as

\[ P_f = P(E_1) + P(E_2) - P(E_1 \cap E_2) + P(E_3) - P(E_1 \cap E_3) \]

\[ -P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3) + ... \]

where \((E_1 \cap E_2)\) is the event that failure occurs in both modes 1 and 2, etc.

Since it is not always an easy task to determine the joint probabilities of more than two failure modes, the following approximation methods can be used to predict the system reliabilities.
• First-order bounds

The probability of failure for the system can be expressed as $P_f = 1 - P(S)$, where $P(S)$ is the probability of survival. For independent failure modes, $P(S)$ can be represented by the product of the mode survival probabilities, or, noting that $P(S_i) = 1 - P(E_i)$, by

$$P_f = 1 - \prod_{i=1}^{n} [1 - P(E_i)]$$

(15)

where, as before, $P_f$ is the probability of failure in mode $i$. This result can be shown to be identical with Eq. 14. It follows directly from Eq. 14 that, if $P(E_i) \ll 1$, then Eq. 15 can be approximated by [Freudenthal et al., 1966]

$$P_f = \sum_{i=1}^{n} P(E_i)$$

(16)

In the case where all failure modes are fully dependent, it follows directly that the weakest failure mode will always govern system failure, irrespective of the random nature of the strength. Hence

$$P_f = \max_{i=1}^{n} [P(E_i)]$$

(17)

Equations 15 or 16 and 17 can be used to define relatively crude bounds on the failure probability of any system of the series types when the failure modes are neither completely independent nor fully dependent. These are Cornell’s first-order bounds:

$$\max_{1 \leq i \leq n} P(E_i) \leq P(\cup_{i=1}^{n} E_i) \leq \sum_{i=1}^{n} P(E_i)$$

(18)
• Second-order bounds

For some practical systems, the above first-order bounds may be too wide to be meaningful. For more accurate estimation, second-order bounds have been developed. There are various second-order bounds in the literature [Kounias, 1968; Vanmarcke, 1973; Hunter, 1976; Ditlevsen, 1979]. Cruse et al (1992) derived second-order bounds which are independent of any ranking of the failure events[1]. The upper bound is

\[
P(\cup_{i=1}^{n} E_i) \leq \left\{ \sum_{i=1}^{n} P(E_i) - \max\left\{ \sum_{i=2}^{n} \max_{1 \leq i \leq n} P(E_i E_j), \max_{i=1}^{n} \sum_{i,j \neq i} P(E_i E_j) \right\} \right\}
\]

(19)

The lower bound is

\[
P(\cup_{i=1}^{n} E_i) \geq \max_{1 \leq j \leq n} \left\{ P(E_j) + \sum_{i=1}^{n} \max_{i \neq j} \left[ (P(E_i) - \sum_{k=1}^{\max(i,j)} P(E_i E_k)); 0 \right] \right\}
\]

(20)

Utilization of the second-order bounds requires evaluation of terms of the form \(P(E_i E_j)\) where \(E_i\) denotes the event "failure in limit state \(i\)". The intersection terms refer to domains such as \(\Omega_1\) shown bounded by the non-linear limit state functions \(g_i(X) = 0(i = 1, 2, 3)\) in Fig. 3. The individual failure mode probabilities in the first-order analysis are determined as

\[P_f = \phi(-\beta)\]

(21)

In standardized independent normal \(X\) space, the linear limit state function is given by

\[g_i(X) = \beta_i + \sum_{j=1}^{n} \alpha_{ij} x_j\]

(22)
where \( n \) is the number of random variables.

The angle between the two limit states provides information about the correlation of the two failure modes. The correlation coefficient is obtained as

\[
\rho_{ij} = \sum_{r=1}^{n} \alpha_{ir} \alpha_{jr} = \cos \psi_{ij}
\]  

(23)

Once \( \beta_i, \beta_j \) and \( \rho_{ij} \) are obtained, the computation of joint probability of failure can be carried out. Eq. 19 and 20 can be used to compute the second-order bounds for system failure probability estimation.

The above method only provides a tool to approximately estimate the failure probability correlation of two different failure modes in a multi-disciplinary system. A more accurate approach would be the imposition of one failure mode on another mode and reanalysis of the latter. For example, consider two failure modes in a heat transfer system: structural failure and heat transfer failure. Structural failure happens when the stress, caused by the fluid pressure and temperature difference between outer and inner surfaces exceeds limiting value of strength. The heat transfer failure happens when the temperature of the contained liquid can not be kept at a certain level. When the thermal failure occurs, the increase in the temperature field also causes changes in stress field. The structural failure probability can be re-estimated under this changed stress field and the result can be considered as the interactive failure probability under influence of heat transfer failure. A numerical examples will be shown for this approach in a later chapter.
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<td><img src="image" alt="Symbol" /></td>
<td>Representation of an event of a failure</td>
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<tr>
<td><img src="image" alt="Symbol" /></td>
<td>OR-gate</td>
</tr>
<tr>
<td><img src="image" alt="Symbol" /></td>
<td>AND-gate</td>
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Figure 6: Standard symbols used in fault tree analysis

**Probabilistic fault tree analysis**

NESSUS system risk assessment (SRA) uses probabilistic fault tree analysis (PFTA). A fault tree is a mathematical construction of assumed component failure modes (bottom events) linked in series or parallel leading to a top event, which denotes system failure. Standard graphical symbols are used to construct the fault tree picture, by describing events and logical connections. These are shown in Fig. 6, and a simple PFTA is shown in Fig. 7.

- Fault Trees with a Single AND-GATE

Consider the fault tree in Fig. 8. Here the top event occurs if and only if all the bottom events $E_1, E_2, ..., E_n$ occur simultaneously. A system with AND-GATE is very similar to a series system structure.
Figure 7: Probabilistic Fault Tree for System Reliability Example
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<td>AND-gate</td>
<td>The AND-gate indicates that the output event $A$ occurs only when all the input events $E_i$ occur.</td>
</tr>
<tr>
<td>OR-gate</td>
<td>The OR-gate indicates that the output event $A$ occurs if any of the input events $E_i$ occur.</td>
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Figure 8: Fault tree with a single AND-GATE and a single OR-gate
Consider the fault tree in Fig. 8. The top event occurs if at least one of the bottom events $E_1, E_2, ..., E_n$ occurs. The structure of this fault tree is similar to the parallel system structure.

A schematic of Version 6.0 of the NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) probabilistic structural analysis computer program is shown in Fig. 9. As shown in the diagram, the NESSUS includes other modules, namely the System Risk Assessment (SRA) and Simulation Finite Element (SIM-FEM) modules. The random field pre-processor (PRE) provides data manipulation needed to express the uncertainties in a random field as a set of uncorrelated random variables. The user-subroutine which defines the response model (UZFUNC) enables
users to define required limit state with the computed response. This study will main-ly use FEM, PFEM and FPI for reliability analysis. The NESSUS program is quite comprehesive with respect to structural reliability estimation. As mentioned in Chapter I, the purpose of this study is to develop a technique by which the NESSUS program can be used for the system reliability analysis of multi-disciplinary systems. The following chapters describe this technique in detail.
CHAPTER III

ANALOGY BETWEEN ENGINEERING SYSTEMS

Introduction

Since NESSUS/FEM program has been mostly applied only to structural analysis, a thermal or a fluid mechanical system needs to be converted through an analogous model to a structural system on which the NESSUS program can be applied for analysis. Then the probability analysis for a heat transfer system or a fluid mechanics system can be carried out by NESSUS. By doing so, a system with heat transfer, fluid flow and mechanical stress problems can be analyzed by NESSUS automatically with FEM, PFEM, FPI and SRA modules for system reliability analysis.

In this chapter, a new methodology is presented for one-dimensional steady-state heat transfer analysis and one-dimensional steady-state uniform flow problem using a structural finite element program. First, the use of the analogous models is introduced for the analysis of systems involving one-dimensional steady-state heat transfer and simple one-dimensional steady-state uniform flow in closed conduit systems.

Heat transfer analysis through structural analogy

One-dimensional steady-state heat transfer

We begin our analysis of one-dimensional, steady-state conduction by discussing heat transfer with no internal generation. The objective is to determine the expressions for temperature distribution and heat transfer rate in common geometries.
The concept of thermal conductivity (analogous to stiffness in stress analysis) is introduced as an aid to solving conduction heat transfer problems. Consider a three dimensional differential volume shown in Fig. 10. The general heat equation is

$$\frac{\partial}{\partial x} (K \frac{\partial T}{\partial x}) + \frac{\partial}{\partial y} (K \frac{\partial T}{\partial y}) + \frac{\partial}{\partial z} (K \frac{\partial T}{\partial z}) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

where $K$ is the thermal conductivity of the material. $K \frac{\partial T}{\partial x}$, $K \frac{\partial T}{\partial y}$, $K \frac{\partial T}{\partial z}$ are related to heat flux in a direction perpendicular to the surface. $\dot{q}$ is the rate at which energy is generated per unit volume of the medium. The density $\rho$ and specific heat $c_p$ are two thermodynamic properties. The product $\rho c_p$ is the volumetric heat capacity. $\rho c_p \frac{\partial T}{\partial t}$ is the time rate of change of the internal (thermal) energy of the medium per unit volume.
If the heat transfer is one-dimensional and steady state, any differentiation with respect to time is equal to zero and there is no internal heat generation, so Eq. 24 reduces to

$$\frac{d}{dx} (KA \frac{dT}{dx}) = 0$$ (25)

The heat flux is a constant, independent of x.

As shown in Fig. 11, a plane wall separates two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{s,1}$, by conduction through the wall, and by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$.

Assuming the thermal conductivity of the material to be constant, Eq. 25 may
be integrated twice to obtain the general solution

\[ T(x) = C_1 x + C_2 \]  \hspace{1cm} (26)

To obtain the constants of integration, \( C_1 \) and \( C_2 \), boundary conditions must be introduced. These are:

\[ T(0) = T_{s,1} \]  \hspace{1cm} (27)
\[ T(L) = T_{s,2} \]  \hspace{1cm} (28)

Applying the condition at \( x = 0 \) to the general solution, it follows that

\[ T_{s,1} = C_2 \]  \hspace{1cm} (29)

Similarly, at \( x = L \)

\[ T_{s,2} = C_1 L + C_2 = C_1 L + T_{s,1} \]  \hspace{1cm} (30)

in which case

\[ \frac{T_{s,2} - T_{s,1}}{L} = C_1 \]  \hspace{1cm} (31)

Substituting into the general solution, the temperature distribution is then

\[ T(x) = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} T_1 \\
T_2 \end{bmatrix} \]

where \( N_1 = 1 - \frac{x}{L}, N_2 = \frac{x}{L} \)

The heat flow can be determined by Fourier's law, that is

\[ q = -KA \frac{dT}{dx} \]  \hspace{1cm} (32)

or

\[ q = -KA \left[ \frac{-(T_1 - T_2)}{L} \right] = \frac{KA}{L} (T_1 - T_2) \]  \hspace{1cm} (33)
Stress analysis of a bar element

Now consider a linear-elastic, constant cross-sectional area (prismatic) bar element shown in Fig. 12. Using Hooke’s law, the differential equation governing the linear-elastic bar behavior is

\[
\frac{d}{dx} \left( ES \frac{dU}{dx} \right) = 0
\]

(34)

where \( U \) is the axial displacement function in the \( x \) direction and \( S \) and \( E \) are cross-sectional area and Young’s modulus of elasticity respectively.

\[
U = \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}
\]

where \( N_1 = 1 - \frac{D}{L} \), and \( N_2 = \frac{L}{D} \).

The strain-displacement relationship is

\[
\epsilon_x = \frac{dU}{dx} = \frac{U_2 - U_1}{D}
\]

(35)
Table 1: Analogous quantities for structural and thermal systems

<table>
<thead>
<tr>
<th>Heat transfer</th>
<th>Structure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux ( q )</td>
<td>Nodal force ( f_1 )</td>
</tr>
<tr>
<td>Temperature ( T(x) )</td>
<td>Displacement ( U(x) )</td>
</tr>
<tr>
<td>Inverse of heat transfer resistance</td>
<td>Structural stiffness ( \frac{ES}{D} )</td>
</tr>
<tr>
<td>Conduction: ( \frac{KA}{L} ), convection: ( hA )</td>
<td></td>
</tr>
</tbody>
</table>

We obtain

\[
F = ES \left( \frac{U_2 - U_1}{D} \right) \tag{36}
\]

Also, by the nodal force sign convention of Fig. 12,

\[
f_1 = -F \tag{37}
\]

So Eq. 36 becomes

\[
f_1 = \frac{ES}{D} (U_1 - U_2) \tag{38}
\]

Analogous modeling between heat transfer and structure

Comparing Eq. 38 with Eq. 33, the similarities become apparent. These two equations indicate a direct analogy between heat transfer and structural analysis. The analogous quantities are listed in the Table 1.

With this analogy, we are able to model a heat transfer problem into a stress analysis problem.
In the plane wall, we refer heat transfer resistance of conduction \( R \) to \( \frac{L}{KA} \), that is

\[
R_{\text{cond}} = \frac{T_{s,1} - T_{s,2}}{q} = \frac{L}{KA}
\]  \hspace{1cm} (39)

Considering the structural system, Hooke's law provides stiffness of the form

\[
k = \frac{f_1}{U_1 - U_2} = \frac{ES}{D}
\]  \hspace{1cm} (40)

Comparing Eqns. 39 and 40, and considering \( \frac{KA}{L} \) and \( \frac{ES}{D} \) as analogous qualities, \( \frac{1}{R_{\text{cond}}} \) can be traced to be analogous to \( K \).

A heat transfer factor may also be associated with convection at a surface. From Newton's law of cooling,

\[
q = hA(T_s - T_\infty)
\]  \hspace{1cm} (41)

where \( h \) is Planck's constant of convection heat transfer coefficient, \( T_s \) is the surface temperature and \( T_\infty \) is the ambient temperature.

The thermal resistance for convection is then

\[
R_{\text{conv}} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}
\]  \hspace{1cm} (42)

The equivalent thermal circuit for the plane wall with convection surface conditions is shown in Fig. 11. The heat transfer rate may be determined from separate consideration of each element in the network, that is,

\[
q = h_1A(T_{\infty,1} - T_{s,1}) = \frac{KA}{L}(T_{s,1} - T_{s,2}) = h_2A(T_{s,2} - T_{\infty,2})
\]  \hspace{1cm} (43)
In terms of the overall temperature difference, $T_{\infty,1} - T_{\infty,2}$, and the effective thermal resistance $R_{eff}$, the heat transfer rate may also be expressed as

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{eff}} \quad (44)$$

Because the conduction and convection resistance are in series and may be summed up, it follows that

$$R_{eff} = \frac{1}{h_1A} + \frac{L}{KA} + \frac{1}{h_2A} \quad (45)$$

Consider a bar consisting of three different materials which are denoted as elements 1, 2 and 3. The effective stiffness for this composite bar is

$$k_{eff} = \frac{1}{\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}} \quad (46)$$

Comparing the above equations Eq. 45 and Eq. 46, the analogy is $k_{eff} \leftrightarrow \frac{1}{R_{eff}}$, that is

$$R_{eff} \leftrightarrow \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad (47)$$

Substituting with Eq. 45, we obtain

$$\frac{1}{h_1A} + \frac{L}{KA} + \frac{1}{h_2A} \leftrightarrow \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \quad (48)$$

where

$$k_1 = \frac{E_1S_1}{D_1} \quad (49)$$

$$k_2 = \frac{E_2S_1}{D_2} \quad (50)$$

$$k_3 = \frac{E_3S_1}{D_3} \quad (51)$$
Substituting the corresponding terms in Eq. 48, we obtain the equivalent quantities

\[ E_1 \leftrightarrow (h_1 A) \left( \frac{D_1}{S_1} \right) \]  

(52)

\[ E_2 \leftrightarrow \left( \frac{K A}{L} \right) \left( \frac{D_2}{S_2} \right) \]  

(53)

\[ E_3 \leftrightarrow (h_2 A) \left( \frac{D_3}{S_3} \right) \]  

(54)

With these analogous quantities, we use the NESSUS/FEM beam element with the \( E \) values replaced by the values involving heat transfer problem outlined above. The boundary conditions for the bar are the end displacements corresponding to the ambient temperature of the wall. After the structural analysis, we get the temperature distribution from the corresponding displacement distribution in the output.

**Heat transfer in composite walls**

Equivalence concepts for thermal-structural analysis may also be used for more complex systems, such as composite walls and radial heat transfer systems. Fig. 13 shows a series composite wall. The one-dimensional heat transfer rate for this system can be expressed as

\[ q = \frac{T_{\infty,1} - T_{\infty,4}}{\sum R} \]  

(55)

where \( T_{\infty,1} - T_{\infty,4} \) is the overall temperature difference and the summation includes all thermal resistances. Hence
Figure 13: Equivalent thermal circuit of a series composite wall
Figure 14: Structural analog for the series composite wall heat transfer

\[ q = \frac{T_{\infty,1} - T_{\infty,4}}{R_1 + R_2 + R_3 + R_4 + R_5} \]  

(56)

\[ = \frac{T_{\infty,1} - T_{\infty,4}}{(1/h_A) + (L_A/K_A) + (L_B/K_B) + (1/h_4)} \]  

(57)

Alternatively, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

\[ q = \frac{T_{\infty,1} - T_{\infty,4}}{(1/h_A) + (L_A/K_A) + (L_B/K_B) + (1/h_4)} \]

(58)

The analogous structural model for this series composite wall heat transfer problem is shown in Fig. 14. The bar consists of five elements with stiffnesses of \( k_1, k_2, k_3, k_4, k_5 \). Using the mechanical structure equivalence for convection and conduction, we obtain

\[ E = \begin{cases} 
(h_A) \left( \frac{B}{L} \right) & \text{for convection} \\
(K_A) \left( \frac{B}{L} \right) & \text{for conduction} 
\end{cases} \]
Figure 15: A heat exchanger for engine oil and refrigerant fluid
Heat transfer in radial systems

Cylindrical and spherical systems often experience temperature gradients in the radial direction only and may therefore be treated as one dimensional.

Fig. 15 shows an example of a heat exchanger, whose inner cylinder is used to store engine oil and the outer cylinder is used to transfer the refrigerant fluid to cool down the oil temperature. The outer insulated covering is assumed to isolate the system from the ambient environment. For steady state conditions with no heat generation, the appropriate form of the heat equation is

\[ \frac{1}{r} \frac{d}{dr} \left( Kr \frac{dT}{dr} \right) = 0 \]  

(59)

The rate at which energy is conducted across any cylindrical surface in the solid may be expressed as

\[ q = -K A \frac{dT}{dr} = -K(2\pi r L) \frac{dT}{dr} \]  

(60)

where \( A = 2\pi r L \) is the area normal to the direction of heat transfer.

The thermal resistance is

\[ R_{\text{eff}} = \frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi KL} + \frac{1}{h_2 2\pi r_2 L} \]  

(61)

which includes both conduction and convection.

The heat transfer rate for a unit length of the cylinder therefore is

\[ q = \frac{T_{\infty,1} - T_{\infty,2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi KL} + \frac{1}{h_2 2\pi r_2 L}} \]  

(62)
Table 2: Analogous quantities for heat transfer in a radial system.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Heat transfer</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_1$</td>
<td>$(2\pi r_1 h_1) \left( \frac{D_1}{S_1} \right)$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$\left( \frac{2\pi K}{\ln(r_2/r_1)} \right) \left( \frac{D_2}{S_2} \right)$</td>
</tr>
<tr>
<td>$E_3$</td>
<td>$(2\pi r_2 h_2) \left( \frac{D_2}{S_2} \right)$</td>
</tr>
</tbody>
</table>

The structural analog for the cylinder is shown in Fig. 15(b). In this case, the mechanical structure equivalence is $k_{eff} \leftrightarrow \frac{1}{R_{eff}}$. The corresponding equivalent quantities are listed in Table 2. The $E$ values are input to NESSUS structural analysis, and the output displacements from NESSUS give the temperature distribution.

**Numerical example for heat transfer solved with NESSUS/FEM**

Fig. 17 shows the sectional view of the cylindrical copper heat exchanger which the engine oil flows through. The copper wall thickness is 0.281 in. The radius to the surface of the insulation pipe covering ($k_i = 0.428\text{Btu}/(h - \text{ft}^2 - ^\circ F)$) is 1.33 in. The fluid in the outer container is controlled at a constant temperature of $70^\circ F$. The forced convection heat transfer occurs between the outer surface of the insulation covering and the flowing fluid with $h = 10\text{Btu}/(h - \text{ft}^2 - ^\circ F)$. The surface temperature at the insulation covering is $35.298^\circ F$. The structural analogy model is used to determine the inside temperature of the tube, assuming steady state, one dimensional, uniform properties in each material, forced convection cooling and negligible thermal radiation. The conductivity coefficient of copper at room...
Figure 16: Analogous model for the heat exchange

The thermal circuit is shown in Fig. 16(a). Fig. 16(b) shows the structural analogous model for this heat transfer problem. Beam element type 98 in NESSUS/FEM element type library is adopted. Three elements represent three heat transfer forms involved in this problem, which are the forced convection between the surface of the insulation covering and the ambient air, conduction through the copper layer, and conduction through insulation covering, respectively. Therefore, in terms of the structural model, we must assign three different material elastic constants for this beam structure. Since the NESSUS/FEM utilizes the Nodal-based data input, two duplicate nodes are used at each boundary between elements 1 and 2, and between elements 2 and 3. The room temperature $70^\circ F$ becomes the boundary displacement.
70.0 at node 1. A concentrated load F at point 3 is -241.66 lb. The length of the structural element is 10.0 units, the sectional area is 1.0. It should be noted that the units used in the structural model here do not have any real meaning in terms of a real structure. They are simply used to facilitate the structural analysis.

The equivalent values are calculated as follows.

\[ E_1 = \left( \frac{2\pi K_c}{\ln(r_2/r_1)} \right) \left( \frac{D_1}{S_1} \right) = \frac{2\pi \times 223}{\ln(0.95/0.669)} \times 10 = 39955.476 \] (63)

\[ E_2 = \left( \frac{2\pi K_i}{\ln(r_0/r_2)} \right) \left( \frac{D_2}{S_2} \right) = \frac{2\pi \times 0.428}{\ln(1.33/0.95)} \times 10 = 79.923 \] (64)

\[ E_3 = \left( 2\pi r_0 h \right) \left( \frac{D_3}{S_3} \right) = 2\pi \times 11.33/12 \times 10.0 \times 10 = 69.63867 \] (65)

\[ F = 2\pi r_0 h(T_\infty - T_3) = 2\pi \times 1.33/12 \times 10(35.298 - 70) = -241.66 \] (66)
NESSUS/FEM uses this data, and gives the output of the displacement distribution in the structure as follows:

\[ U_1 = 70.000 \]
\[ U_2 = 35.298 \]
\[ U_3 = 35.298 \]
\[ U_4 = 5.0614 \]
\[ U_5 = 5.0614 \]
\[ U_6 = 5.0009 \]

Converting the above displacement information to the equivalent temperature distribution, we obtain:

\[ T_\infty = 70.000^\circ F \]
\[ T_1 = 35.298^\circ F \]
\[ T_2 = 5.0614^\circ F \]
\[ T_3 = 5.0009^\circ F \]

The data and the output files are shown in Appendix A.

**Fluid flow analysis through structural analogy**

Equation of motion for fluid flow

The Bernoulli equation gives a relationship between pressure, velocity, and position or elevation in a flow field. Normally, these properties vary considerably in the flow, and the relationship between them if written in differential form is quite complex. The equation can be solved exactly only under very special conditions. There-
fore, in most practical problems, it is often more convenient to make assumptions to simplify the descriptive equations. The Bernoulli equation for steady, incompressible flow along a streamline with no friction (no viscous effects) is written as [10]

$$\frac{p}{\rho} + \frac{V^2}{2} + gz = C$$

(67)

where

- $p$ is fluid pressure
- $\rho$ is the density of the fluid
- $V$ is the flow velocity
- $g = 32.174 \text{ ft/s}^2$, and $z$ is height.

For a horizontal pipe shown in Fig. 18, $z_1 = z_2$. From continuity, $A_1V_1 = A_2V_2$. Because $D_1 = D_2$, then $A_1 = A_2$, and therefore, $V_1 = V_2$. The Bernoulli's equation reduces to

$$p_1 = p_2$$

(68)
This result is not a proper description of the situation, however. For flow to be maintained in the direction indicated in Fig. 18, $p_1$ must be greater than $p_2$ in an amount sufficient to overcome friction between the fluid and the pipe wall. In order to apply Bernoulli's equation and obtain an accurate description, we must modify the equation with a friction term.

Consider flow in a pipe as shown in Fig. 19. A control volume that extends to the wall (where the friction force acts) is selected for analysis.

Note that a circular cross section is illustrated, but the results are general until we substitute specific equations for the geometry of the cross section. The forces acting on the control volume are pressure normal to the surface and shear stress acting at the wall. The momentum equation is [10]

$$\sum F_z = \int \int V_z \rho V_n dA$$

(69)
where

\[ V_z \] is the fluid velocity along the longitudinal direction

\[ V_n \] is the nominal fluid velocity

Since the flow out of the control volume equals the flow in, the right-hand side of this equation is zero. The sum of the forces is

\[
pA - \tau_w Pdz - (p + dp)A = 0
\]

(70)

where

\[ A = \text{cross-sectional area} \]

\[ Pdz = \text{the surface area (perimeter times length) over which the wall shear } \tau_w \text{ acts} \]

The equation reduces to

\[
\tau_w Pdz + Adp = 0
\]

(71)

Rearranging and solving for pressure drop, we get

\[
\frac{dp}{dz} = -\frac{4\tau_w}{D_h}
\]

(72)

We have thus expressed the pressure drop per unit length of the conduit in terms of the wall shear and the hydraulic diameter. Eq. 72 is a general expression for any cross section. It is convenient to introduce a friction factor \( f \), which is customarily defined as the ratio of friction forces to inertia forces:

\[
f = \frac{4\tau_w}{\frac{1}{2} \rho V^2}
\]

(73)
where $V$ is the average flow velocity.

By substitution into Eq. 72, we obtain

$$dp = -\frac{\rho V^2 f dz}{2D_h}$$

(74)

Integrating this expression from point 1 to point 2 a distance $L$ apart in the conduit yields

$$V^2 = \frac{2D_h}{\rho f L} \Delta p$$

(75)

Eq. 75 gives the relationship between the velocity and the pressure drop in the duct due to friction. This equation can be applied to two flow regimes - laminar and turbulent flow. However, caution must be exercised when determining the friction factor $f$.

This equation can also be applied to flow through noncircular cross section such as rectangular duct and annulus. Fig. 20 shows the laminar flow in an annulus.
The annulus flow area is bounded by the inside surface of the outer duct (radius \( R_1 \)) and the outside surface of the inner duct (\( R_2 \)). We define the ratio of these diameters as

\[
k = \frac{R_2}{R_1}
\]  

in which \( 0 < k < 1 \).

The friction factor used in Eq. 75 is defined as [10]

\[
\frac{1}{f} = \frac{R_e}{64} \left[ \frac{1 + k^2}{1 - k} + \frac{1 + k}{\ln(k)} \right]
\]

where \( R_e = \frac{\rho V^2}{\mu} (1 - k) \).

Compare Eq. 75 with Eq. 38 concerning the beam structure subjected to the end nodal force, as discussed in previous section:

\[
f_1 = \frac{ES}{D} (U_1 - U_2)
\]

We are now able to set up the analogous quantities listed in Table 3.
Numerical example of flow in a tube solved with NESSUS/FEM

Consider the refrigerant flow in a copper tube as an example to demonstrate how NESSUS/FEM can be applied to problems in fluid mechanics.

A horizontal copper duct as shown in Fig. 18 with inside radius of 0.669 in, and 1,200 in in length. If the inflow pressure $p_1$ is 1838.7 psi, assuming the refrigerant is Freon F-12 under a temperature of 5° F, $\rho$ is 0.0499 lb/in$^3$. The friction factor $f$ is assumed to be 0.03, and $V$ is 15.5 in/sec. The objective is to obtain the outflow pressure $p_2$ using NESSUS/FEM.

First of all, we need to identify all the equivalent quantities for structural analysis. We assume a single element beam structure subjected to a concentrated force equal to -240.25 units. The beam element has a section of 0.1 in $\times$ 0.1, and a length of 1.0. The boundary condition is an initial displacement of 1838.3 units at node 1. Again, it should be mentioned that the units used here do not have real meaning in terms of a real structure. According to Table 3, the analogous quantities can be obtained as follows

$$E = \left(\frac{2D_h}{\rho fL}\right) \left(\frac{D}{S}\right) = \frac{2 \times 2 \times 0.669}{0.0499 \times 0.03 \times 1200} \times 10 = 14.896$$

$$f_1 = V^2 = 15.5^2 = 240.25$$

$$F = -240.25$$

This data is input to NESSUS/FEM, and the displacement at point 2 is obtained as 1677.4. Converting this displacement to the fluid model, we get the output pressure $p_2 = 1677.4$ psi.

The data and the output files are shown in Appendix B.
CHAPTER IV

MULTI-DISCIPLINARY SYSTEM RELIABILITY ANALYSIS

Introduction

After transforming the heat transfer and fluid mechanics problems into corresponding structural analog models and using NESSUS/FEM to perform the finite element analysis, we can define the individual failure modes in NESSUS/FPI. Then NESSUS/PFEM can be employed to integrate FEM and FPI programs to obtain the failure probability and CDF for each failure mode. The failure mode for heat transfer problem would be defined as, for example, the event that the temperature at a certain location is lower or higher than the required temperature. The failure mode for fluid flow would be defined as the flow pressure exceeding a certain pressure level, and the structural failure is defined as the stress exceeding either the ultimate strength or the yield strength of the material.

Upon the completion of failure probability analyses of individual failure modes, the system failure analysis can be pursued. The different failure modes involved in a system have different impacts on the overall performance of a system. Some types of failure such as structural failure are critical to the system. If the material used to construct the main parts of the system fails, the whole system can no longer function. Such failure is called critical failure. Other failures modes such as thermal failure of a heat exchanger do not destroy the system but degrade the performance of the system.
Such failure is referred to as functional failure. The function of fluid flow will fail when the outflow pressure rises higher than the designed value, but the system can still be working until the pressure increases to the level which will cause the system to shut down. The individual failure modes can also be correlated to each other. For example, the temperature field in the thermal failure mode affects the stress field in the structural mode. The flow pressure definitely has impact on the stress. However, in some cases, the component-level events in a system is considered as independent events. In the example which will be discussed later in this chapter, the thermal failure and fluid flow failure modes do not share correlated input parameters, so they are considered as independent of each other.

Using the analogy method, the thermal and fluid flow problems are analyzed similar to the structural model by means of NESSUS. For physically correlated events, the failure mode of one event is imposed on the other. In a system consisting of structural, thermal and fluid flow modes, the thermal and flow failures are imposed into the structural failure analysis to study the impact of correlated events. The failure probability of the whole system is then estimated based on the output from the above analyses.

**Individual failure analysis**

**Structural failure mode**

For a copper duct of a heat exchanger shown in Fig.17 in Chapter III, the structural failure mode is defined as that when the tensile stress exceeds the yield strength $f_{yield} = 8.0$ ksi. In finite element modeling, we use Element type 153 in the NESSUS/FEM file. This element is a four-noded quadrilateral lying in the global
$zr$-plane which is defined by cylindrical coordinates.

This structure is subjected to two types of load - fluid pressure from the inside flow and the stress caused by temperature difference between the outer surface and the inner surface.

One convenient feature of the NESSUS/PFEM is that we can impose different temperatures at the inner surface of the pipe and obtain the different probability results under different temperature conditions. This enables us to investigate the effect of different temperature levels on the structural failure probability. By doing so, the relationship between the failure modes in two disciplines - heat transfer and structural mechanics - is established. This is a significant step toward the system reliability analysis with physically correlated failure modes. This will be demonstrated in a later section.

For this structural failure model, we first suppose that the temperature failure (which will be described in the following section) did not occur, that is, the temperature at the inner surface of the duct is below $5.0009^\circ F$. Given that the inner surface temperature is $4.0000^\circ F$, using the FEM we obtain the outer surface temperature as $4.0605^\circ F$.

Also, we assume that the outlet flow pressure is under 1677.4 psi which enables the system to work properly. We assume that the outlet flow pressure is 1577.4 psi.

We input this temperature and flow pressure profile in the structure FEM data file, with the random variables defined in Table 4.
Table 4: Random variables for structural model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean value</th>
<th>Distribution</th>
<th>C.O.V.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure P</td>
<td>1577.4 psi</td>
<td>Normal</td>
<td>0.2</td>
</tr>
<tr>
<td>Modulus of elasticity E</td>
<td>1.7×10^7 psi</td>
<td>Normal</td>
<td>0.1</td>
</tr>
<tr>
<td>Coefficient of thermal expansion α</td>
<td>9.5×10^-6</td>
<td>Normal</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 5: CDF corresponding to different tensile strength levels

<table>
<thead>
<tr>
<th>Z-level (strength)</th>
<th>CDF</th>
<th>Z-level (strength)</th>
<th>CDF</th>
</tr>
</thead>
<tbody>
<tr>
<td>-206.89941 psi</td>
<td>0.00000017</td>
<td>4385.4964 psi</td>
<td>0.81593991</td>
</tr>
<tr>
<td>558.49989 psi</td>
<td>0.00002067</td>
<td>5150.8957 psi</td>
<td>0.97128351</td>
</tr>
<tr>
<td>1323.8992 psi</td>
<td>0.00096767</td>
<td>5916.2950 psi</td>
<td>0.99813412</td>
</tr>
<tr>
<td>2089.2985 psi</td>
<td>0.01786435</td>
<td>6681.6943 psi</td>
<td>0.99995188</td>
</tr>
<tr>
<td>2854.6978 psi</td>
<td>0.13566610</td>
<td>7447.0935 psi</td>
<td>0.99999952</td>
</tr>
</tbody>
</table>
The result attached in Appendix C indicates that the structural reliability when
the heat exchanger is working properly in thermal and fluid aspects is 0.99999999.
The failure probability is expressed as $1 - P_{\text{reliability}}$. Therefore, the structural failure
probability is $1.0 \times 10^{-9}$.

The key word \textit{response type} in the NESSUS input data file, FPI section, is set
equal to 3 which means that the response quantity used in limit state function is
stress. The corresponding keyword \textit{analysis type} in FPI section is first set equal to
1 which means that the probability analysis is for a single Z-level. The Z-level in this
case is 8,000 psi. The probability result will be under the condition of $\sigma < 8,000$
psi, i.e., the structural reliability of the system under certain thermal and fluid flow
working conditions.

The CDF is obtained by using PFEM by setting \textit{analysis type} in FPI section
equal to 0 which automatically generates a set of different values of $Z_0$ (i.e., Z levels
for a series of stress valus) for probability analysis. The CDF values corresponding
to different strength Z-levels are shown in Table 5.

The CDF chart is shown in Fig. 21. It should be noted that the first line of
the data which contains negative Z-level is eliminated because negative stress is
considered impractical in this model. The input and output files are attached in
Appendix C as well.
Figure 21: CDF of structural reliability of refrigerant duct

Table 6: Random variables for thermal model

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean Value</th>
<th>Distribution</th>
<th>Coefficient of Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_e$</td>
<td>223.0</td>
<td>Normal</td>
<td>0.1</td>
</tr>
<tr>
<td>$K_i$</td>
<td>0.428</td>
<td>Normal</td>
<td>0.1</td>
</tr>
<tr>
<td>$h$</td>
<td>11.3</td>
<td>Normal</td>
<td>0.1</td>
</tr>
</tbody>
</table>
Figure 22: CDF of internal fluid temperature of refrigerant duct
Using the same example of a heat exchanger as in Fig. 17, we define a failure event when the inside temperature is higher than 5°F, because the refrigerant will not function properly beyond 5°F which is considered as a failure in the device we studied. First the input data for NESSUS/PFEM is set up to obtain the reliability under this failure mode, then the data file is set up with different Z-levels to obtain the CDF, which provides reliability estimate corresponding to different temperature levels. The random variables $K_c$ and $K_i$ and $h$ for heat transfer are defined in Table 6.

In order to use NESSUS/PFEM, the analogous quantities $E_1$, $E_2$, $E_3$ and $F$ are calculated from Eqs. 63, 64, 65, 66. Because the distribution of the random variables $K_c$ and $K_i$ and $h$ is normal and $E_1$, $E_2$, $E_3$ are linear to $K_c$ and $K_i$ and $h$, the distribution of random variables $E_1$, $E_2$, $E_3$ is also normal. The mean values and standard deviations of of $E_1$, $E_2$ and $E_3$ are input to NESSUS/PFEM.

The $Z$-level is 5.0°F, so $P(Z < Z_0)$ is the probability the device can keep the inside fluid temperature under 5.0°F, which is the thermal reliability of the system. We set up the keyword in FPI section analysis type equal to 1 which means the probability analysis is performed for a single Z-level. The result is attached in Appendix D. The thermal reliability of this device is 0.9099214. Therefore, the thermal failure probability is $9.00786 \times 10^{-2}$.

The CDF is obtained by setting up the FPI keyword analysis type equal to 0 which automatically generates a set of different values for $Z_0$ (i.e., Z-levels). The
Table 7: Random variables for flow model

<table>
<thead>
<tr>
<th></th>
<th>Mean value</th>
<th>Distribution</th>
<th>Coefficient of variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_h$</td>
<td>0.669</td>
<td>Normal</td>
<td>0.1</td>
</tr>
<tr>
<td>$V$</td>
<td>18.71</td>
<td>Normal</td>
<td>0.1</td>
</tr>
</tbody>
</table>

CDF output is shown in Appendix D and the CDF curve is shown in Fig. 22.

**Failure mode in fluid flow**

Next we consider the one-dimensional fluid flow in a duct of a heat exchanger. The failure mode is defined as the pressure at a certain point along the duct rising above the value at which the system cannot function properly.

The example of a duct in a heat exchanger shown in Fig. 16 is used. The only difference is that $V$ is assumed to be 18.7 in/sec. We define the failure mode when pressure rises above 1677.4 psi. The $Z$-level is therefore 1677.4 psi. The keyword *response type* is set as 1 for the displacement output which is the analogy of the pressure. The random variables related to fluid flow are defined in Table 7. The analogous quantities for use in NESSUS/PFEM are calculated according to Table 3 as follows:

$$E = \left( \frac{2D_h}{\rho f L} \right) \left( \frac{D}{S} \right) = \frac{2 \times 2 \times 0.669}{0.0499 \times 0.03 \times 1200} \times 10 = 14.896$$

$$f_1 = V^2 = 18.7^2 = 350.0$$

$$F = -f_1 = -350.0$$
Figure 23: CDF of fluid pressure of refrigerant in the duct
Since $E$ is a linear function of $D_h$, its distribution is also normal. But since $f_1 = V^2$, the distribution of $f_1$ is actually chi-square ($\chi^2$). However, we have used the normal distribution for $f_1$ in this study as an approximation. The friction factor $f$ is assume to be a constant.

A NESSUS/PFEM input data file is compiled. The reliability is obtained by setting the keyword analysis type equal to 1, and the CDF is obtained by setting it to a value of 0 which automatically generates a set of different values for $Z_0$. The PFEM input and output files are shown in Appendix E and the CDF curve is shown in Fig. 23.

The reliability is 0.98670241 and therefore the failure probability of the output flow pressure being higher than 1677.4 psi is $1.329759 \times 10^{-2}$.

**Multi-disciplinary system reliability**

After the individual failure modes are identified and analyzed, the system reliability analysis can be pursued. Fig. 24 shows a device which is used to transfer refrigerant fluid through a copper duct. The duct is installed in an enclosed chamber which is maintained at a constant temperature of 70°F. The thickness of the copper wall is 0.281 in. The radius to the surface of the insulation pipe covering (mean value of $K_i$ equals to 0.428 $Btu/(h \cdot ft^2 \cdot \circ F)$, c.o.v. equals to 0.1 ) is 1.33 in. Forced convection heat transfer occurs with $h = 10Btu/(h \cdot ft^2 \cdot \circ F)$ (mean value with c.o.v. equals to 0.1). The thermal conductivity of copper $K_c$ has a mean value of 233.0 $Btu/(h \cdot ft^2 \cdot \circ F)$ with a c.o.v. 0.1. The surface temperature at the insulation covering is 35.298°F. The inflow pressure $p_1$ is designed to be 1838.7
psi. The refrigerant is Dichlorodifluoromethane (Freon F-12) under a temperature of 4°F. \( \rho \) is 0.0499 lb/in\(^3\). The friction factor \( f \) is assumed to be 0.03, and \( V \) is 18.71 in/sec (assuming \( V^2 \) is normal distribution and has a c.o.v. of 0.1). The inner radius of the duct has a mean value of 0.669 in and a c.o.v. of 0.1.

The above data are the same as the model shown in Fig. 17 of Chapter III, which are used in FEM analysis and reliability estimation of individual failure modes. The system shown in Fig. 24 is simply the combination of the previous individual models which have been analyzed in different disciplines. The following is to demonstrate how the analysis results of the individual failure modes can be integrated into the analysis of a whole system.

The failure of the system consists of the individual failure modes in three disciplines: structural failure, thermal failure and fluid mechanics failure.

- First of all, the duct should work without any damage to the structure, i.e. the duct should be structurally sound without yied or crack. If yield occurs, then structural failure is assumed to occur. We denote the structural failure as \( E_1 \). The structural failure is a critical failure in this system.

- The refrigerant liquid this device transports is sensitive to temperature changes. The requirement is that the temperature cannot be higher than 50°F for the next process to proceed. If the temperature of the liquid rises higher than 50°F, then thermal failure occurs, which we refer to as \( E_2 \). The thermal failure is a non-critical functional failure in this system.

- It is required that the fluid flow be maintained at a certain pressure at the
Thermal chamber

Input valve

Outlet to the next process

Chamber with consistent temperature

Insulation covering

Copper duct

P1

P2

70°F

Figure 24: Refrigerant duct through a chamber
ends which enables the refrigerant fluid to maintain a steady speed to provide constant volume in the device. If the flow pressure rises higher than 1677.4 psi, the flow failure occurs, which is referred to as $E_3$. The flow failure is a non-critical functional failure in this system.

From the previous section, the probability of the individual failure modes of the system shown in Fig. 24 have been obtained, which are:

\[
P(E_1) = 1.0000000 \times 10^{-9}
\]
\[
P(E_2) = 9.0078600 \times 10^{-2}
\]
\[
P(E_3) = 1.3297590 \times 10^{-2}
\]

As was indicated in the previous section, we can also impose one failure mode upon the other. In this case we can impose the thermal failure (which happens when the fluid temperature rises above $5^\circ F$) and the fluid pressure failure (which happens when the outlet fluid pressure rises above 1677.4 psi) upon the structure respectively. In the FEM file for the structure model, the corresponding data are modified to impose those failures.

First, we assume that the temperature failure occurs while the fluid pressure is still lower than 1677.4 psi, say 1577.4 psi, i.e., the fluid flow is operation in safe mode. By redefining the temperature profile in the FEM data deck as $6.06^\circ F$ (for example) in the inner layer of the wall and $6.12^\circ F$ in the outer layer of the wall, which mean the thermal failure occurs, we impose the thermal failure to the structural model. The PFEM result gives us the structural failure probability under the condition that the thermal failure occurs. In this case, the structural reliability is 0.999, therefore.
the failure probability $P(E_1/E_2) = 4.0 \times 10^{-8}$.

Now we impose the fluid pressure failure upon the structural model. The fluid mechanical failure occurs when the pressure at the outlet rises above 1677.4 psi, say 1777.4 psi. The structural FEM file is modified by redefining the pressure profile according to this failure pressure. It should be noted that the temperature profile should remain under the normal working condition, which is that the temperature in the inner layer of the wall of duct is under 5°F, say 4°F, i.e., the thermal aspect of the system is operating in the safe zone. The result indicates that in this case, the structural reliability is 0.99999633, therefore, the conditional probability, $P(E_1/E_3) = 3.67 \times 10^{-6}$.

Next, both thermal and fluid mechanical failures are imposed that is, the fluid temperature rises above 5°F, and the outlet flow pressure rises above 1677.4 psi. Modifying the input FEM data deck in structural PFEM file with inner surface temperature of 6.06°F, and the fluid pressure of 1777.4 psi, we can get the result of the structural reliability of 0.99999227, which means, $P(E_1/E_2E_3) = 7.73 \times 10^{-6}$.

The conditional probabilities of structural failure have been obtained as

\[
P(E_1/E_2) = 4.00 \times 10^{-8};
\]

\[
P(E_1/E_3) = 3.67 \times 10^{-6};
\]

\[
P(E_1/E_2E_3) = 7.73 \times 10^{-6};
\]

**System reliability computation**

System reliability analysis can be performed in two different ways, depending on the definition of the system failure. In the first (traditional) method, we define that
the system failure occurs when any component-level failure occurs. In the system involving structural, thermal, fluid mechanical failure modes, i.e., $E_1$, $E_2$ and $E_3$, the system failure can be illustrated in a fault tree shown in Fig. 25.

As discussed in Chapter II, the probability of system failure $P(E)$ can be obtained using the following equations:

$$P(E) = P(E_1 \cup E_2 \cup E_3)$$

The above expression can be expanded as:

$$P(E) = P(E_1) + P(E_2) - P(E_1 \cap E_2) + P(E_3) - P(E_1 \cap E_3) - P(E_2 \cap E_3) + P(E_1 \cap E_2 \cap E_3)$$

Since the joint probability is not always available, an approximate method is to consider the individual failure modes as independent and ignore the correlations. In our case however, the conditional probabilities have been calculated. Therefore, the
system failure probability can be computed as

\[
P(E) = P(E_1) + P(E_2) + P(E_3) - P(E_1/E_2)P(E_2) - P(E_1/E_3)P(E_3) \\
- P(E_2/E_3)P(E_3) + P(E_1/E_2E_3)P(E_2E_3)
\]  

(81)

For systems involving many failure modes, approximation methods are used to predict the system failure probability (or reliability), such as first-order bounds or second-order bounds [1].

Since no correlation is assumed between the failures of flow pressure and fluid temperature, we assume \( P(E_2 \cap E_3) \) is equal to \( P(E_2)P(E_3) \) in our analysis.

Substituting the numerical results from the previous discussion into Eq. 81, we obtain the probability of the system failure of the heat exchanger, \( P(E) = 0.10217832 \).

As mentioned before, the above failure probability is an estimation of system failure in case any failure occurs which includes both critical and non-critical functional failures. Now we will pursue the probability estimation for the system critical failure which, in our case, is structural failure. During the service cycles, the thermal and fluid mechanical failures may be non-critical, i.e., their occurrence does not cause total system failure. They will cause the system to fail in some functions as designed, such as keeping the fluid under certain temperature or keeping the outlet fluid pressure under certain value. However, if the system keeps operating, the changes in temperature and fluid pressure will cause progressive damage to the structure due to load redistribution. The estimation of critical structural failure of the system has to consider the progressive damage caused by all components. Structural reanalysis
Table 8: Structural failure probability under various temperatures

<table>
<thead>
<tr>
<th>T(°)</th>
<th>0.0</th>
<th>5.0</th>
<th>10.0</th>
<th>15.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(E_1)$</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

is used to account for the effect of non-critical damage on critical failure mode. In the refrigerant model, a reanalysis procedure is performed to accurately estimate the failure region segments for structural failure mode affected by progressive damage caused by thermal and fluid pressure changes within the system. The overall structural failure probability is obtained through the union of the failure region segments defined by each limit-state function.

We can also impose a series of temperatures under which the system may be operating upon the structural model to examine the temperature impact on the structural failure probability. Just as we did before, the failure probability is obtained as $(1 - \text{Reliability})$. In this case, we still assume inner fluid pressure is 1577.4 psi, which means that the fluid flow mode of the system is operating in the safe zone. The results are shown in Table 8.

Table 8 shows that when the fluid pressure is not considered as a random variable in perturbation for probability analysis, the temperature changes do not have a significant impact on structural reliability of the system.

We can also get the structural failure probability under different pressure conditions by defining a series of the pressure profiles in the FEM data deck for structural
Table 9: Structural failure probability under various pressures

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>2700.0</th>
<th>2900.0</th>
<th>3000.0</th>
<th>3200.0</th>
<th>3300.0</th>
<th>3400.0</th>
<th>3420.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(E_1)$</td>
<td>0.000</td>
<td>1.000</td>
<td>1.920</td>
<td>7.687</td>
<td>9.195684</td>
<td>4.06468</td>
<td>4.92622</td>
</tr>
<tr>
<td></td>
<td>$\times 10^{-9}$</td>
<td>$\times 10^{-6}$</td>
<td>$\times 10^{-3}$</td>
<td>$\times 10^{-2}$</td>
<td>$\times 10^{-1}$</td>
<td>$\times 10^{-1}$</td>
<td>$\times 10^{-1}$</td>
</tr>
</tbody>
</table>

model. The results are listed on Table 9. In this case, we assume that the inner temperature is 4°F.

It should be noted that the NESSUS/PFEM input file $gfun.dat$ is different from the previous structural PFEM file in which the pressure is defined as a random variable. In the program [Ang and Tang, 1984] to calculate the union of the region segments, the random variables once defined can not be changed for different limit states. Since in the pressure profile the different pressure levels are presented, pressure should not be defined as a random variable. Therefore only two random variables are involved in $gfun.dat$ - modulus of elasticity $E$ and coefficient of thermal expansion $\alpha$. The $gfun.dat$ and various $gfun.mov$ files are shown in Appendix F.

There are two ways of quantifying the effect of progressive damage on critical failure. The first is simply to compute the variation of critical failure probability with respect to progressive damage. This is shown in Fig. 26 for various pressure levels.

An alternate way is to compute the progressive damage on overall critical failure probability. If each critical failure limit state segment for each progressive damage gives the event $E_i$, then the overall critical failure probability is $P(\bigcup_{i=1}^{n} E_i)$, where $n$
Figure 26: Structural failure probability for various pressures.
is the number of limit states ( \( n = 7 \) in this case).

The output files \( gfun.mov \) provide g-functions defining different limit-states for each level of damage as

\[
g_i = \beta_i + \sum_{i=1}^{n} \alpha_i u_i^*
\]

(82)

From Table 8, it is clear that temperature variations do not have any significant effect on the structural failure probability. Therefore, only pressure variations are considered as follows:

The parameters provided by \( gfun.mov \) for the structural limit states corresponding to fluid pressure profile are as follows:

<table>
<thead>
<tr>
<th>Pressure (psi)</th>
<th>( \beta )</th>
<th>( \alpha_1 )</th>
<th>( \alpha_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2700.0</td>
<td>6.825696</td>
<td>0.999887</td>
<td>-0.015057</td>
</tr>
<tr>
<td>2900.0</td>
<td>5.721588</td>
<td>0.999886</td>
<td>-0.015088</td>
</tr>
<tr>
<td>3000.0</td>
<td>4.619647</td>
<td>0.999886</td>
<td>-0.015071</td>
</tr>
<tr>
<td>3200.0</td>
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<tr>
<td>3300.0</td>
<td>1.328801</td>
<td>0.999887</td>
<td>-0.015021</td>
</tr>
<tr>
<td>3400.0</td>
<td>0.236641</td>
<td>0.999888</td>
<td>-0.014959</td>
</tr>
<tr>
<td>3420.0</td>
<td>0.018494</td>
<td>0.999887</td>
<td>-0.015003</td>
</tr>
</tbody>
</table>

The above data provides parameters for 7 g-functions. Using the above data to calculate the union of the region defined by a group of g-functions [Ang and Tang, 1984], we obtain the probability of structural failure involving the progressive damage caused by fluid pressure. The probability is defined by a lower and upper bounds, which in this case are both 0.492611. The data and output files are shown in
Appendix G. Those bounds provide the overall structural failure estimation when the system experiences various levels of progressive damage. The fluid pressure changes in a range from 2700.0 psi to 3420.0 psi.

The above two methods provide practical tools for multi-disciplinary system reliability estimation using NESSUS. With multiple impositions of one mode on the other mode, a close approximation to the failure domain can be constructed, and the critical failure probability can be obtained through the union of the failure region defined by the various limit-states.
CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusion

This report has demonstrated the application of equivalence concepts to the reliability analysis of multi-disciplinary systems using NESSUS. A thermal-structural-fluid system is used to illustrate the proposed methodology. The analogous model is a very powerful tool to analyze the one-dimensional steady state problem in heat transfer and fluid mechanics by converting those models into a structural model. Then the NESSUS probability analysis program can be implemented and the precise system reliability can be evaluated. Both traditional and progressive system failure probability methods using NESSUS provide practical tools for multi-disciplinary system reliability analysis.

Recommendations for future research

This research project demonstrated how the NESSUS program could be applied for reliability analysis of engineering systems involving different disciplines, such as structure, heat transfer and fluid mechanics. The current models are based on the condition of one-dimensional, steady-state for both heat transfer and fluid mechanics. More complex systems could be treated in the similar way. However, the scope of application of this methodology is largely dependent on the ability of NESSUS/FEM to deal with problems in different disciplines under more complicated situations, for
instance, a thermal or a flow model in two or three dimensions and non-steady state
conditions. For more sophisticated systems, the need for separate FEM program
may be inevitable. Either a new source code should be developed or the existing
commercial softwares could be integrated into the program. Several users of NESSUS
have already integrated its FPI module to other FEM analysis program such as
ANSYS and NASTRAN. Nevertheless, the use of equivalent concepts helps to obtain
a quick estimate of multi-disciplinary system reliability through the use of NESSUS.

The idea of progressive damage imposition to quantify nonlinear system reliability
effects has previously been applied to structural mechanics problems [1]. This study
extends this concept to multi-disciplinary systems. This appears to be a practical
methodology for system reliability analysis when failure modes (even in different
disciplines) have physical relationship with each other. The methodology should be
pursued further for application to other, more complicated engineering systems.
Appendix A

FEM FILES FOR HEAT TRANSFER OF A HEAT EXCHANGER

(Refer to Chapter III, Fig.16)

NESSUS/FEM input file

exp2fem.dat  Wed May 14 20:37:04 1997  1

*FEM
C ... HEAT TRANSFER FOR A REFRIGERANT PIPE
C
C ... PARAMETER DATA
C
*BOUND 20
*CONE 0
*DEPF
*MODE 6
*EXPL 4
*ELMN 3
**
*FORC 1
*FRER 4 4
*FRIN
*BRAM
*END
** MODEL DATA
**
*COOR
  1 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
  2 0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
  3 0.0000 12.0000 0.0000 0.0000 0.0000 1.0000
  4 0.0000 20.0000 0.0000 0.0000 0.0000 1.0000
  5 0.0000 30.0000 0.0000 0.0000 0.0000 1.0000
  6 0.0000 40.0000 0.0000 0.0000 0.0000 1.0000

*ELMN 98
  1 1 2
  2 3 4
  3 5 6

*DENT
  1 2
  3 4

*BOUND
  6 1 0.0000
  1 1 0.0000
  1 2 10.0000
  1 3 0.0000
  1 4 0.0000
  1 5 0.0000
  1 6 0.0000

*BRAM 1
  1 6 1.0000 1.0000

*ITERA 0 1
  0 0.0500

*PROP 99
  1 2 1.0000 49.63687 0.3000 0.0000 2.58-4
  3 4 1.0000 79.3230 0.3000 0.0000 2.58-4
  5 6 1.0000 39955.5476 0.3000 0.0000 2.58-4

*FORC 6 2 -241.44

*FIN
  TOTAL MODE

*END
NESSUS/FEM output file

exp2fem.out Wed May 14 20:50:14 1997 1

DATE: 14-5-1997 20:50 - LEVEL 6.0 (109) - DATED June, 30, 1992

C ... HEAT TRANSFER FOR A REFRIGERANT PIPE

*********************************************************
** PERTURBATION DATA 
*********************************************************

<table>
<thead>
<tr>
<th>VARIABLE NO.</th>
<th>MEAN VALUE</th>
<th>STANDARD DEVIATION</th>
</tr>
</thead>
<tbody>
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<td>0.00000D+00</td>
<td>0.00000D+00</td>
</tr>
<tr>
<td>2</td>
<td>0.00000D+00</td>
<td>0.00000D+00</td>
</tr>
<tr>
<td>3</td>
<td>0.00000D+00</td>
<td>0.00000D+00</td>
</tr>
<tr>
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<td>0.00000D+00</td>
</tr>
<tr>
<td>5</td>
<td>0.00000D+00</td>
<td>0.00000D+00</td>
</tr>
<tr>
<td>6</td>
<td>0.00000D+00</td>
<td>0.00000D+00</td>
</tr>
</tbody>
</table>

PERTURBATION 4 HAS NO EFFECT ON THIS ANALYSIS

DATABASE UPDATE, INC. 10 PER. 4 ITER. 0 CPTIME= 0.31 SEC

HEAT TRANSFER FOR A REFRIGERANT PIPE

TOTAL DISPLACEMENTS: 0, TOTAL TRANSIENT TIME: 0.00000D+00

MODE COMP. 1 COMP. 2 COMP. 3 COMP. 4 COMP. 5
0 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
1 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
2 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
3 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
4 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
5 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00
6 0.00000D+00 0.00000D+00 0.00000D+00 0.00000D+00

HEAT TRANSFER FOR A REFRIGERANT PIPE

END OF INCREMENT, INC. 0 PERT. 0 ITER. 0 CPTIME= 0.34 SEC

HEAT TRANSFER FOR A REFRIGERANT PIPE

NASA/CR—2001-210969
Appendix B

FEM FILES FOR FLUID FLOW IN A DUCT

(Refer to Chapter III, Fig.18)

NESSUS/FEM input file


*FEM
C...FLOW IN A CONSTANT DIAMETER PIPE (WITH A/L=0.1)
C
C...PARAMETER DATA
C
*BOUND 10
*CONS 0
*DISP
*MODE 2
*ELSM 1
98
*FORC 1
*PRNT 4 4
*PRIN
*BEAM
*END
C.**** MODEL DATA
**COOR
1 0.0000 0.0000 0.0000 0.0000 1.0000
3 0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
ELEM 98 1 2
*BOUND
1 3 0.0000
2 3 0.0000
1 1 0.0000
1 2 0.1850
1 3 0.0000
1 4 0.0000
1 5 0.0000
1 6 0.0000
*BEAM 1
1 2 1.0000 1.0000
*ITERA 0 4
40 0.0000
*PROP 98
1 2 1.0000 14.8960 0.3000 0.0000 1.5E-4
*FORC 3
2 3 -240.25
*PRIN
*TOTA NODE
*END
NESSUS/FEM output file

flowfeml.out Sat Jun 28 22:27:00 1997

1 DATE: 28-6-1997 22:26 PAGE: 9

*******************************
** PERTURBATION NO. 1 **
*******************************

SUMMARY OF PERTURRATION DATA
*******************************

VARIABLE NO. 1 MEAN VALUE 0.00000D+00 STANDARD DEVIATION 0.00000D+00
PERTURBED VALUE 0.00000D+00
VARIABLE NO. 2 MEAN VALUE 0.00000D+00 STANDARD DEVIATION 0.00000D+00
PERTURBED VALUE 0.00000D+00
VARIABLE NO. 3 MEAN VALUE 0.00000D+00 STANDARD DEVIATION 0.00000D+00
PERTURBED VALUE 0.00000D+00
VARIABLE NO. 4 MEAN VALUE 0.00000D+00 STANDARD DEVIATION 0.00000D+00
PERTURBED VALUE 0.00000D+00
PERTURBATION ASSEMBLY, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.27 SEC
*** PERTURBATION 1 HAS NO EFFECT ON THIS ANALYSIS

DATABASE UPDATE, INCR. 0 PERT. 4 ITER. 0 CPTIME= 0.27 SEC
1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)
DATE: 28-6-1997 22:26 RESULTS PAGE: 3

TOTAL DISPLACEMENTS INCREMENT 0 TOTAL TRANSIENT TIME 0.00000D+00

MODE COMP. 1 COMP. 2 COMP. 3 COMP. 4 COMP. 5
COMP. 6
1 0.00000D+00 0.18307D+04 0.00000D+00 0.00000D+00 0.00000D+00
3 0.00000D+00 0.16714D+04 0.00000D+00 0.00000D+00 0.00000D+00
9 0.00000D+00

1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)
DATE: 28-6-1997 22:26 PAGE: 10

END OF INCREMENT, INCR. 0 PERT. 0 ITER. 0 CPTIME= 0.25 SEC
1C ... FLOW IN A CONSTANT DIAMETER PIPE (with A/L=0.1)

STOP DUE TO END OF INPUT FILE
Appendix C

PFEM FILES FOR STRUCTURAL RELIABILITY

(Chapter IV, Data is shown in Fig.16 of Chapter III)

NESSUS/PFEM input and output files for single Z-level

```
*PFEM
C ... PROBABILISTIC ANALYSIS FOR STRUCTURAL RELIABILITY OF EXP1
*INVERTIX
  1 2 3 4 5 6 7 8 9
*START
  1 2 3 4 5
*LOAD 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
*END

*DEFINE
  1 2 3 4 5

*DEFINE PRESSURE 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5

*DEFINE PROPERTY 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5

*DEFINE ELEM 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5

*DEFINE MODE 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5

*DEFINE TEMPERATURE 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5

*DEFINE COORDINATE 1 2 3 4 5
  0.1 0.2 0.3 0.4 0.5
```
Taylor series expansion of the form

\[ f(x) \approx A_0 + A_1 x + A_2 x^2 + \ldots + A_n x^n \]

Where:

- \( A_0 \) is the constant term
- \( A_1, A_2, \ldots, A_n \) are the design sensitivities
- \( x_1, x_2, \ldots, x_n \) are the random variables

---

**Taylor series coefficients**

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<th>A coefficient</th>
<th>Value</th>
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<td>-0.128338+00</td>
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<td>3</td>
<td>0.169730+07</td>
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**Random expansion coefficients**

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<th>Sensitivity</th>
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<tr>
<td>0.170000+08</td>
<td>0.361849+00</td>
</tr>
<tr>
<td>0.390000+05</td>
<td>0.787302-02</td>
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**Performing probabilistic analysis with PPI**

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<td>0.4000000000-04</td>
<td>0.961237658+01</td>
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---

NASA/CR—2001-210969
NESSUS/PFEM input and output file for CDF

```
EXP2SCDF.DAT Tue Jun 24 19:19:57 1997 1

*PFEM
C....PROBABILISTIC ANALYSIS FOR STRUCTURAL RELIABILITY OF EXP2
*PROCEDURE
  *CONDO 0
  *STATIST 0
  *RESPTTy 3
  *COMP 3
  *MODE 44
  *VENT 3
  1.2.3
  *PANVAR 3
  1.2.3
*END
*ENTRY
  *COMPUTATIONAL 1 3
1.3.3
*ENTRY
  *DEFINE
    *DEFINE 1
    PFR 0.157740000+04 0.315400000+00 NORMAL
    PRESSURE
    35 98 1.0
*DEFINE 2
END
0.170000000-08 0.170000000-07 NORMAL
PROP 153
  1 98 0.0 1.0 0.0 0.0 0.0
*DEFINE 3
ALPA
  0.993000000-05 0.993000000-05 NORMAL
PROP 153
  1 98 0.0 0.0 0.0 1.0 0.0
*DEFINE 3
*DEFINE 3
*DEFINE 3
*DEFINE 3
*DEFINE 3
*DEFINE 3
C....PARAMETER DATA
C
*CONS 0
*EPSF
*MODE 58
*ELM 28
*BOUND 20
*PRESSURE
*TEMPERATURE
*LOC 2 1 3
*VENT 10 10
*PRESSURE
*PROPERTY
*FREQ
*END
C....MODEL DATA
*CORE

-5.000 0.95
```

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**exp2scdf.dat**

```
10 10 39 40 11
11 11 40 41 12
12 12 41 42 13
13 13 42 43 14
14 14 43 44 15
15 15 44 45 16
16 16 45 46 17
17 17 46 47 18
18 18 47 48 19
19 19 48 49 20
20 20 49 50 21
21 21 50 51 22
22 22 51 52 23
23 23 52 53 24
24 24 53 54 25
25 25 54 55 26
26 26 55 56 27
27 27 56 57 28
28 28 57 58 29

*PRESSURE*
30 58 0.1577400E+04

*TEMPERATURE*
1 29 4.04
10 58 4.00

*BOOM*
15 1 0.0
44 1 0.0

*ITERA*
1 1
40 0.000

*PROP*
153
58 0.10000+01 0.17000E+08 0.340 0.95000E+05 0.3168 8.00E+06

*FIN*

**END**
```

**exp2scdf.out**

```
10 10 39 40 11
11 11 40 41 12
12 12 41 42 13
13 13 42 43 14
14 14 43 44 15
15 15 44 45 16
16 16 45 46 17
17 17 46 47 18
18 18 47 48 19
19 19 48 49 20
20 20 49 50 21
21 21 50 51 22
22 22 51 52 23
23 23 52 53 24
24 24 53 54 25
25 25 54 55 26
26 26 55 56 27
27 27 56 57 28
28 28 57 58 29

*PRESSURE*
30 58 0.1577400E+04

*TEMPERATURE*
1 29 4.04
10 58 4.00

*BOOM*
15 1 0.0
44 1 0.0

*ITERA*
1 1
40 0.000

*PROP*
153
58 0.10000+01 0.17000E+08 0.340 0.95000E+05 0.3168 8.00E+06

*FIN*

**END**

**TAYLOR SERIES EXPANSION OF THE FORM**

\[ y = a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \]

**WHERE:**

\[ a_0 \text{ IS THE CONSTANT TERM} \]
\[ a_1, a_2, \ldots, a_n \text{ ARE THE DESIGN SENSITIVITIES} \]
\[ x_1, x_2, \ldots, x_n \text{ ARE THE RANDOM VARIABLES} \]

**TAYLOR SERIES COEFFICIENTS & RANDOM EXPANSION VARIOUS POINTS**

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<td>0.168750E+07</td>
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**PERFORMING PROBABILISTIC ANALYSIS WITH FPF**

**COV RESULTS**

```
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<th>PROBABILITY</th>
<th>ITER. NO.</th>
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| 0 | 0.2000099E+03 | 0.51000008E+01 | 0.7610321E-04 | 0 | 0
| 0 | 0.5849398E+03 | 0.41000080E+01 | 0.2968971E-04 | 0 | 0
| 0 | 0.1023999E+04 | 0.2100000E+01 | 0.1467922E-03 | 0 | 0
| 0 | 0.2493939E+04 | 0.1000000E+02 | 0.1784437E-01 | 0 | 0
| 0 | 0.2894599E+04 | 0.1000000E+02 | 0.1146000E+00 | 0 | 0
| 0 | 0.4583489E+04 | 0.1000000E+02 | 0.9835399E-09 | 0 | 0
| 0 | 0.5102097E+04 | 0.1000000E+02 | 0.9750351E-09 | 0 | 0
| 0 | 0.5962010E+04 | 0.1000000E+02 | 0.9750351E-09 | 0 | 0
| 0 | 0.6081941E+04 | 0.1000000E+02 | 0.9595188E-09 | 0 | 0
| 0 | 0.7447053E+04 | 0.1000000E+02 | 0.9595188E-09 | 0 | 0
```

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Appendix D

PFEM FILES FOR THE HEAT TRANSFER RELIABILITY

(Chapter IV, data is shown in Fig.16 in Chapter III)

NESSUS/PFEM input and output files for single Z-level

```plaintext
exp2tempfem.dat  Sun Jun 22 01:05:39 1997  1
PFEM
C --- PROBABLISTIC ANALYSIS FOR TEMPERATURE FAILURE OF EXP Z-LEVEL
DEFINE
  *COMP 0
  *DATATYPE 0
  *REPTYPE 1
  *CONF 2
  *PROP 6
  *PERT 3
  1.1.3
  *VARVAR 3
  1.2.1
END
DEFINE
  *COMPUTATIONAL 3 3
END
DEFINE
  *DEFINE 1
END
0.195554760000E+05 0.395554760000E+04 NORMAL
PROP 98
5 6 0.0 1.0 0.0 0.0 0.0
DEFINE 3
END
0.5887580000E+02 0.5887580000E+01 NORMAL
PROP 98
3 4 0.0 1.0 0.0 0.0 0.0
DEFINE 3
END
0.765398E+03 0.765398E+01 NORMAL
PROP 99
1 3 0.0 1.0 0.0 0.0 0.0
*PERT 1
1 0.1
2 0.1
*PERT 3
3 0.1
END
END
C *********************************************************
PFEM
C --- HEAT TRANSFER FOR A REAGREGANT PIPE EXP2
C
C
C ...
PARAMETER DATA
C
*CARD 10
*COMP 0
*THWT 6
*DISP 6
*EMG3 3
93
*PERT 10 10
*PROPERTIES
*PROPERTIES
*PROPERTIES
*PROPERTIES
*NRE
*ERAN
END
C **** MODEL DATA
*COORD 1
  0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
```

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**ELEN 98**

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**PROP 98**

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<tbody>
<tr>
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<tr>
<td>1</td>
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<td>6</td>
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</tr>
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</table>

**END**

---

exp3tempfems.out  Sun Jun 22 01:06:16 1997

AS IS THE CONSTANT TERM
A1, A2, ..., AN ARE THE DESIGN SENSITIVITIES
X1, X2, ..., XH ARE THE RANDOM VARIABLES

<table>
<thead>
<tr>
<th>TAYLOR SERIES COEFFICIENTS</th>
<th>RANDOM EXPANSION</th>
<th>NUM. DESIGN VARIABLE POINT (NPP)</th>
<th>SENSITIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.75556E-02</td>
<td>1</td>
<td>0.39955E+00</td>
</tr>
<tr>
<td>1</td>
<td>0.15117E+00</td>
<td>2</td>
<td>0.56074E+00</td>
</tr>
<tr>
<td>2</td>
<td>0.39176E+00</td>
<td>3</td>
<td>0.78139E+00</td>
</tr>
</tbody>
</table>

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

<table>
<thead>
<tr>
<th>CDP RESULTS</th>
<th>PROBABILITY</th>
<th>ITER. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.30000000E+00</td>
<td>0.13404E+00</td>
<td>0.929921E+00</td>
</tr>
</tbody>
</table>

NASA/CR—2001-210969  80
NESSUS/PFEM input and output files for CDF

```
*PFEM
C ... PROBABILISTIC (CDF) ANALYSIS FOR TEMPERATURE FAILURE OF EXP2
*DEFINITION
  *COOR 0
  *DATATYPE 0
  *KESPTYPE 1
  *COMP 2
  *MODE 6
  *PERT 1
  *ANVAR 1
  *END
*DEFINE
  *COMPUTATIONAL 1 3
  1.3 3
  *END
*DEFINE
  *DEFINE 1
ENDC3
0.399554456008E+05 0.399554456008E+05 NORMAL
PROP 98
  5 6 0.0 1.0 0.0 0.0 0.0
*DEFINE 2
ENDC3
0.3867580098E+02 0.3867580098E+02 NORMAL
PROP 98
  3 4 0.0 1.0 0.0 0.0 0.0
*DEFINE 3
ENDC3
0.785398E+02 0.785398E+02 NORMAL
PROP 98
  1 2 0.0 1.0 0.0 0.0 0.0
  *PERT 1
  1 0.1
  *PERT 2
  1 0.1
  *PERT 3
  1 0.1
*END
*END
C
C *****************************************
*PFEM
C ... HEAT TRANSFER FOR A REFRIGERANT PIPE EXP2
C
C
C ... PARAMETER DATA
C
  *BONW 20
  *CINV 0
  *DINV
  *MODE 4
  *DINV
  *EINV 3
  *FNCI
  *PERT 10 10
  *PERT 10 10
  PROPERTIES
  PROPERTIES
  PROPERTIES
  *PRIN
  *END
C ... MODEL DATA
  *COOR 1
  0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
```

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exp2temp.dat.out

Mon May 19 22:40:36 1997

AO IS THE CONSTANT TERM
AI, A2, ..., AN ARE THE DESIGN SENSITIVIES
X1, X2, ..., XN ARE THE RANDOM VARIABLES

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

<table>
<thead>
<tr>
<th>CDF RESULTS</th>
<th>PROBABILITY</th>
<th>ITER. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.176122108-06</td>
<td>0</td>
</tr>
<tr>
<td>-0.177079098-03</td>
<td>0.206687116-04</td>
<td>0</td>
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<tr>
<td>-0.17664356E-01</td>
<td>0.978571221-03</td>
<td>0</td>
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<tr>
<td>0.39137358E+02</td>
<td>0.813539816-00</td>
<td>0</td>
</tr>
<tr>
<td>0.39137358E+02</td>
<td>0.999955555-00</td>
<td>0</td>
</tr>
<tr>
<td>0.39137358E+02</td>
<td>0.999955555-00</td>
<td>0</td>
</tr>
<tr>
<td>0.39137358E+02</td>
<td>0.999955555-00</td>
<td>0</td>
</tr>
</tbody>
</table>
Appendix E

PFEM FILES FOR RELIABILITY OF FLUID FLOW

(Chapter IV, data is show in Fig.18 of Chapter III)

NESSUS/PFEM input and output files for a single Z-level

```
flowpemz.dat Sat Jun 28 22:36:52 1997

*PFEM
C .... PROBABILISTIC ANALYSIS FOR FLOW FAILURE
*DEFINE
   *COND 0
   *DATATYPE 0
   *RESERVED 1
   *COND 0
   *FUNC 2
   *PERM 2
   1.2
   *ANVAR 2
   1.2
*END
*DEFINE
   *COMP 1 2
   *END
*DEFINE
   *DEFINE 1
   *END
14.489 1.4894 NORMAL
PROP 98 1 2 0.0 1.0 0.0 0.0 0.0
*DEFINE 2
VELO
-500.0 15.0 NORMAL
FORC 2 2 1.00
*PERT 1 1 0.1
*PERT 2 2 0.1
*END
*END
C .................................................................
*PFEM
C .... FLOW IN A CONSTANT DIAMETER PIPE (with a/L=0.1)
C
C
C
C ... PARAMETER DATA
C
BOND 10
*COND 0
*DISP
*MODE 2
*ELEM 98
*FUNC 1
*PERT 10 10
VELOCITY FORC
*PRIN
*REAS
*END
C ... MODEL DATA
*COND
1 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
1 0.0000 10.0000 0.0000 0.0000 0.0000 1.0000
2 0.0000
3 1 0.0000
2 1 0.0000
1 1 0.0000
1 1 18.00
1 3 0.0000
```

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TAYLOR SERIES EXPANSION OF THE FORM

\[ G = A_0 + A_1X_1 + A_2X_2 + \ldots + A_NX_N \]

WHERE:

- \( A_0 \) is the constant term
- \( A_1, A_2, \ldots, A_N \) are the design sensitivities
- \( X_1, X_2, \ldots, X_N \) are the random variables

---

**flowpams.out**

```
flowpams.out  Sat Jun 28 22:37:13 1997  3

1 0.157735E+02  1 0.649840E+02  0.500000E+00
2 0.671318E+00  2 -0.350000E+00 -0.500000E+00

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

COP RESULTS

<table>
<thead>
<tr>
<th>Z</th>
<th>U</th>
<th>PROBABILITIES</th>
<th>ITER. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.167740E+04</td>
<td>0.321460E+03</td>
<td>0.986732E+01</td>
<td>0</td>
</tr>
</tbody>
</table>
NESSUS/PFEM input and output files for CDF


*PFEM
C ... PROBABILISTIC ANALYSIS FOR FLOW FAILURE
*DEFINE
  +COND 0
  +DATAETYPE 0
  +RESULTYPE 1
  +COND 2
  +MODE 1
  +PERT 1
  1.1
  +RANDVAR 1
  1.2
  +END
*DEFINE
  +COMPUTATIONAL 1 2
  1.2
  +END
*DEFINE
  +RESULTYPE 1
  +COND 1
  +DATAETYPE 1.4896 NORMAL
  +PROP 1.4896
  1 2 0.0 1.0 0.0 0.0 0.0
  +DEFINE 2
  +VELC 1 -350.0 35.0 NORMAL
  +FORCE 1 2 1.00
  +PERT 1 0.1
  +PERT 2 0.1
  +END
  +END
C **********************************************************************
*PFEM
C ... FLOW IN A CONSTANT DIAMETER PIPE (WITH A/L=0.1)
C
C
C肋 PARAMETER DATA
C
  +BOUND 30
  +CONS 0
  +DISP
  +MODE 3
  +ELEN 1
  +98
  +POOR 1
  +PERT 10 10
  +VELOCITY FORCE
  +PRIM
  +BEAM
  +END
C C**** MODEL DATA
+COND 1
  1 0.0000 0.0000 0.0000 0.0000 0.0000 0.0000 1.0000
  2
  +ELEN 98
  1 1 2
  +BOUND
  1 1 0.0000
  2 1 0.0000
  1 1 0.0000
  1 2 109.7
  1 3 0.0000

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DESIGN SENSITIVITIES

TAYLOR SERIES EXPANSION OF THE FORM

\[ G = a_0 + a_1 x_1 + a_2 x_2 + \ldots + a_n x_n \]

WHERE:

- \( a_0 \) is the constant term
- \( a_1, a_2, \ldots, a_n \) are the design sensitivities
- \( x_1, x_2, \ldots, x_n \) are the random variables

TAYLOR SERIES COEFFICIENTS

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>POINT (NPP)</th>
<th>SENSITIVITY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RANDOM EXPANSION WRT DESIGN</td>
<td>WRT RANDOM</td>
</tr>
<tr>
<td>0</td>
<td>0.160374E+04</td>
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</tr>
</tbody>
</table>

PERFORMING PROBABILISTIC ANALYSIS WITH FPI

<table>
<thead>
<tr>
<th>Z</th>
<th>U</th>
<th>PROBABILITY</th>
<th>ITER. NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4356155E+04</td>
<td>0.3100000E+01</td>
<td>0.1701221E-04</td>
<td>0</td>
</tr>
<tr>
<td>1.4453155E+04</td>
<td>0.3100000E+01</td>
<td>0.2648716E-04</td>
<td>0</td>
</tr>
<tr>
<td>1.5007564E+04</td>
<td>0.3100000E+01</td>
<td>0.94767122E-03</td>
<td>0</td>
</tr>
<tr>
<td>0.15397963E-04</td>
<td>-2.1600000E+01</td>
<td>0.17846437E-01</td>
<td>0</td>
</tr>
<tr>
<td>0.15875161E+04</td>
<td>-2.1600000E+01</td>
<td>0.13564612E+00</td>
<td>0</td>
</tr>
<tr>
<td>0.16331539E+04</td>
<td>0.3900000E+00</td>
<td>0.81753951E-04</td>
<td>0</td>
</tr>
<tr>
<td>0.16448158E+04</td>
<td>0.3900000E+00</td>
<td>0.97135515E-03</td>
<td>0</td>
</tr>
<tr>
<td>0.17007558E+04</td>
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<td>0.99816412E-00</td>
<td>0</td>
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<tr>
<td>0.17321955E+04</td>
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<td>0.17457548E+04</td>
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<td>0.99999952E-00</td>
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</table>

flowcdf.out Sat Jun 28 22:46:36 1997 3

1
2
Appendix F

GFUN.DAT AND GFUN.MOV FILES OF IMPOSITIONS

(Chapter IV, data is show in Fig.16 of Chapter III)

NESSUS/PFEM input gfun.dat and output gfun.mov files

<table>
<thead>
<tr>
<th>gfun.dat</th>
<th>Tue Jul 1 15:39:34 1997</th>
</tr>
</thead>
</table>

---

NASA/CR—2001-210969

87
gfun.dat
Tue Jul 1 15:39:34 1997

18 18 47 48 19
19 19 46 49 20
20 20 49 50 21
21 21 50 51 22
22 22 51 52 23
23 23 52 53 24
24 24 53 54 25
25 25 54 55 26
26 26 55 56 27
27 27 56 57 28
28 28 57 58 29

*PRESSURE
20 58 3300.0

*TEMPERATURE
1 29 4.04
20 58 4.0

*EOD

END

**----------------------------**

**HEAT EXCHANGER OF EXP2 WITH DATASETS**

**FUNCTION 1**

**DATASETS 3**

**METHOD 1**

**PRINT 3**

**ANALYSIS 1**

**END**

**ELEVATION 0**

**EOD**

gfun.bso

Tue Jul 1 15:28:37 1997

CONDITION # NODE COMPONENT LAYER

INCRA 0 44 3 1

1 MEAN, MEAN, STD. DEV.
0.654873964 0.654873964 0.319634843

SECOND OF PROBABILITY LEVELS 1

LVL. 0.0. PROB. 1. 0.890800080 0.82758 1.000000000

DESEIGN POINT COORD., SENSITIVITY FACTORS, STD. DEV. FROM MEAN VALUES

10. PROB. = 0.890800080 1.000000000

MCDA 0.319634843 0.999687 0.82758

ALPHA 0.999687 0.0.0592180 -0.015077 0.102775

***********************

CDF RESULTS

0.80000000E+04 0.823453664 0.0.0.0.000028 0.0.0.000028 0.0.0.000028

Iter. No. 0

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88
### Condition #1

#### Mode Component Layer

| INCR | 44 | 3 | 1 |

- **Median, Mean, Std. Dev.**
  - Mode: 0.711460E-04, 0.714058E-04, 0.213598E-03

- **Number of Probability Levels**: 1

- **CDF Results**
  - Probability: 0.908431E-03
  - Iteration No.: 0

---

### Condition #2

#### Mode Component Layer

| INCR | 44 | 3 | 1 |

- **Median, Mean, Std. Dev.**
  - Mode: 0.711460E-04, 0.714058E-04, 0.213598E-03

- **Number of Probability Levels**: 1

- **CDF Results**
  - Probability: 0.908431E-03
  - Iteration No.: 0

---

### Condition #3

#### Mode Component Layer

| INCR | 44 | 3 | 1 |

- **Median, Mean, Std. Dev.**
  - Mode: 0.711460E-04, 0.714058E-04, 0.213598E-03

- **Number of Probability Levels**: 1

- **CDF Results**
  - Probability: 0.908431E-03
  - Iteration No.: 0

---

**NASA/CR—2001-210969**
Appendix G

DATA AND OUTPUT FILES FOR CALCULATION OF UNION PROBABILITY

(Chapter IV)

temp.dat Tue Jul 1 16:23:17 1997

1. 1.0000000000
2. 1.0000009, 1.000000091
3. 6.825696
4. 5.721588
5. 4.619647
6. 2.423429
7. 1.320801
8. 0.236641
9. 0.348497

7. 2.1
8. 1.0000000000
9. 1.0000009, 1.000000091
10. 6.825696
11. 5.721588
12. 4.619647
13. 2.423429
14. 1.320801
15. 0.236641

********** BOUNDS **********
Lower bound = 0.4926211
Upper bound = 0.4926211

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BIBLIOGRAPHY


**Title and Subtitle:**
Multidisciplinary System Reliability Analysis

**Authors:**
Sankaran Mahadevan and Song Han

**Performing Organization:**
Vanderbilt University
Civil and Environmental Engineering
2201 W. End Avenue
Nashville, Tennessee 37240-0001

**Funding Numbers:**
WU-705-30-11-00
NAG3-1372

**Abstract:**
The objective of this study is to develop a new methodology for estimating the reliability of engineering systems that encompass multiple disciplines. The methodology is formulated in the context of the NESSUS probabilistic structural analysis code, developed under the leadership of NASA Glenn Research Center. The NESSUS code has been successfully applied to the reliability estimation of a variety of structural engineering systems. This study examines whether the features of NESSUS could be used to investigate the reliability of systems in other disciplines such as heat transfer, fluid mechanics, electrical circuits etc., without considerable programming effort specific to each discipline. In this study, the mechanical equivalence between system behavior models in different disciplines are investigated to achieve this objective. A new methodology is presented for the analysis of heat transfer, fluid flow, and electrical circuit problems using the structural analysis routines within NESSUS, by utilizing the equivalence between the computational quantities in different disciplines. This technique is integrated with the fast probability integration and system reliability techniques within the NESSUS code, to successfully compute the system reliability of multidisciplinary systems. Traditional as well as progressive failure analysis methods for system reliability estimation are demonstrated, through a numerical example of a heat exchanger system involving failure modes in structural, heat transfer and fluid flow disciplines.

**Subject Terms:**
Probabilistic analysis code; System reliability analysis; System reliability computation; Tensile stress; Yield strength; Structure analysis; Thermal-structural fluid system

**Supplementary Notes:**
Project Manager, Christos C. Chamis, Research and Technology Directorate, NASA Glenn Research Center, organization code 5000, 216-433-3252.

**Abstract (Maximum 200 words):**

The objective of this study is to develop a new methodology for estimating the reliability of engineering systems that encompass multiple disciplines. The methodology is formulated in the context of the NESSUS probabilistic structural analysis code, developed under the leadership of NASA Glenn Research Center. The NESSUS code has been successfully applied to the reliability estimation of a variety of structural engineering systems. This study examines whether the features of NESSUS could be used to investigate the reliability of systems in other disciplines such as heat transfer, fluid mechanics, electrical circuits etc., without considerable programming effort specific to each discipline. In this study, the mechanical equivalence between system behavior models in different disciplines are investigated to achieve this objective. A new methodology is presented for the analysis of heat transfer, fluid flow, and electrical circuit problems using the structural analysis routines within NESSUS, by utilizing the equivalence between the computational quantities in different disciplines. This technique is integrated with the fast probability integration and system reliability techniques within the NESSUS code, to successfully compute the system reliability of multidisciplinary systems. Traditional as well as progressive failure analysis methods for system reliability estimation are demonstrated, through a numerical example of a heat exchanger system involving failure modes in structural, heat transfer and fluid flow disciplines.