Cirrus parcel model comparison project phase 1:
The critical components to simulate cirrus initiation explicitly

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Abstract

The Cirrus Parcel Model Comparison Project, a project of the GCSS (GEWEX Cloud System Studies) Working Group on Cirrus Cloud Systems, involves the systematic comparison of current models of ice crystal nucleation and growth for specified, typical, cirrus cloud environments. In Phase 1 of the project reported here, simulated cirrus cloud microphysical properties are compared for situations of "warm" (-40°C) and "cold" (-60°C) cirrus, both subject to updrafts of 4, 20 and 100 cm s⁻¹. Five models participated. The various models employ explicit microphysical schemes wherein the size distribution of each class of particles (aerosols and ice crystals) is resolved into bins or treated separately. Simulations are made including both the homogeneous and heterogeneous ice nucleation mechanisms. A single initial aerosol population of sulfuric acid particles is prescribed for all simulations. To isolate the treatment of the homogeneous freezing (of haze droplets) nucleation process, the heterogeneous nucleation mechanism is disabled for a second parallel set of simulations.

Qualitative agreement is found for the homogeneous-nucleation-only simulations, e.g., the number density of nucleated ice crystals increases with the strength of the prescribed updraft. However, significant quantitative differences are found. Detailed analysis reveals that the homogeneous nucleation rate, haze particle solution concentration, and water vapor uptake rate by ice crystal growth (particularly as controlled by the deposition coefficient) are critical components that lead to differences in predicted microphysics.

Systematic bias exists between results based on a modified classical theory approach and models using an effective freezing temperature approach to the treatment of nucleation. Each approach is constrained by critical freezing data from laboratory studies,
but each includes assumptions that can only be justified by further laboratory research. Consequently, it is not yet clear if the two approaches can be made consistent. Large haze particles may deviate considerably from equilibrium size in moderate to strong updrafts (20-100 cm s\(^{-1}\)) at -60°C when the commonly invoked equilibrium assumption is lifted. The resulting difference in particle-size-dependent solution concentration of haze particles may significantly affect the ice particle formation rate during the initial nucleation interval. The uptake rate for water vapor excess by ice crystals is another key component regulating the total number of nucleated ice crystals. This rate, the product of particle number concentration and ice crystal diffusional growth rate, which is particularly sensitive to the deposition coefficient when ice particles are small, modulates the peak particle formation rate achieved in an air parcel and the duration of the active nucleation time period.

The effects of heterogeneous nucleation are most pronounced in weak updraft situations. Vapor competition by the heterogeneously nucleated ice crystals may limit the achieved ice supersaturation and thus suppresses the contribution of homogeneous nucleation. Correspondingly, ice crystal number density is markedly reduced. Definitive laboratory and atmospheric benchmark data are needed for the heterogeneous nucleation process. Inter-model differences are correspondingly greater than in the case of the homogeneous nucleation process acting alone.
1. Introduction

The Cirrus Parcel Model Comparison Project (CPMC) is a project of the GEWEX (Global Energy and Water Cycle Experiment) Cloud System Study Program (GCSS) Working Group on Cirrus Cloud Systems (WG2). The primary goals of this project are to assess the current understanding of cirrus microphysical modeling and to identify cirrus model sensitivities to the state of our knowledge of nucleation and microphysics at low temperature. CPMC also gives the Idealized Cirrus Model Comparison Project (ICMCP) of GCSS WG2 a benchmark estimate of the cirrus initiation characteristics. Furthermore, the findings of this project are expected to provide improvements for models with simpler cirrus microphysical modules.

Simulations of cloud initiation processes and the evolution of hydrometeor size distribution by explicit microphysical schemes date back to the 1940s and 50s (e.g., Howell 1948; Mordy 1958). The direct application of explicit microphysical schemes to simulate cirrus clouds did not begin until the late 1980s after the vast area-coverage of cirrus was recognized, and the effects of cirrus on the global radiation budget were addressed (e.g., Liou 1986; Ramanathan et al. 1989). Sassen and Dodd (1988) compared lidar data with parcel model results to infer the homogeneous nucleation rate of ice in supercooled cloud droplets within the temperature range of -34.3°C and -37.3°C. Heymsfield and Sabin (1989) conducted a numerical study of homogeneous freezing of ammonium sulfate solution droplets at water subsaturated conditions between -40°C and -50°C. They reported that the predicted number concentration of ice crystal depended on temperature, parcel cooling rate, and cloud condensation nuclei (CCN) distributions. Parameterization of homogeneous freezing of supercooled cloud droplets and CCN solution droplets for
use in regional-scale cloud models via a bulk microphysical scheme was developed with the aid of parcel model simulations (DeMott et al. 1994). These simulations relied on an explicit microphysical scheme, in which the number distribution of ice particles is resolved into bins, in contrast to the use of bulk parameterizations. The parcel model approach was also adopted in evaluating the potential impact of volcanic aerosols on cirrus microphysics (Jensen and Toon 1992), and in the study of orographic clouds where laminar flow produces these clouds. For example, model results were compared with in situ measurements of orographic clouds (Jensen et al. 1998; Lin et al. 1998), and the dominant nucleation mode under various background conditions was investigated (Spice et al. 1999; DeMott et al. 1997).

Jensen et al. (1994a,b) used a one-dimensional model, which included the vertical transport of particles, to study the initiation as well as the development of cirrus. Model results were sensitive to parameters controlling the homogeneous nucleation rate; namely, the activation energy and ice-solution surface tension. These and other authors (Heymsfield and Sabin 1989; DeMott et al. 1994) have also demonstrated that cirrus microphysical properties, unlike those of stratus, are not very sensitive to the total CCN number concentration. This is a consequence of (i) the fact that homogeneous nucleation of ice in supercooled solution droplets acts first and foremost on the larger end of the CCN size distribution, and (ii) the fraction of the CCN population that participates in homogeneous freezing is usually quite small, being vapor-limited by the ice formation process itself. Explicit microphysical schemes have been incorporated into fully-coupled multi-dimensional models to investigate contrail evolution (Gierens and Jensen 1996; Khvorostyanov and Sassen 1998c), dynamic-radiative-microphysical interaction in cirrus
anvils (Lin 1997), and fundamental issues of nucleation and ice particle growth with implications for cirrus cloud development (Khvorostyanov and Sassen 1998a).

Many studies have ignored heterogeneous nucleation as a mechanism for cirrus formation, primarily because of the current lack of knowledge regarding the abundance and activation of ice nuclei in the upper troposphere. However, it is becoming apparent that the conditions observed for cirrus formation (Heymsfield and Miloshevich 1995; Heymsfield et al. 1998) cannot be satisfied by simply incorporating the existing knowledge of the freezing of sulfate particles via homogeneous freezing schemes (DeMott 2001). This differs from conclusions that were initially inferred from model findings by Heymsfield and Miloshevich (1993). Sassen and Benson (2000) indicated that heterogeneous ice nucleation can strongly modulate the cirrus formation process, but only if ice nuclei concentrations and activations are similar to the most favorable conditions measured at the Earth's surface. So, heterogeneous nucleation must ultimately be parameterized in some manner. This project attempts to clarify the range of effects to be expected from incorporating various available heterogeneous nucleation schemes that are either based on data or expectations.

Qualitative agreement between models are frequently demonstrated in the papers cited above; however, the quantitative discrepancy margin has yet to be assessed. It is important to do a systematic comparison of these models because of the sensitivity of cloud microphysical and radiative properties to the cloud initiation processes. The questions we hope to answer from this exercise are; what is the range of model responses to a highly-idealized situation; what are the key components in the models causing discrepancies in the simulation results; and what kind of bias would these discrepancies
introduce if various microphysical schemes are adopted in multi-dimensional models.

The intercomparison study described here took place at the GCSS WG2 workshop at Geestacht, Germany on 19-21 May 1999. Five parcel modeling groups from the UK and USA participated in the intercomparison. A brief description of the participant models is given in Section 3.

Participants were asked to submit a series of relatively simple calculations involving an ascending parcel lifted by a superimposed updraft without mass and heat exchange with the environment. The primary focus was on the homogeneous nucleation process operating in isolation (hereafter, HN-ONLY). A secondary focus was the combination of the homogeneous and heterogeneous ice nucleation mechanisms included in cirrus parcel models (hereafter, ALL-MODE). A detailed description of the project is given in Section 2. Cirrus initiation requirements and cloud properties are presented and the implications of the model results discussed in Section 4 followed by a summary in Section 5.

2. Simulation protocols

We focus on the nucleation regimes of the warm and cold cases studied in the WG2 ICMCP (Starr et al. 2000), and test the models under a range of imposed updraft conditions (0.04, 0.2, 1 m s\(^{-1}\)). All parcel models were run in the "closed" Lagrangian mode, i.e., neither particle fallout nor mixing with the environment was allowed. Nucleation and ice crystal growth were forced through an externally imposed constant rate of lift, \(W\), with consequent adiabatic cooling. The parcel cooling rate is formulated as

\[
\frac{dT}{dt} = -\frac{g}{C_p} W + \dot{Q}_{\text{LH}}. \tag{1}
\]
where $Q_{LH}$ is the latent heat release due to diffusional growth of hydrometeors. The environmental pressure was specified as:

$$\frac{d\ln P}{dz} = -\frac{g}{R_d[T_{\text{ini}} - \Gamma(z - z_{\text{ini}})]},$$

(2)

where the environmental lapse rate, $\Gamma$, approximates the ice pseudo-adiabatic lapse rate for the assumed conditions. The background haze particles\(^1\) are assumed to be aqueous sulfuric acid particles with a number concentration of 200 cm\(^{-3}\), lognormally distributed with a pure H\(_2\)SO\(_4\) mode radius of 0.02 micron and a distribution width corresponding to $\sigma = 2.3$ with pure H\(_2\)SO\(_4\) density as 1.841 g cm\(^{-3}\). The initial conditions for the 14 runs are tabulated in Table 1. In each of the HN-ONLY runs (Wh004, Wh020, Wh100, Ch004, Ch020, and Ch100), only homogeneous nucleation mode is allowed to form ice crystals; the other mode is turned off. Both nucleation modes are switched on in ALL-MODE runs (Wa004, Wa020, Wa100, Ca004, Ca020, and Ca100). Participants were asked to run two additional simulations, Wh020L and Ch020L, by setting $\lambda = 2$ (to be discussed further in Section 3a), or tailoring the nucleation rate calculation so that the predicted freezing is in agreement with $\lambda = 2$. Chen et al. (2000) noted that $\lambda = 2$ agreed with their data, and those presented by Koop et al. (1998). This same $\lambda$, or equivalent theoretical treatment, served as a tool to diagnose the relative importance of nucleation rate versus the treatment of haze and ice particle growth in determining the source of model differences in the predicted $N_i$.

Heterogeneous nucleation could be the dominant ice formation mode in cirrus in slow updraft conditions (e.g., DeMott et al. 1997; Spice et al. 1999; Sassen and Benson 2000).

\(^1\)Similar aerosol number distribution was used in Jensen et al. (1994a).
However, significant differences exist in treatments of the heterogeneous ice nucleation processes. Thus, major inter-model differences might be expected in the predicted cirrus microphysical properties. Definitive laboratory and atmospheric benchmark data are only now being developed. Despite the still primitive nature of this subject, it is important to compare the frequently used schemes and assess their features.

The shape of ice crystals may also affect the simulation. In order to simplify the comparison, participants assumed ice crystals to be spherical with constant ice density of 0.9 g cm$^{-3}$. The assumption of bulk ice density may be an acceptable approximation for the small crystal sizes present in most of the simulations (e.g., Heymsfield and Sabin 1989), but it is expected that the simulation results will be sensitive to this assumption (Heymsfield, personal communication). Participants also assured that time steps were small enough not to cause numerical artifacts. For the present calculations, ice particle aggregation and direct radiation effect on diffusional growth are ignored.

The depth of parcel lift used in this study was set to assure that parcels underwent complete transition through the nucleation regime to a stage of approximate equilibrium between ice mass growth and vapor supplied by the specified updrafts. This was deemed to provide the best comparison of the models and their sensitivities to critical parameters.

3. Model descriptions

A brief summary of the model characteristics is given in Table 2. Hereafter, we will refer to these models as the C, D, J, L, S models, respectively, as denoted in the table. According to the method in which particles are handled, these models are classified as either particle-tracing models or bin models (Young 1993). In the particle-tracing method,
the evolution of each individual particle is traced (model S). In bin models (models C, D, J and L), particles are grouped according to particle size or mass to expedite numerical integration. Individual particle histories are lost in the grouping process. Among the five models, J and L have been incorporated into fully-coupled multi-dimensional models. The major components in parcel models, from a numerical standpoint, are the time-integration scheme and size-coordinate scheme. From the standpoint of cloud physics, the major components are the specification and treatment of the saturation vapor pressure, diffusional growth of ice and haze particles, equilibrium haze size and solution concentration, and heterogeneous and homogeneous nucleation schemes (Fig. 1). The latter three components are highlighted in this section.

a. Homogeneous nucleation of unactivated solution particles

Much progress has been made in estimating homogeneous nucleation rate of ice in pure water droplets, $J_w$, by theoretical, laboratory and field measurement studies (e.g., Sassen and Dodd 1988; DeMott and Rogers 1990; Pruppacher 1995; Jeffery and Austin 1997). The nucleation rate $J_w$ can be formulated theoretically or fitted parametrically to droplet temperature only (e.g., Heymsfield and Sabin 1989; Jeffery and Austin 1997; and Khvorostyanov and Sassen 1998b). Consensus on the temperature dependence of $J_w$ in the temperature regime where pure water freezes homogeneously has been reached in the community. The estimate of the nucleation rate of ice in solution droplets, $J_{haze}$, however, remains an active research area. This is critically important for cirrus modeling because most cirrus clouds form below the homogeneous freezing temperature of pure water ($\sim -40^\circ C$) and at relative humidities below saturation with respect to the liquid
phase, where liquid water exists only as a component of solution droplets.

The homogeneous nucleation rate of ice in solution droplets has been derived in various studies using (i) the modified classical theory approach (model J) or (ii) an empirical approach referred to as the effective freezing temperature (hereafter, $T_{eff}$ approach, models C, D, L, and S). In either case, calculations depend critically on the use of laboratory data describing properties of solution particles at cirrus temperatures. The modified classical theory approach computes nucleation rate using the equation (Pruppacher and Klett 1997)

$$J_{haze} = 2N_{c}\left(\frac{\rho_w B_z T}{\rho_h h}\right)\left(\frac{\sigma_{i/s}}{B_z T}\right)^{1/2}\exp\left[-\left(\frac{\Delta F_{actv} + \Delta F_{germ}}{B_z T}\right)\right],$$

(3)

where $\sigma_{i/s}$, $\Delta F_{actv}$, and $\Delta F_{germ} = \frac{4\pi\sigma_{i/s}}{3} \left(\frac{2\sigma_{i/s}}{B_c \ln(T_0 + T) + \frac{1}{2}(T_0 + T) R_c \ln a_{\omega}}\right)^2$ are the surface tension across the ice-solution interface, activation energy, and ice-germ formation energy, respectively. All symbols are defined in Notation. Solution effects, primarily compositional effects on $\sigma_{i/s}$, serve to raise the critical germ radius and the required germ formation energy. In order to use (3), laboratory data on $\sigma_{i/s}$ and $\Delta F_{actv}$ as functions of temperature and composition are needed. Seldom are both quantities available from direct measurement for a given solution. Typically, direct data on one parameter is used and the other is constrained by measurements of $J_{haze}$. Thus, in model J, recent direct data on $\sigma_{i/s}$ was incorporated and $\Delta F_{actv}$ was inferred from laboratory measurements of $J_{haze}$ for sulfuric acid haze particles following the method of Tabazadeh et al. (2000).

The $T_{eff}$ approach attempts to directly link measured nucleation rates of ice in solution particles to nucleation rates of equivalent-sized pure water droplets via a
parameter called the effective freezing temperature. $T_{eff}$ is defined as

$$T_{eff} = T + \lambda \Delta T_m,$$  

(4)

as introduced by Sassen and Dodd (1988) such that

$$J_{haze} = J_w(T_{eff}).$$  

(5)

In (4), $\Delta T_m$ is the equilibrium melting point depression (positive valued), which depends on solution concentration, and $\lambda$ is an empirical coefficient to account for additional suppression ($\lambda > 1$) or enhancement ($\lambda < 1$) of nucleation (freezing) temperature due to non-ideal interaction between ions and condensed water. Nucleation rate is extremely sensitive to $\Delta T_m$ and $\lambda$; thus, these two factors must be specified with care. The values of $\lambda$ are constrained by, and in fact have their source in, laboratory data, as first described by Sassen and Dodd (1988). Although Sassen and Dodd (1988) noted that an average $\lambda$ for different solutions was around 1.7, the value for specific solutions may range from just greater than 1 to about 2.5 (DeMott 2001). The existence of relation (4) as a descriptor of laboratory data on the freezing of a large variety of solution droplet types has never been explained theoretically from first principles.

The homogeneous nucleation rates for sulfuric acid solution droplets computed by these methods are compared as functions of temperature in Figure 2. In this figure, the same equilibrium-sized haze particle data provided by Paul DeMott is used in each model where $\lambda$ is set to 2 and the fitted $\Delta T_m$ by DeMott et al. (1997) is adopted. The curves for models using the $T_{eff}$ approach agree fairly well, e.g., 1°C difference in temperature where $J_{haze}V = 1$ s$^{-1}$ for a 5% weight percent solution droplet. This is not surprising because, as long as these schemes use an identical value of $\lambda$ and the same formula to compute
\( \Delta T_m \), the differences in nucleation rate is solely determined by the differences in the functional specification of the homogeneous nucleation rate of ice in pure water \( J_w \). This figure suggests that, at the critical freezing condition \( (J_{\text{haze}} V = 1 \text{ s}^{-1}) \), the formulation differences for \( J_w \) in the \( T_{\text{eff}} \) group at most contribute an absolute 1% difference in the critical solute weight percent (hereafter wt%).

The classical approach to determine \( J_{\text{haze}} \), as in model J, can be interpreted as a \( T_{\text{eff}} \) scheme with varying \( \lambda \). Given temperature, the classical approach would obtain smaller \( J_{\text{haze}} V \) compared to the \( T_{\text{eff}} \) approach when solution is dilute (e.g., \( T = -45^\circ\text{C}, \text{wt}\% = 5 \)). The opposite is found when solution is more concentrated (e.g., \( T = -45^\circ\text{C}, \text{wt}\% = 15 \)). Given solute wt\%, model J produces larger \( J_{\text{haze}} V \) than those using \( T_{\text{eff}} \) approach when temperature is warm (e.g., \( T = -35^\circ\text{C}, \text{wt}\% = 5 \)). The condition reverses as temperature decreases (e.g., \( T = -45^\circ\text{C}, \text{wt}\% = 5 \)). Therefore, the intrinsic \( \lambda \) in the classical approach varies inversely with solution concentration and temperature (Fig. 3). Also following this figure, the inferred \( \lambda \Delta T_m \) from model J at these two temperature regions does not approach 0 as solute wt\% decreases to 0. This may be a potential problem.

For a given temperature, \( J_{\text{haze}} V \) computed by the two approaches shows high sensitivity to the haze particle solute wt\% (Fig. 2). However, the gradient of \( J_{\text{haze}} V \) to solute wt\% in the \( T_{\text{eff}} \) approach is much greater. For instance, at \( T = -45^\circ\text{C} \), \( J_{\text{haze}} V \) computed by model J ranges roughly from \( 10^{-8} \) to 1 s\(^{-1} \) for solute wt\% from 15 to 5%; whereas the \( T_{\text{eff}} \) approach models render a range from \( 10^{-21} \) to 10 s\(^{-1} \) for the same solute wt\% range. In the next section, we will show that this difference in the sensitivity of \( J_{\text{haze}} V \) to solution concentration is the dominant factor leading to the systematic differences in the freezing haze size distributions between the two approaches.
It is not known if the behavior predicted by classical nucleation theory as treated in model J is a consequence of the way that theory was constrained by a single laboratory data set, or if it reflects a real expected consequence of solution effects on freezing. Laboratory measurements usually focus on the $J_{haze}V = 1 \text{ s}^{-1}$ region. Nucleation rate data over a wide range of values, particularly data points beyond the critical freezing condition, are needed for solutions of similar composition to address this issue.

Differences in $\lambda$ affect the required humidity to trigger nucleation. If $\lambda$ is the only varying component, it's expected that homogeneous nucleation is first triggered in the model with the smallest $\lambda$ because particles freeze at higher solute wt% which entails a lower RH$_w$ condition. Therefore, the range of wt% correspond to a given $\lambda\Delta T_m$ might be used to estimate the triggering RH$_w$ condition, at least, to a first order basis. It is shown from Fig. 3 that this wt% range increases with $\lambda\Delta T_m$. Meanwhile, the required $\lambda\Delta T_m$ to trigger nucleation increases with a decrease in temperature. We may then conclude that the nucleation triggering RH$_w$ becomes more sensitive to $\lambda$ as temperature decreases. This will be further demonstrated in the model results (Section 4).

b. Heterogeneous nucleation

Little constraint was imposed on formulating heterogeneous nucleation because theoretical and experimental understanding is still quite poor. For the ALL-MODE simulations, both parameterized and explicit heterogeneous nucleation calculation are used. Models C and L employ ice saturation ratio dependent parameterizations of activated IN as

$$N_{IN} = A(S_i - 1)^B,$$  \hspace{1cm} (6)
following Spice et al. (1999); and

\[
N_{IN} = \exp[\alpha + 100\beta(S_i - 1)],
\]

following Meyers et al. (1992), respectively. Note that the values of the coefficients mentioned in this section are listed in Notation. Hence, number concentrations of ice crystals produced by heterogeneous nucleation are controlled solely by the peak \( S_i \) in the simulations.

At relatively high peak \( S_i \) condition, the parameterized \( N_{IN} \) by these two formulas may exceed the range of 0.1 to 100 \( L^{-1} \) measured by Rogers et al. (1998) from aircraft, c.f., \( N_{IN} \) becomes 1000 \( L^{-1} \) when \( S_i \) is 1.38 (Eq. 6) or 1.48 (Eq. 7). Although most of the flights were spent at temperature between -35 to -60°C, Rogers et al. (1998) processed conditions from about -15 to -40°C and from ice saturation to \(~15\% \) water supersaturation. Since activation of IN depends on temperature and humidity, it is possible that more IN are active at lower temperature than those considered in the measurements by Rogers et al. (1998). Nevertheless, the formulas Eq. 6 and 7 are expected to represent a maximum heterogeneous nucleation impact.

Models D and S treat heterogeneous freezing in an explicit fashion. Haze particles of the given \( H_2SO_4 \) aerosol distribution are subject simultaneously to heterogeneous and homogeneous nucleation. The unfrozen haze particles are tracked during the entire simulation. Model D assumes that all heterogeneous ice nuclei were freezing nuclei contained within 10\% of the cloud condensation nuclei (CCN) population at sizes above 0.1 \( \mu m \). Further, these particles were assumed to contain 50\% insoluble matter (bulk density of 1.9 \( g \ cm^{-3} \)), the rest being \( H_2SO_4 \). These choices have a basis in observations
from DeMott et al. (1998) at temperatures down to -35°C. The average freezing nuclei spectrum from that study was employed and extrapolated to cirrus temperatures in this study. Thus, the fraction of ice nuclei from each haze category $k$ (> 0.1 $\mu$m) were predicted according to

$$F_k = \frac{a}{F_{>0.1\mu m}}(T_0 - T_{eff,k})^b,$$

where $F_{>0.1\mu m}$ is the fraction of CCN with size greater than 0.1 $\mu$m compared to the total CCN. This treatment was expected to yield the most conservative (minimal) estimate of heterogeneous ice nuclei concentrations in cirrus.

Model S assumes that the ice particle formation due to heterogeneous nucleation follows a $T_{eff}$ modified Fletcher function given by Khvorostyanov and Sassen (1998a) as

$$\frac{dN_i}{dt} = \sum_k A_{s0}B_s \exp(-A_s z) \exp[B_s(T_0 - T_{eff,k})] \frac{dT}{dt}.$$  \hspace{1cm} (9)

Note that this equation introduces a height dependent term into the concentration of ice nuclei in the upper troposphere. A higher than normal value of $B_s$ (0.8 compared to 0.4 to 0.6 of the "typical cirrus" in Sassen and Benson (2000)) is arbitrarily used to simulate the condition in which heterogeneous nucleation competes most favorably. Note that model S, using the particle tracing scheme, records the evolution of a total of 20000 particles, or a parcel of volume 100 cm$^3$ at the starting point of simulation. Therefore, its minimum detectable change in ice number concentration is ~0.01 cm$^{-3}$.

c. Haze particle size and solution concentration

There are differences in the specification of haze size and solute wt% (Table 2). These differences may significantly affect model results, e.g., nucleation triggering humidity, ice particle formation rate, and freezing haze distributions.
The volume of a haze particle is given by

$$V = m_s \frac{1 + \frac{1000}{M_s c}}{\rho_s(M)}$$

(10)

where \(M\), \(m_s\), and \(\rho_s\) are molality (mol of solute per 1000 g of water), solute mass, and solution density, respectively, in C. G. S. units. Within the temperature and moisture ranges considered in this study, the volume of equilibrium-sized haze particles of a given solute mass varies within a factor of 10. Following Eq. 10 and Fig. 2, the sensitivity of \(J_{haze}V\) to haze volume is far less than to solution molality, temperature, and \(\lambda\). However, if haze radius is the prognostic or diagnostic variable in a model, and the water mass of haze is derived by \(m_w = \rho_s V - m_s\), solution density is not a trivial issue. The difference in \(\rho_s\) would significantly affect the obtained solute wt%. Measurements of the H$_2$SO$_4$ solution density for temperature below 0°C were not available until the work by Myhre et al. (1998). They showed that extrapolation of existing \(\rho_s\) data which were measured above 0°C matches well with their measurements. Participants of this project used different formulae for solution density, explicitly or implicitly (Fig. 4). We suggest that solution density, in future studies, be specified with care.

Haze particles are in equilibrium with the environment except (i) when temperature is greater than -42°C and the required RH$_w$ to trigger homogeneous nucleation is close to, or greater than, unity (Heymsfield and Sabin 1989); and (ii) in a strong updraft associated with a cold environment (to be discussed more in the following). When equilibrium is assured, the haze particle size can be obtained by iteration either using (e.g. models D
\[ S_w = a_w \exp\left( \frac{2\sigma_{s/a}}{\rho_w R_w T_r} \right) \]  

where \( a_w = \exp\left( -\frac{\nu \Phi_s m_s}{m_w M_w} \right) \), or the dilute solution approximation (\( \ln S_w \approx S_w - 1; \Phi_s \approx 1; m_w \gg m_s; \rho_s \approx \rho_w \), e.g. models C and S\(^3\)):

\[ S_w - 1 = -\frac{3\nu (M_s / M_w) m_s}{4\pi \rho_w r^3} + \frac{2\sigma_{s/a}}{\rho_w R_w T_r}. \]  

If using a temperature-independent \( a_w \) formula (Chen 1994), the haze equilibrium radius is not very sensitive to temperature within the cirrus temperature range (-45 to -65 °C) considered in this project, especially for haze with solute mass greater than \( 10^{-15} \) g. In other words, Kelvin’s effect (or the curvature effect) is controlled by the value of solute mass and is not sensitive to a 10% change in either temperature nor haze surface tension \( \sigma_{s/a} \) within this temperature range. If using a temperature-dependent \( a_w \) formula (Clegg and Brimblecombe 1995), the required \( S_w \) for the equilibrium condition fluctuates within ±0.005 in this temperature range. This temperature and molality dependent water activity is adopted by models D and J.

Figure 5 compares solute wt% computed by models C (Eq. 12) and L (Eq. 11) as the solid and dashed lines, respectively. The obtained equilibrium haze solute wt% by model C can be a few percent lower (higher) than that obtained by model L in the high (low) ambient humidity RH\(_w\) condition. Note that substantial differences in solute wt%.

\(^2\)Model L used fitted functions based on (11).

\(^3\)Model S uses equation (50) of Fukuta and Walter (1970) to compute the equilibrium haze radius. Yet, the model sets \( \Phi_s = 1, \nu = 3 \). Model S renders equilibrium haze solute wt% close to that of model C, which also sets \( \nu = 3 \), when solution is dilute.
exist even at RH_w \sim 95\%, probably because the assumption that \Phi_s = 1, \nu = 3 in the
dilute approximation deviates significantly from the \alpha_w used by models D and L. Curves
of equilibrium haze volume for the two approaches gradually become divergent as RH_w
decreases.

The equilibrium haze solute wt\% by model S coincides with those of model C when
humidity is high. However, they deviate from model C as humidity decreases. This
trend results from the differences in solution density specification in the two models and
demonstrates the importance of \rho_s when acquiring equilibrium solution concentration from
haze radius. This result has obvious consequences on \lambda \Delta T_m and thus nucleation rate. In
the cases that haze particles are approximately in equilibrium with the environment (will
be discussed further in the Section 4a1), Fig. 5 may provide useful information about the
nucleation triggering RH_w.

Multi-dimensional models cannot afford to iterate. Thus, further simplification is
required for parcel models which are modules of multi-dimensional models. Model L uses
fitted functions based on Eq. 11. Model J neglects Kelvin's effect and sets \alpha_w = S_w,
for which the equilibrium haze solute wt\% becomes only a function of environment S_w.
As shown in Fig 5, Kelvin's effect results in 1 to 2\% solute wt\% differences between
haze containing solute 10^{-15} and 10^{-12}g. This is a significant difference (Fig. 2). Hence,
ignoring Kelvin's effect enhances the freezing possibility of particles with small solute
mass and leads to the broadening of the freezing haze particle size distribution.

The equation for diffusional growth of haze particles is given by (Fukuta and Walter 1970)

\[
\frac{dm_w}{dt} = \frac{4\pi r (S_w - S_w^*)}{R_s T c_{s,w} D^* + (\frac{I_w}{R_s T} - 1) \frac{L_w}{TK} \rho_w \rho_* S_w^*},
\]  

(13)
where $S^*_{w}$ is obtained by substituting the instantaneous haze radius and solute wt% into
Eqs. (11) or (12) depending on the models. The heat transfer contribution term (the
second term in the denominator) is more than an order of magnitude smaller than the
mass transfer term (the first term in the denominator) in the temperature range considered
here. Thus, the following discussion focuses only on the effect of the mass transfer
term and its sensitivity to condensation coefficient, $\beta_w$. The diffusional growth rate
decreases exponentially with temperature and reflects the strong temperature dependence
of saturation water vapor pressure. The response time scale to the deviation from
equilibrium can be considerably greater than one model time step in a cold environment.
The stronger the updraft, the wider the gap between these two time scales may be.
Therefore, haze with high solute mass become more concentrated than the corresponding
equilibrium-size particles in a cold environment and strong updraft conditions.

Haze particle growth rate is sensitive to $\beta_w$, which modifies the value of $D'$ and has
to be included with caution. Both models C and S use $\beta_w = 0.05$ resulting in considerable
delaying of haze growth in several simulations. In contrast, diffusional growth of haze
particles does not cause noticeable effects on haze solute wt% in simulations by model L
because a much higher $\beta_w$ is used.

Figure 6 shows haze solute wt% with respect to solute mass by model C just before
homogeneous nucleation is triggered. Large haze particles deviate from the equilibrium
condition (approximately the 0.04 m s$^{-1}$) as the strength of updraft increases. The
haze particles that are most dilute are not necessarily the largest ones when the limit of
diffusional growth is imposed (e.g., the Ch100 curve). In comparison, the deviation from
equilibrium is not as significant in the warm cases. Haze solution concentration is close
to being in equilibrium with the environment in the slow and moderate updraft cases (Wh004 and Wh020). Note that ±1% in wt% at wt% = 20% results in approximately a 3°C difference in $T_{eff}$ when $\lambda = 2$ (Fig. 3).

4. Model results and discussion

A parcel lifted by an updraft goes through four stages (Fig. 7). In the first stage, the parcel cools adiabatically and expands. Haze particles grow, but uptake of vapor by these particles is negligible.

The nucleation regime starts as the first ice crystals form. We arbitrarily define the level at which ice particle number concentration $N_i$ reaches 1 L$^{-1}$ as the beginning of the nucleation regime or cirrus cloud base $z_b$ (relative to the parcel initial height $z_{ini}$). Ice water content (IWC) and $N_i$ increases afterwards. In the beginning of this stage, $\text{RH}_i$ tendency ($\frac{d\text{RH}_i}{dt}$) is positive but decreasing in magnitude due to vapor uptake by growing ice crystals. Homogeneous nucleation, if activated, shuts off shortly after $\text{RH}_i$ reaches its peak value. The homogeneous nucleation regime takes place within relatively a small range of vertical displacement of the parcel.

The next regime features a rapid decrease in $\text{RH}_i$ corresponding to the rapid increase of IWC. $\text{RH}_i$ tendency starts from 0, falls to a minimum, then increases asymptotically to zero. The equilibrium regime begins when $\frac{d\text{RH}_i}{dt} \sim 0$. In general, the parcel’s temperature tendency $\frac{dT}{dt}$ is close to ice moist-adiabatic in the equilibrium regime, although this may depend on the value of $\text{RH}_i$ at equilibrium (hereafter, $\text{RH}_{i,eq}$).

Comparison of $N_i$ generated by the different models is the center piece of this project (Fig. 8). Note that $N_i$ at 800 m, where little additional nucleation is occurring, above
$z_{\text{ini}}$ is compared. Temperature differences among models at this level are negligible. In the HN-ONLY cases, to a first order approximation, the logarithm of ice crystal number concentration increases quasi-linearly with the logarithm of updraft speed. Increase of $N_i$ with $W$ was first demonstrated by Heymsfield and Sabin (1989) and Sassen and Dodd (1989) and followed by others. This figure demonstrates that while all models manifest this trend of $N_i$ with $W$, the quantitative differences among models can not be overlooked and have to be studied in detail. The predicted $N_i$ by models D, S and L are close. The predicted $N_i$ by models J and C tend to form the lowest and highest bounds in the six cases, respectively. The difference in $N_i$ among the models reach as large as a factor of 25. Comparing the predicted $N_i$ in these two temperature regimes, generally speaking, more ice crystals are generated in the cold cases. This tendency was also first reported by the aforementioned pioneer works. Jensen and Toon (1994) conjectured that the inverse trend of ice crystal growth rate with temperature was the primary cause.

The cloud base RH$_i$, $z_b$ and $\Delta$RH$_i$ (peak RH$_i$ minus RH$_i$ at $z_b$) increase with the updraft speed in the HN-ONLY cases (Fig. 9). Cirrus forms in a narrower range of altitude in the warm cases ($360 < z_b < 400$ m, Fig. 9b) than in the cold cases ($325 < z_b < 410$ m, Fig. 9a). Following the research of Lin et al. (1998) on cloud properties sensitivity in lift-limited situations, if the depth of updraft region is incidentally between the minimum cloud base height and the maximum nucleation-ceasing altitude of all models, the discrepancy in the predicted $N_i$ would be greatly enhanced.

The corresponding range of cloud base RH$_i$ in the warm cases is also smaller than that in the cold cases. The primary factor of the increasing sensitivity of the cloud base RH$_i$ as temperature decreases in the four $T_{\text{eff}}$-approach models has been discussed in
section 3a: the range of critical freezing $RH_w$ corresponding to a given range of $\lambda$ increases with the decrease of ambient temperature. Generally speaking, smaller $\lambda$ entails lower cloud base height. Other factors (e.g. differences in solution density, specification of haze particle solution concentration, saturation water vapor pressure over water) may also affect the cloud base $RH_i$. This is why in the cold cases, models C and D using $\lambda = 1.5$ obtain different cloud base $RH_i$, and in the warm cases the sequence of the time that nucleation is triggered does not follow the values of $\lambda$.

The cloud base $RH_i$, $z_b$, and peak $RH_i$ in the ALL-MODE cases vary even more because of our respective unbounded choices of heterogeneous nucleation (Fig. 10). The impact of heterogeneous nucleation on lowering $N_i$, peak $RH_i$, and cloud formation altitude is extremely sensitive to the onset conditions for nucleation and the subsequent rate of ice formation. Switching on heterogeneous nucleation, the peak $RH_i$ is lower in all but the case Wa100 by model S, possibly because of the height dependency in ice nuclei concentration. The predicted $N_i$ is reduced in all but the case Ca004 by model S, which will be discussed further below.

Here we define the $N_i$ reduction ratio as the $N_i$ of the ALL-MODE case over that of the corresponding HN-ONLY case (Table 3). The small $N_i$ reduction ratio in models C and L may be traced to their sensitivity to $S_i$ that is not mitigated by any attachment to aerosol number concentration or size and the low $RH_i$ onset condition for heterogeneous nucleation (e.g., $S_i > 1$ in model L). The difference of $N_i$ between ALL-MODE and HN-ONLY cases by model D is only prominent in the warm and slow updraft case (Wa004). The weak influence of heterogeneous nucleation in model D is tied to two factors. First, the maximum IN concentration predicted by the formulation are no more
than a few hundred per liter in the range of conditions simulated. Secondly, the lowering of the onset RH\textsubscript{i} for ice formation is limited by solute concentration via the use of effective freezing temperature in the immersion freezing mechanism.

The \( N_i \) reduction ratios of model S are significantly higher at the 0.04 m s\textsuperscript{-1} cases. The predicted \( N_i \) in Ca004 is even larger than that of Ch004; \( N_i \) of Ca004 increases exponentially with height from 200 m above the starting level (not shown). It is possible that the anomaly behavior for this single model run results from the large value of \( B_s \) used. However, this large \( B_s \) is the only value capable of producing significant heterogeneous nucleation, in view of the height-dependent term in Eq. 9. More study is required to understand this anomaly.

Recent model intercomparison projects demonstrate that the benchmark is not necessarily the median or the average of model results. With this in mind, we investigate the reasons causing the noted differences, emphasizing the HN-ONLY simulations.

\textit{a. Causes of variability in the \( \lambda \)-fixed simulations}

The number density of haze particles freezing in time interval \( \Delta t \) is formulated as

\[
\frac{dN_i}{dt} \Delta t = \sum_k N_{\text{haze},k}(t) [1 - \exp(-J_{\text{haze},k}V_k \Delta t)],
\]

where the summation in \( k \) is the integration over haze particle size bins, and \( \frac{dN_i}{dt} \) is termed as ice particle formation rate afterwards. It can be shown that the predicted ice particle number density is simply

\[
N_i = \sum_k N_{\text{haze},k}(t = 0)(1 - \exp(-\int_0^t J_{\text{haze},k}V_k dt)).
\]
predicted $N_i$. Given temperature, size and solute wt% of a haze particle, the difference in formulating the homogeneous freezing nucleation would affect the nucleation rate on one single haze particle level. Haze particle specification and evolution, together with homogeneous freezing nucleation formulation, determine the instantaneous ice particle formation rate. Given an uplift speed, the diffusional growth rate of ice crystals in combination with all of the above components control the water vapor uptake rate, and thus the evolution of the parcel RH$_w$, which then determines the predicted $N_i$. The results of the $\lambda$-fixed simulations are examined to understand these interactions and feedback in depth.

1) The level at which homogeneous nucleation is triggered

We now discuss the results of the simulations for which participants set the coefficient $\lambda = 2$ (or an approximation of this in model J). These simulations (Ch020L and Wh020L) sought to minimize differences in predicted nucleation rates as a source of discrepancies in the results. It was found that the nucleation regimes of participant models occurred within the temperature range of -43.2 to -44.2°C and -63.2 to -64.2°C in the two simulations. The effect of temperature variation on nucleation rates within this 1°C range is secondary compared to the evolution of RH$_w$. Therefore, it is justified to analyze model results according to the $z - z_b$ level. The results of this analysis are shown in Fig. 11.

The range of triggering RH$_w$ was reduced as expected to less than 2% in Wh020L and 5% in Ch020L in comparison to 3% and 8% in simulations Wh020 and Ch020. Note that participants use different formulae to compute saturation vapor pressure over water and ice. These formulae gradually diverge as temperature becomes lower than -45°C so we
use RH\textsubscript{w} here. Its effects on predicted $N_i$ will be further explored in the Phase 2 of this project.

There are still significant differences in $N_i$ among the models. It can be seen that $N_i$ is only slightly changed by the value of $\lambda$ when comparing $N_i$ of cases $\lambda = 2$ with the corresponding $\lambda$-unspecified cases. In the cold cases, the increase in $\lambda$ results in a slight decrease in the predicted $N_i$ in models D and L whereas the predicted $N_i$ by models C and S are not much affected. Contrarily, in the warm cases, the increase in $\lambda$ results in a slight increase in $N_i$ in the four models of the $T_{eff}$ approach family. Additional study of the effect of $\lambda$ is planned in Phase 2 of this project. In the light of these results, $\lambda$ is not the dominant factor controlling the number concentration of ice crystals nucleated.

In Wh020L, haze particles are almost in equilibrium with the environment. Nucleation is triggered at around RH\textsubscript{w} = 95.5% in models C, J and S, suggesting that aerosols with solute wt\% around 8-9\% in models C and S and 11\% in model J start to freeze at this condition (Fig. 5). As shown in Fig. 5, in models D and L, the equilibrium-sized aerosols of solute wt\% 8-9\% are not formed until ambient RH\textsubscript{w} is around 97\%. This is probably the reason why the nucleation triggering RH\textsubscript{w} by models C and S is about 1\% lower than those of models D and L.

Compared with Wh020L, the order of nucleation triggering RH\textsubscript{w} in Ch020L switches among the four models using $T_{eff}$ approach. Models D and L have the lowest nucleation triggering RH\textsubscript{w} in this case. As shown in Fig. 5, if models S and C turn off diffusional growth of haze particles and use equilibrium-sized particles instead, models S and C would have a lower or similar triggering RH\textsubscript{w} than those of models D and L, respectively. It is suspected that the diffusional growth of haze particles may delay the nucleation being
2). The nucleation regime

At the beginning of the nucleation stage in simulation Wh020L, ice particle formation rates are close among the four models of the $T_{eff}$ approach group (Fig. 11). However, models C and D reach much higher $\text{RH}_w$ that leads to higher instantaneous nucleation rates, and maintain the peak ice formation rate longer than the other two models. The ranges between $\text{RH}_w$ at $z_b$ and peak $\text{RH}_w$ thus are higher in these two models. As a consequence, model C has the highest $N_i$ followed by model D.

The curve grouping of ice particle formation rates in simulation Ch020L are not exactly the same as those in Wh020L. From the start of the nucleation stage in this simulation, the $N_i$ curves of models D and L distinctly separate from those of models C and S. This grouping incidentally coincides with the grouping according to the haze particle specifications. Haze particles are in equilibrium or close to equilibrium in models D and L while large haze particles are more concentrated than the corresponding equilibrium values in models C and S. The ice particle formation rates in models C and S are more than one order greater than those by models D and L. Yet, the nucleation regime in model S does not sustain as long as in model C; a similar observation is noted when comparing results of model D and L. Near the end of the nucleation regime, model C has the highest $N_i$. Model S has slightly larger $N_i$ than model D, despite the fact that the former has a much higher initial ice particle formation rate. The long nucleation duration in model D eventually produces a value of $N_i$ that is comparable to model S. The reason for these discrepancies among $T_{eff}$ group is a complicated combination of differences
in haze specification (discussed in this paragraph), deposition coefficient, and numerical
procedures (to be discussed in this section later).

The ice particle formation rate by model J is smaller than the rest. The range of
RH \_w between \_z_b and peak is also wider in simulations by model J. From Fig. 12, the
freezing haze number distribution in simulations by model J is much broader. These three
distinct features of model J might result from a combination of the following elements.
The homogeneous nucleation scheme of model J is less sensitive to solution concentration
compared to the others; the curvature effect is ignored when computing haze size; and
with ice particle formation rate an order of magnitude smaller than the others, and with
the long nucleation duration, the average size of ice crystals in model J can be much
greater when nucleation stops.

To a first order approximation, the logarithm of ice particle formation rate is
parabolically centered at peak \( \frac{dN_i}{dt} \) (Fig. 11), suggesting that the curve of ice particle
formation rate along the z axis can be approximated by a normal distribution. Therefore,
the ice crystals nucleated \( N_i = \int_0^t \frac{dN_i}{dt} dt \) can be approximated as \( N_i = \sqrt{\frac{2}{\pi}} \frac{dN_i}{dt} \_p \Delta t \), where
\( \Delta t \) is the width of the curve and \( \frac{dN_i}{dt} \_p \) is the peak value of ice particle formation rate.
The evolution of ice water content, which is the cumulative water vapor uptake, as
aforementioned, will control the peak RH \_w and \( \frac{dN_i}{dt} \_p \). The vapor uptake rate at any
instance is equal to the summation of ice crystal growth rate times \( N_i \). The growth rate
of small ice crystals \( (r < 10 \mu m) \), under the influence of the kinetic effect, is quite sensitive
to the deposition coefficient \( \beta_i \). Sensitivity tests on \( \beta_i \) have been performed to check its
effect in isolation. It is found by both models D and L that varying \( \beta_i \) from 0.04 to 1, the
range of \( \beta_i \) values specified by participants (Table 2), would result in about a factor of 4~5
and 9~12 variation in the predicted $N_i$ in Wh020L and Ch020L, respectively (Fig. 13). Nucleation begins at the same RH$_w$, but simulations with small $\beta_i$ reach high RH$_w$ and trigger freezing of many small haze particles. The predicted $N_i$ decreases as $\beta_i$ increases from 0.04 to 0.2; it only adjusts marginally to $\beta_i$ when 0.2 < $\beta_i$ < 1. It is worth noting that by adjusting $\beta_i$ alone, model D was able to obtain $N_i$ predicted by model J.

A systematic bias in $N_i$ and IWC relationships among models is prominent (Fig. 14). At a given $N_i$ level, model C uptook the least amount of water vapor while model J the most. Eventually, model C (J) stopped producing ice particles at much higher (lower) $N_i$ compared with other models. The mismatch between $\beta_i$ values and the $N_i$ and IWC bias suggests that the difference in $\beta_i$ is a crucial but not the only component to result in this systematic bias. It is conjectured that an intrinsic characteristic of the numerical component, e.g., bin characteristic, would also affect the ensemble growth rate. By increasing the frequency to rebin ice particle size distribution (every 5 m to every 0.5 s), the predicted $N_i$ of model C is significantly reduced. Results after revision by model C will be discussed more in the forthcoming paper about Phase 2 of CPMC.

b. All-mode simulations versus homogeneous-nucleation-only simulations

Heterogeneous nucleation is a possible explanation for the discrepancy between the observed threshold RH$_w$ required for cirrus formation and the theoretically derived threshold RH$_w$ to trigger homogeneous nucleation in H$_2$SO$_4$ or (NH$_4$)$_2$SO$_4$ solution particles (Heymsfield and Miloshevich 1995; DeMott et al. 1997). Cirrus microphysical and radiative properties are affected by the dominant nucleation mode in cloud initiation because ice particle formation rates of the two modes are expected to be distinct. First of
all, the threshold humidity for heterogeneous nucleation of ice in a water-subsaturated and ice-supersaturated environment may be much less than that for homogeneous nucleation. If the upward motion is confined within a thin layer, in which the maximum humidity that may be achieved by an ascending parcel is between the threshold humidity of the two modes, the presence of IN will determine if cirrus forms or not. If the vertical displacement of parcels is big enough to provide the necessary condition to trigger homogeneous nucleation, the presence of IN may partially or completely suppress homogeneous nucleation and change the cloud properties depending on ice nuclei characteristics (Sassen and Benson 2000). Generally speaking, the biggest difference in $N_i$ between the ALL-MODE and HN-ONLY simulations takes place when the updraft speed is slow to moderate and homogeneous nucleation is completely suppressed (Fig. 8 and Table 3). Complete suppression of homogeneous nucleation is achieved when the uptake of excess water vapor by ice crystals formed by heterogeneous nucleation sufficiently lowers the maximum humidity achieved by the parcel. Hence for a given cooling rate, a larger difference between the threshold humidity of the two modes would provide particles more time to grow and thus a more favorable condition for suppression. Of all ALL-MODE simulations by model D, homogeneous nucleation is completely suppressed only in case Wa004. It is found that the difference of threshold humidity of the two modes in model D is the smallest. This directly relates to the conservative estimate of the concentrations of available heterogeneous ice nuclei (as a function of T and RH) made in model D.

Information about detailed ice crystal size distributions enables us to compute the
volume absorption coefficient of $b_{abs}$ as

$$
b_{abs} = \sum_k Q_{abs,k} \pi r_k^2 N_{ice,k},
$$

(16)

where $Q_{abs}$ is the absorption efficiency obtained from Mie theory assuming spherical ice particles. The absorption efficiency at wavelength 10.75 $\mu$m increases from 0.27 for ice particles $r = 1\mu$m to 1.12 for particles $r \sim 15\mu$m, and gradually decreases to $\sim 1.0$ as particles radius equals 250 $\mu$m.

From Fig. 15, on average, the absorption coefficient increases with updraft speed. The difference in absorption coefficient among models in the HN-ONLY simulations can be more than 100%. Except for simulation Ca100 by models C and S, $b_{abs}$ of HN-ONLY cases is higher than that of their ALL-MODE counterparts; maximum differences exceeding 100% are found between ALL-MODE and HN-ONLY simulations. Though the uncertainty of estimated ice crystal number concentration is 25-fold in the extreme, the uncertainty of estimated optical properties is about twofold for a given cooling rate.

Ice crystal size distribution information also gives us a unique chance to check the calculation of cloud optical properties using effective radius. Current parameterization of cloud optical properties usually incorporates $b_{abs}$ in the form

$$
b_{abs} = \kappa(d_e)IWC,
$$

(17)

where $\kappa$ and $d_e$ are the mass absorption coefficient and the effective length of the ice crystal distributions, respectively (e.g., Ebert and Curry 1992; Fu et al. 1998). In Fig. 15, $r_e$ is computed according to $r_e = \frac{\sum_k r_k^4 N_k}{\sum_k r_k^2 N_k}$ (Ebert and Curry 1992). We do not have to worry about the definition of $r_e$ used because in our simple simulations assuming spherical particles, all types of definition of $d_e$ converge (McFarquhar and Heymsfield 1998). Note
that the mass absorption coefficient decreases as effective radius increases only in the range of $r_e$ greater than ~20 $\mu$m.

The maximum difference in IWC among models for the cold or the warm cases is less than 15%. It is evident that the difference in IWC can not explain the large difference in $b_{abs}$ for two simulations with similar $r_e$. The effective radii of case Ch020 obtained by models D and S are about the same. However, $b_{abs}$ of the two models calculated according to Eq. 16 shows significant difference. We found that the ice crystal distribution by model D is much broader than that by model S in this case.

The difference of $N_i$ between ALL-MODE and HN-ONLY is the least when $W = 1$ m s$^{-1}$. However, except for model D, $b_{abs}$ in cases Ch100 and Ca100 for all models are close unlike the values of $b_{abs}$ in cases Wh100 and Wa100 (Fig. 15). We speculate that the growth rate difference in these two temperature regions is the major factor. The growth rates of ice crystals in the warm cases are much higher than those in the cold cases. As a consequence, ice crystal distributions in Wa100 are considerably broader than their corresponding Wh100 simulations. This is not so in the cold cases.

The mass weighted terminal velocity $V_t = \frac{\sum_k V_{t,k} m_k N_k}{\sum_k m_k N_k}$ is also plotted in Fig. 15. As far as ice mass flux is concerned, the model-to-model differences in $V_t$ are negligible in the $W = 1$ m s$^{-1}$ cases. Ice mass flux is weighted favorably to the amount of large particles. Percentage-wise, the resultant difference in ice mass flux between HN-ONLY and ALL-MODE cases is bigger than that in $b_{abs}$.

The $R_{H_{\text{eq}}}$ (see Section 4) of parcel models provides GCM and cirrus models, which use a moisture-adjustment scheme to convert excess water vapor to cloud ice mass (e.g., Starr and Cox, 1985; Krueger et al., 1995), with a guideline to set the threshold $R_{H_{\text{eq}}}$. It
can be shown that the RH_{i,eq} is proportional to W and inversely related to N_i. Figure 16 demonstrates the relation between these three components at 800 m above the starting point of the simulations. The RH_{i,eq} is well within 110% except for cases Wa020 and Ca020 by model L. However, the moisture-adjustment time scale, the time interval between the peak RH_i and RH_{i,eq}, also has to be considered to render a complete view. Note the moisture-adjustment time scale defined in this paper is conceptually similar to the phase relaxation time defined in Khvorostyanov and Sassen (1998a), yet it differs from the phase relaxation time because the latter is defined according to the local gradient. The moisture-adjustment time almost decreases exponentially with updraft speed. For example, at W = 0.04, 0.2, 1 m s^{-1}, the time scales are roughly 30 ~ 100 min, 6 ~ 40 min and 1 ~ 6 min, respectively (Fig. 15). Therefore, RH_i higher than 110% can be frequently detected in regions of small to moderate cooling rates. This suggests that models using time steps of around 1-hr may set a nucleation RH_i and a threshold RH_i to simulate cold cirrus in a weak (synoptic) ascent; models using smaller time steps face a more difficult modification.

5. Summary

Broad qualitative agreement has been found in the simulations wherein homogeneous nucleation was the only nucleation mechanism activated. The model predicted cirrus microphysical and optical properties are strongly influenced by the updraft speed. However, significant differences were found in the predicted ice crystal number concentrations N_i and ice crystal size distributions. For a given cooling rate, there was up to a factor of 25 difference in N_i, for the conditions simulated, resulting in factors of 2 to 3 differences in
infrared absorption coefficient and ice mass flux at the end of simulation (800 m above the starting point of the parcel ascent).

Detailed examination of the output of HN-λ-fixed simulations revealed that the homogeneous nucleation formulation, haze particle solution concentration, and ice crystal growth rate were critical factors to affect the water vapor uptake rate, the peak ice supersaturation the parcel achieved, and, subsequently, the predicted $N_i$. Our chief findings can be summarized as follows:

- The parameter $\lambda$ mainly affects the required initiation humidity to trigger homogeneous nucleation. Preliminary results show that the predicted $N_i$ is marginally affected by a reasonable choice of $\lambda$.

- Ignoring Kelvin's effect would result in the broadening of the freezing haze size distribution, which represents an enhancement of the model sensitivity to the existence of small haze particles. It is also found that the modified classical scheme produces broader freezing haze size distributions than the $T_{eff}$ approach.

- The equilibrium assumption of haze particle size is violated in strong updrafts in a cold environment. Assuming equilibrium may result in underestimate of ice particle formation rate at the beginning of the cloud initiation stage.

- The condensation/evaporation coefficient for ice, which directly affects the growth rate of small ice crystals, is a critical factor determining the predicted $N_i$.

Disagreement in formulating the homogeneous nucleation of condensed solution particles by the $T_{eff}$ approach and the classical approach indicates that the critical freezing
condition measured in laboratory work provides only a part of information necessary to construct a consistent way to formulate the homogeneous nucleation rate. These results highlight the need for new laboratory and field measurements to infer the correct values for the critical quantities in the cirrus regime.

In general, discrepancies among models were greater in the ALL-MODE simulations. No attempt was made to scrutinize the causes of differences in ALL-MODE simulations due to lack of strict restriction to formulate heterogeneous nucleation. Nevertheless, it was confirmed that the expected effect of a heterogeneous nucleation process is to lower \( N_i \) and the \( \text{RH}_i \) required for cloud initiation, and increase the average size of ice crystals formed in cirrus clouds compared to the case of a singular homogeneous freezing process. Clearly, new measurements of ice nuclei activation in cirrus conditions are warranted. Moreover, it is important to come to grasp with the effects of solution droplet strength and chemistry on the activation of ice nuclei.

This study was conducted based on a single CCN distribution. The sensitivity of predicted cirrus microphysical and optical properties to the assumed CCN distribution must be addressed in order to advance this study. In Phase 2 of the CPMC, now underway, the effects of varying input aerosol distributions (e.g., number concentration, mode radius, and distribution width) are taken into account. Sensitivity of model results to CCN composition will indirectly be made by altering the value of \( \lambda \).
Notation

- **A**: Coefficient in the heterogeneous nucleation scheme used by model C \([2.7 \times 10^{10} \text{ m}^{-3}]\)
- **A_s**: Coefficient in the modified Fletcher eq. used by model S \([10^{-2} \text{ m}^{-3}]\)
- **A_z**: Coefficient in the modified Fletcher eq. used by model S \([7.5 \times 10^{-4} \text{ m}^{-1}]\)
- **a**: Coefficient in the immersion freezing parameterization by model D \([1.3 \times 10^{-23}, \text{ dimensionless}]\)
- **a_w**: Water activity
- **B**: Coefficient in the heterogeneous nucleation scheme used by model C \([10.6, \text{ dimensionless}]\)
- **B_z**: Coefficient in the modified Fletcher eq. used by model S \([0.8 \text{ °C}^{-1}]\)
- **b**: Coefficient in the immersion freezing parameterization by model D \([11.75, \text{ dimensionless}]\)
- **b_{abs}**: Volume absorption coefficient \([\text{m}^{-1}]\)
- **B_s**: Boltzmann constant
- **C_p**: Specific heat capacity for dry air at constant pressure \([\text{J kg}^{-1} \text{ K}^{-1}]\)
- **d_e**: Effective length for a distribution of particles for optical properties calculation \([\text{m}]\)
- **D'**: Coefficient of diffusion of water vapor in air modified by kinetic effect (See Eq. (13-14) of (Pruppacher and Klett 1997)) \([\text{m}^2\text{s}^{-1}]\)
- **e_{s,w}**: Saturation water vapor pressure over a flat pure water surface \([\text{Pa}]\)
- **F_k**: Number fraction of haze particles frozen in bin \(k\)
- **\(\Delta F_{actv}\)**: Activation energy for diffusion of a water molecule across the water-ice boundary \([\text{J}]\)
\( \Delta F_{\text{germ}} \)  Energy of ice-germ formation [J]

\( F_{>0.1\mu m} \)  CCN fraction of haze size greater than 0.1 \( \mu m \)

\( g \)  Acceleration due to gravity

\( h \)  Planck constant

\( \text{IWC} \)  Ice water content [kg m\(^{-3}\)]

\( J_w \)  Homogeneous nucleation rate of ice in pure water [m\(^{-3}\) s\(^{-1}\)]

\( J_{\text{haze}} \)  Homogeneous nucleation rate of ice in solution droplet [m\(^{-3}\) s\(^{-1}\)]

\( K' \)  Coefficient of diffusion of water vapor in air modified by kinetic effect (See Eq. (13-20) of (Pruppacher and Klett 1997)) [J m\(^{-1}\) s\(^{-1}\) K\(^{-1}\)]

\( L_m \)  Average latent heat of fusion of water between \( T \) and \( T_0 \) [J kg\(^{-1}\)]

\( L_v \)  Latent heat of condensation [J kg\(^{-1}\)]

\( M \)  Molality [mol per 1000 g of water]

\( M_s \)  Molecular weight of solute

\( M_w \)  Molecular weight of water

\( m_s \)  Solute mass

\( m_w \)  Water mass

\( N_c \)  Number of monomers of water in contact with unit area of the ice surface [m\(^{-2}\)]

\( N_{\text{haze},k} \)  Number concentration of haze particles in bin \( k \) [m\(^{-3}\)]

\( N_i \)  Number concentration of ice crystals (total) [m\(^{-3}\)]

\( N_{\text{ice},k} \)  Number concentration of ice crystals in bin \( k \) [m\(^{-3}\)]
\( N_{IN} \) Number concentration of activated ice nuclei \([m^{-3}]\)

\( P \) Pressure [Pa]

\( Q_{abs} \) Absorption efficiency

\( Q_{LH} \) Diabatic heating due to latent heat release \([K\ s^{-1}]\)

\( R_d \) Gas constant for dry air \([J\ kg^{-1}\ K^{-1}]\)

\( R_v \) Gas constant for water vapor \([J\ kg^{-1}\ K^{-1}]\)

\( R_{H_i} \) Relative humidity wrt. ice

\( R_{H_w} \) Relative humidity wrt. water

\( r \) Radius [m]

\( r_e \) Effective radius for optical properties [m]

\( S_i \) Water vapor saturation ratio wrt. ice

\( S_w \) Water vapor saturation ratio wrt. water

\( T \) Temperature [K]

\( T_0 \) Reference temperature 273.15 K

\( T_{eff} \) Effective freezing temperature [K]

\( \Delta T_m \) Melting point depression [K]

\( t \) Time [s]

\( \Delta t \) Time interval [s]

\( V \) Volume \([m^3]\)
\( V_t \) Terminal velocity [m s\(^{-1}\)]

\( W \) Updraft speed [m s\(^{-1}\)]

\( z \) Height [m]

\( z_b \) Height at which ice number concentration becomes 1 L\(^{-1}\) [m]

\( \alpha \) Coefficient in the heterogeneous nucleation scheme used by model L \([7.5468, \text{dimensionless}]\)

\( \beta \) Coefficient in the heterogeneous nucleation scheme used by model L \([0.1296, \text{dimensionless}]\)

\( \beta_i \) Deposition coefficient

\( \beta_w \) Condensation coefficient

\( \lambda \) See section 3.

\( \Gamma \) Lapse rate [K m\(^{-1}\)]

\( \rho_i \) Ice density [kg m\(^{-3}\)]

\( \overline{\rho_i} \) Average ice density between \( T \) and \( T_0 \) [kg m\(^{-3}\)]

\( \rho_s \) Solution density [kg m\(^{-3}\)]

\( \rho_w \) Water density [kg m\(^{-3}\)]

\( \sigma_{a/s} \) Surface tension across the air and solution interface [J m\(^{-2}\)]

\( \sigma_{i/s} \) Surface tension across the ice and solution interface [J m\(^{-2}\)]

\( \nu \) Dissociation constant for ions in solution

\( \Phi_s \) Molal osmotic coefficient

Subscripts \( eq, ini, k \) and \( p \) denote equilibrium, the initial condition, category \( k \) and the peak value, respectively.
Acknowledgments.

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E. Jensen, NASA/Ames Research Center, Moffett Field, CA 94035

Received ________________
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Figure 12. The number distribution of the haze particles frozen in the lifting process. The initial haze number distribution is denoted by the thin solid curves.

Figure 13. The same as Fig. 11 except this is a sensitivity test on \( \beta_i \) performed by model L.

Figure 14. Ice water content versus \( N_i \). Note that the parcel height increment between two consecutive markers is 5 m.

Figure 15. Volume absorption coefficient at wavelength 10.75 \( \mu \)m (panels [a] and [c]), effective radius (panels [b] and [f]), mass weighted ice terminal velocity (panels [c] and [g]) at 800 m above the starting point of the simulations, and the time interval between peak RH\(_i\) achieved by the parcel and the equilibrium RH\(_i\) (panels [d] and [h]). The filled bars indicate ALL-MODE simulations while the unfilled bars denote HN-ONLY simulations.

Figure 16. The predicted ice number concentration versus the equilibrium RH\(_i\).
Table 1. Simulation identifiers and initial conditions. RH<sub>i,ini</sub> = 100% in every simulation.

<table>
<thead>
<tr>
<th>simulation identifiers</th>
<th>z&lt;sub&gt;ini&lt;/sub&gt; (km)</th>
<th>P&lt;sub&gt;ini&lt;/sub&gt; (mb)</th>
<th>T&lt;sub&gt;ini&lt;/sub&gt; (°C)</th>
<th>W (m s&lt;sup&gt;-1&lt;/sup&gt;)</th>
<th>Γ (°C m&lt;sup&gt;-1&lt;/sup&gt;)</th>
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<tbody>
<tr>
<td>All-MODE</td>
<td>HN-ONLY</td>
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<td></td>
<td></td>
<td></td>
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<td>warm</td>
<td>Wa004</td>
<td>Wa004</td>
<td>8.3</td>
<td>340</td>
<td>-40</td>
</tr>
<tr>
<td>cases&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Wa020</td>
<td>Wa020</td>
<td>8.3</td>
<td>340</td>
<td>-40</td>
</tr>
<tr>
<td></td>
<td>Wa100</td>
<td>Wa100</td>
<td>8.3</td>
<td>340</td>
<td>-40</td>
</tr>
<tr>
<td>cold</td>
<td>Ca004</td>
<td>Ch004</td>
<td>13.4</td>
<td>170</td>
<td>-60</td>
</tr>
<tr>
<td>cases&lt;sup&gt;a&lt;/sup&gt;</td>
<td>Ca020</td>
<td>Ch020</td>
<td>13.4</td>
<td>170</td>
<td>-60</td>
</tr>
<tr>
<td></td>
<td>Ca100</td>
<td>Ch100</td>
<td>13.4</td>
<td>170</td>
<td>-60</td>
</tr>
<tr>
<td>λ-fixed&lt;sup&gt;b&lt;/sup&gt;</td>
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<td>Wh020L</td>
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<td>340</td>
<td>-40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ch020L</td>
<td>13.4</td>
<td>170</td>
<td>-60</td>
</tr>
</tbody>
</table>

<sup>a</sup>λ, if any, is determined by the participants.

<sup>b</sup>λ = 2. Note that model J incorporates parameters adjusted to the laboratory data of Koop et al. (1998) in all λ-fixed and HN-ONLY cases.
Table 2. Participant cirrus parcel models.

<table>
<thead>
<tr>
<th>Organization</th>
<th>UKMO</th>
<th>CSU</th>
<th>ARC</th>
<th>GSFC</th>
<th>U. Utah</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigator</td>
<td>Cotton (C)</td>
<td>DeMott (D)</td>
<td>Jensen (J)</td>
<td>Lin (L)</td>
<td>Sassen (S)</td>
</tr>
<tr>
<td>Time step (s)</td>
<td>0.01</td>
<td>10/W</td>
<td>adjusting</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(W in cm/s)</td>
<td>10&lt;sup&gt;-5&lt;/sup&gt; to 0.02</td>
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<table>
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<tr>
<th>Ice number distribution</th>
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<td>Bin number</td>
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<tr>
<td>Bin characteristic&lt;sup&gt;a&lt;/sup&gt;</td>
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<tr>
<td>Bin advection</td>
</tr>
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<table>
<thead>
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<th>Haze number distribution</th>
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</thead>
<tbody>
<tr>
<td>Bin number</td>
</tr>
<tr>
<td>Haze size&lt;sup&gt;b&lt;/sup&gt;</td>
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</table>

Homogeneous nucleation
<table>
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<tr>
<th>Organization</th>
<th>UKMO</th>
<th>CSU</th>
<th>ARC</th>
<th>GSFC</th>
<th>U. Utah</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investigator</td>
<td>Cotton (C)</td>
<td>DeMott (D)</td>
<td>Jensen (J)</td>
<td>Lin (L)</td>
<td>Sassen (S)</td>
</tr>
</tbody>
</table>

| $T_{eff}$ | yes | yes | no | yes | yes |
| $\lambda^c$ | 1.5 | 1.5 | 1.0 | 1.7 |
| $\beta_i^d$ | 0.24 | 0.04 | 1 | 0.1 | 0.36 |

Heterogeneous nucleation

- $S_i$ dependent
- Immersion
- Not
- $S_i$ dependent
- $T_{eff}$ dependent

- Deposition
- Freezing
- Submitted
- Deposition
- Modified

- Nucleation
- Nucleation
- Nucleation
- Fletcher eq.

References

- Spice et al. (1999)
- DeMott et al. (1994a)
- Jensen et al. (1994)
- Lin (1997)
- Sassen and Dodd (1988)
- DeMott et al. (1998)
- Tabazadeh et al. (2000)
- Khvorostyanov and Sassen (1998b)

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aDiscrete versus continuous binning indicates if assuming that particles have exactly the same size in a bin or it allows a certain size distribution of particles in a bin.

bNote that $r_{eq}$ versus $\frac{dr}{dr}$ denotes either using the equilibrium-sized haze approximation or computing the diffusional growth of haze particles explicitly.
Native value of λ assumed in model.

βi is the deposition coefficient.

Table 3. Ni reduction ratio, which is defined as Ni of the ALL-MODE divided by Ni of the HN-ONLY simulation, 

$\frac{N_i,\text{ALL-MODE}}{N_i,\text{HN-ONLY}}$.

<table>
<thead>
<tr>
<th>Model</th>
<th>C</th>
<th>D</th>
<th>L</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca004</td>
<td>0.039</td>
<td>0.849</td>
<td>0.091</td>
<td>6.500</td>
</tr>
<tr>
<td>Ca020</td>
<td>0.114</td>
<td>0.958</td>
<td>0.073</td>
<td>0.137</td>
</tr>
<tr>
<td>Ca100</td>
<td>0.791</td>
<td>0.979</td>
<td>0.901</td>
<td>0.177</td>
</tr>
<tr>
<td>Wa004</td>
<td>0.083</td>
<td>0.079</td>
<td>0.190</td>
<td>0.667</td>
</tr>
<tr>
<td>Wa020</td>
<td>0.036</td>
<td>0.825</td>
<td>0.108</td>
<td>0.283</td>
</tr>
<tr>
<td>Wa100</td>
<td>0.524</td>
<td>0.931</td>
<td>0.559</td>
<td>0.287</td>
</tr>
</tbody>
</table>
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800 m above starting point (cold case)

800 m above starting point (warm case)