An aerodynamic design procedure that uses neural networks to model the functional behavior of the objective function in design space has been developed. This method incorporates several improvements to an earlier method that employed a strategy called parameter-based partitioning of the design space in order to reduce the computational costs associated with design optimization. As with the earlier method, the current method uses a sequence of response surfaces to traverse the design space in search of the optimal solution. The new method yields significant reductions in computational costs by using composite response surfaces with better generalization capabilities and by exploiting synergies between the optimization method and the simulation codes used to generate the training data. These reductions in design optimization costs are demonstrated for a turbine airfoil design study where a generic shape is evolved into an optimal airfoil.

INTRODUCTION

Artificial neural networks have been widely used in aeronautical engineering. Recent aerodynamic applications include, for example, flow control, estimation of aerodynamic coefficients, compact functional representations of aerodynamic data for rapid interpolation, grid generation, aerodynamic design and reduction of wind tunnel test times by using neural nets to interpolate between measurements (Refs. 1-9). Neural network applications in...
Aeronautics are not limited to aerodynamics. Hajela and Berke (Ref. 10) provide a review of a variety of neural network applications in structural analysis and design.

A new approach to aerodynamic design optimization that is based on neural networks is presented in Refs. 11 & 12. This method offers several advantages over traditional optimization procedures. First, neural networks are particularly suitable for multidimensional interpolation of data that lack structure. They can provide a greater level of flexibility than other methods in dealing with design in the context of unsteady flows, partial and complete data sets, combined experimental and numerical data, inclusion of various constraints and rules of thumb, and other issues that characterize the aerodynamic design process. Second, neural networks provide a natural framework within which a succession of numerical solutions of increasing fidelity incorporating more and more of the relevant flow physics can be represented and utilized subsequently for optimization. Third, and perhaps most important, neural networks offer an excellent framework for multidisciplinary design optimization. Simulation tools from various disciplines can be integrated within this framework. Rapid trade-off studies across one or many disciplines can also be performed.

Another attractive feature of this neural network-based design system (ref. 12) is that it can make efficient use of distributed and parallel computing resources. The method lends itself to multi-tiered parallelism. At the coarsest level, multiple CFD simulations can be performed simultaneously and independently on multiple processors. In situations where individual simulations are computationally intensive, each simulation can also be partitioned across multiple processors. In addition, neural network training can be distributed over multiple processors to further accelerate the design process.

The design method of Ref. 12 incorporates the advantages of both traditional response surface methodology (RSM) and neural networks by employing a strategy called parameter-based partitioning of the design space. Starting from the reference design, a sequence of response surfaces based on both neural networks and polynomial fits are constructed to traverse the design space in search of an optimal solution. The procedure combines the power of neural networks and the economy of low-order polynomials (in terms of number of simulations required and network training requirements).
This method was used in Ref. 12 to reconstruct the shape of a turbine airfoil given the pressure distribution and some relevant flow and geometry parameters. The shape of the airfoil was not known a priori. Instead, it was evolved from a simple curved section of nearly uniform thickness. The evolved optimal airfoil closely matched the shape of the original airfoil that was used to obtain the pressure distribution. This constituted a "blind" test of the design methodology.

This method was also used in Ref. 13 to redesign a generic gas generator turbine to improve its unsteady aerodynamic performance. Although the turbine was originally designed to operate in the high-subsonic regime, an unsteady analysis showed very strong interaction effects including an unsteady shock in the axial gap region between the stator and rotor rows. The method yielded a modified design that was very close to the reference design and achieved the same work output with better unsteady aerodynamic performance by eliminating the unsteady shock.

The method of Ref. 12 was used again in Ref. 14 to enhance the unsteady aerodynamic characteristics of a transonic turbine stage. Design optimization resulted in a weakened stator trailing edge shock which in turn resulted in significant reductions in the dynamic loads on the stator and rotor airfoils and also eliminated unsteady boundary layer separation on the rotor suction surface. These improvements in aerodynamic characteristics were obtained without a reduction in turbine work output. The results presented in Refs. 13 and 14 add to the successful application of the neural net-based design method to design in a steady flow environment and demonstrate the versatility of the method.

The evolution of a turbine airfoil from a simple curved section of nearly uniform thickness that is described in Ref. 12 required about 40 CPU hours on a single processor of a CRAY-C90 to generate the training data (CFD simulations) and to train the neural networks. Clearly, a reduction in the amount of CPU time required for design optimization would increase the usability of the method in fully three-dimensional design optimization. Improvements to the method of Ref. 12, that significantly accelerate the optimization process, will be presented in this paper. This new method has reduced the total CPU requirements for the turbine airfoil evolution study from 40 hours to 24 minutes (a reduction of two orders of
magnitude). The results are presented in this abstract. Details of the design procedure will be included in the final paper.

EARLIER DESIGN METHOD

The design method of Ref. 12 uses a sequence of response surfaces based on both neural networks and polynomial fits to traverse the design space in search of the optimal solution. A technique called parameter-based partitioning of the design space is used, where the functional dependence of the variables of interest (e.g., pressure) with respect to some of the design parameters is represented using neural networks and the functional dependence with respect to the remaining parameters is represented using polynomials. The power of neural networks and the economy of low-order polynomials (in terms of the number of simulations required and network training requirements) are thus effectively combined. The method can be viewed as a variant of Response Surface Methodology (Ref. 15 & 16) where the response surfaces are constructed using both neural networks and polynomials. Traditional RSM uses only low-order polynomials in constructing the response surfaces.

The method of Ref. 12 uses polynomial approximations on multidimensional simplexes. An s-dimensional simplex is a spatial configuration of s dimensions determined by s+1 equispaced vertices that lie on a hypersphere of unit radius. By this definition a two-dimensional simplex is an equilateral triangle that is circumscribed by a unit circle. This approach assumes that the local variation of the design objective function can be accurately represented using low-order polynomials, which is often the case. The polynomial fit on this simplex together with the trained neural network represents a composite response surface. The optimization procedure then uses a sequence of such composite response surfaces to traverse the design space in search of the optimal solution.

Parameter-based partitioning of the design space is accomplished in the following manner. Assume that the flow variable being modeled in order to compute the objective function is the surface pressure on an airfoil. Since the variation of the pressure along the airfoil surface is typically far more complicated than the variation with small changes in geometric parameter values, a neural network is used only to represent pressure variation in physical
space. The three-layer neural network (with two-hidden layers) shown in Fig. 1 is used for this purpose. The first node in the input layer is a bias node (input of 1.0). The second set of nodes is used to specify the physical location. Figure 3 shows a third set of input nodes that may be used in cases where the functional behavior of the pressure with some of the geometric parameters is "complex" and one wishes to use the neural network to represent this behavior.

The variation of the surface pressure with geometry parameters is approximated using polynomials. If a linear variation is assumed, the points at which the pressure data are determined are located at the vertices of a simplex of dimension equal to the number of geometry parameters.

The optimization strategy of Ref. 12 to evolve an optimal turbine airfoil starting from the initial design can be summarized as follows:

1. Populate the design space in the vicinity of the initial design. The initial design serves as the centroid of the first simplex in the optimization process. A simplex in design space is constructed around this centroid and computational fluid dynamics (CFD) analyses at each of the vertices are obtained.

2. Train the neural networks and compute the polynomial coefficients to define the composite response surface. The input nodes of the neural nets will typically contain parameters that correspond to the physical location on the airfoil and those geometric parameters that give rise to "complex" variations of the surface pressure. The neural nets are trained and the polynomial coefficients that define the pressure variation within the simplex are computed. The trained neural networks in combination with the polynomial fit then constitute the composite response surface.

3. Search the region of the design space represented by the composite response surface. A conjugate gradient method was used in this study to perform this constrained search. Geometrical and other constraints can be easily incorporated within this search procedure. In addition, constraints that limit the search procedure to the volume of the simplex are also incorporated in the search.

4. Relocate the simplex. If the local optimum obtained in the previous step lies on the boundaries of the simplex then this
point is chosen as the new centroid and steps 1-4 are repeated until the search culminates inside the simplex. However, the process can be stopped at any time when the design is deemed adequate.

5. **Validate the design.** As a final step in the process the aerodynamic analysis is carried out for the geometry corresponding to the optimal design to determine the adequacy and quality of the design.

Additional details of the design procedure can be found in Ref. 12.

**CURRENT DESIGN METHOD**

The results presented in Refs. 12-14 demonstrate both the ability of the method in solving some complex aerodynamic design optimization problems as well as its versatility in being able to handle different objective functions and constraints. However, the applicability of this approach to even more complex optimization problems such as the design of three-dimensional aerodynamic surfaces in the context of steady and unsteady flows, designing for multiple operating conditions and multidisciplinary optimization requires considerable reductions in both the number of optimization steps required to obtain the optimal shape and the CPU requirements for generating the training data.

Reductions in costs can be achieved in the following ways:

1. Developing composite response surfaces with improved generalization capabilities (in this case, composite response surfaces that are accurate both inside and outside the given simplex).

2. Exploitation of synergies between the optimization method and the simulation codes such that the simulations can be obtained much more rapidly.

Ideally one would also like to formulate the method so that it is easy to use whether one is trying minimize a scalar valued function or a vector valued function (as in inverse design or multiple point design).
The final paper will discuss the progress made in these areas that has resulted in a substantially different composite response surface methodology (improvements to the method of Ref. 12).

AIRFOIL GEOMETRY PARAMETERIZATION

Geometry parameterization and prudent selection of design variables are among the most critical aspects of any shape optimization procedure. Since this study focuses on airfoil design, the ability to represent various airfoil geometries with a common set of geometrical parameters is essential. Variations of the airfoil geometry can be obtained by varying these parameters. Geometrical constraints imposed for various reasons, such as structural, aerodynamic (e.g., to eliminate flow separation), etc., should be included in this parametric representation as much as possible. Additionally, the smallest number of parameters should be used to represent the family of airfoils.

The method used here for parameterization of the airfoil geometries is described in Ref. 12 and is reviewed here for completeness. Figure 2 illustrates the method for a generic airfoil. Some salient features of the method are noted below:

1. The leading edge is constructed using two different ellipses, one for the upper surface and one for the lower surface. The eccentricity of the upper ellipse and the semi-minor axes of both ellipses are specified as geometric parameters \( e_u, t_u, \) and \( t_l \), respectively. All other related parameters can be determined analytically. The major axes of both ellipses are aligned with the tangent to the camber line at the leading edge. This tangent is initially aligned with the inlet flow but is allowed to rotate as the design proceeds. The angles \( \alpha_u \) and \( \alpha_l \) determine the extent of the region in which the leading edge is determined by these ellipses. The two ellipses meet in a slope continuous manner.

2. The trailing edge can also be constructed in a similar manner with the major axes of the ellipses aligned with the tangent to the camber line at the trailing edge. However, in this study the trailing edge was defined using a single circle. The angles \( \beta_u \)
and $\beta_1$ determine the extent of the region in which the trailing edge is represented by this circle.

3. The region of the upper surface between the upper leading edge ellipse and the trailing edge circle is defined using a tension spline. This tension spline meets the leading edge ellipse and the trailing edge circle in a slope continuous manner. Additional control points for the tension spline are introduced as necessary. These points provide better control over the shape of the upper surface. The lower surface of the airfoil between the lower leading edge ellipse and the trailing edge circle is obtained in a similar manner.

As in Ref. 12, a total of 13 geometric parameters were used to define the turbine stator airfoil in the current study. These parameters are listed below:

1. Leading edge and trailing edge airfoil metal angles (2 parameters).
2. Eccentricity of upper leading edge ellipse (1 parameter).
3. Angles defining the extent of the leading edge ellipses (2 parameters).
4. Semi-minor axes values at the leading edge (2 parameters).
5. Angles defining the extent of the trailing edge circle (2 parameters).
6. Airfoil stagger angle (1 parameter).
7. Airfoil y-coordinate values at about 50% chord on the upper and lower surfaces (2 parameters).
8. Airfoil y-coordinate value at about 75% chord on the upper surface (1 parameter).

This method of generating the airfoil surface provided a smooth shape transition from a curved constant thickness section to the optimal airfoil. The intermediate airfoil shapes required by the optimization procedure were obtained by smoothly varying some or all of these 13 parameters.
RESULTS

The current neural-net based design method is used here to evolve a turbine airfoil from a simple curved section of nearly uniform thickness. This is the same airfoil that was used in Ref. 12 and thus allows a direct comparison of results and design method efficacy. The target pressure distribution was supplied by Pratt & Whitney (P&W) (Private Communication, F. Huber, 1997). This pressure distribution was obtained at the midspan of a turbine vane from a modern jet engine. In addition to the target pressure distribution, the inlet total pressure and temperature, inlet and exit gas angles, the exit Mach number, the axial chord, the trailing edge circle radius, the radius of the midspan section and the number of vanes in the row were also provided by P&W.

The computational method used to compute the flow in the stator row is a third-order-accurate, iterative-implicit, upwind-biased scheme that solves the time-dependent Euler and Reynolds-averaged, thin-layer, Navier-Stokes equations. The region of interest is discretized using multiple grids; an inner “O” grid that contains the airfoil and an outer “H” grid that conforms to the external boundaries. A typical turbine stator airfoil and a representative computational grid are shown in Fig. 3. Details regarding the CFD solution methodology can be found in Ref. 17.

Evolution of a generic shape into an optimal airfoil

The current methodology uses a sequence of composite response surfaces to enable a search of the design space thus permitting the use of initial designs that are far from the optimal design. To illustrate this capability the initial geometry was chosen to be a nearly constant thickness curved section. The inlet and exit metal angles for this curved plate were initially set equal to the corresponding gas angles. This initial geometry is shown in Fig. 4 by the airfoil marked A.

The composite response surface was used to represent the functional dependence of the airfoil surface pressure on the airfoil
axial location as well as the geometric parameters. The sum of the squared error or objective function to be minimized was then defined as

$$\text{SSE} = \sum_{i=1}^{\text{imax}} (P_i - p_i)^2$$

where $P_i$ is the target pressure, $p_i$ is the pressure at the same axial location for the airfoils generated during the optimization process, and $\text{imax}$ is the total number of airfoil surface points at which the target pressure is defined.

The current design methodology was then applied to obtain the optimal geometry. Figure 4 shows the progression of the airfoil geometry as the optimal design is approached. Figure 5 compares the corresponding pressure distributions with the target pressure distribution (the surface pressure and the axial location on the airfoil are normalized using the inlet total pressure and the airfoil axial chord, respectively). An additional feature that was incorporated into the design process was the use of CFD solutions of different fidelities. The design process was carried out using solutions to the Euler equations until the geometry denoted C in Fig. 4 was obtained. Subsequently, solutions to the Reynolds-averaged Navier-Stokes equations were used to achieve the final design shown as airfoil D in Figure 4. A similar technique was also used in Ref. 12 to reduce design costs.

Design optimization was performed with only 6 variables ($t_u, t_l, \alpha_u, \alpha_l, \beta_u$ and $\beta_l$) until the airfoil marked C was obtained. Thereafter all 13 of the variables mentioned before were used to represent the airfoil surface. A similar technique was also used in Ref. 12 to reduce design costs. Figures 4 and 5 show a smooth transition of the curved section to the optimal airfoil geometry. An excellent agreement between the target pressure distribution and the computed pressure distribution (corresponding to airfoil D) is obtained at the end of the optimization process. Airfoils B, C and D were obtained after 2, 4 and 6 optimization steps, respectively.
Figure 6 shows the variation of the mean square error as a function of the cumulative computing time (on a single processor CRAY-C90) used to generate the grids, perform the CFD simulations, train the neural networks and search for the optimal airfoil. The current method required 24 minutes to reduce the mean square error to 3.1E-5 whereas the method of Ref. 12 required 40 CPU hours to achieve this value of error. Thus, in this case the new design method decreased computational requirements by a factor of 100.

**Design in the context of multiple operating points**

As mentioned earlier, the current method has been formulated so that it is easy to use whether one is trying minimize a scalar valued function or a vector valued function (as in inverse design or multiple point design). The final paper will include an example of a turbine vane operating at different conditions. The current method will be used to obtain a vane shape that is optimal in some sense for all the operating conditions (minimal total pressure losses).

**SUMMARY**

An aerodynamic design procedure that uses neural networks to model the functional behavior of the objective function in design space has been developed. This method incorporates several improvements to an earlier method that employed a strategy called parameter-based partitioning of the design space in order to reduce the computational costs associated with design optimization. As with the earlier method, the current method uses a sequence of response surfaces to traverse the design space in search of the optimal solution. The new method yields significant reductions in computational costs by using composite response surfaces with better generalization capabilities and by exploiting synergies between the optimization method and the simulation codes used to generate the training data. These reductions in design optimization costs are demonstrated for a turbine airfoil design study where a generic shape is evolved into an optimal airfoil. A hundred-fold decrease in design cost was achieved with the current design method.
REFERENCES


Figure 1. Schematic of the three-layer feed-forward neural network used in Ref. 12.
Figure 2. Typical airfoil geometry showing location of control points on the airfoil surface and the defining angles used in the geometry parameterization.
Figure 3. Representative turbine airfoil geometry and computational grid used in the CFD simulations.
Initial design. A. Simple curved plate to the optimal shape. B. Initial simple curved plate to the optimal shape. C. Initial simple curved plate to the optimal shape.
Convergence of Design Optimization

Figure 6. Reduction of the mean square error in surface pressure with the cumulative computing time used in the design optimization process. CPU time in single