Roughness Perception of Haptically Displayed Fractal Surfaces

Michael A. Costa and Mark R. Cutkosky

‡Autonomy and Robotics Area
NASA Ames Research Center
M/S 269-3
Moffett Field, CA 94035
macosta@cdr.stanford.edu

‡Dexterous Manipulation Laboratory
Building 560, Panama Mall
Stanford, CA 94305-2232
cutkosky@cdr.stanford.edu

ABSTRACT

Surface profiles were generated by a fractal algorithm and haptically rendered on a force feedback joystick. Subjects were asked to use the joystick to explore pairs of surfaces and report to the experimenter which of the surfaces they felt was rougher. Surfaces were characterized by their root mean square (RMS) amplitude and fractal dimension. The most important factor affecting the perceived roughness of the fractal surfaces was the RMS amplitude of the surface. When comparing surfaces of fractal dimension 1.2-1.35 it was found that the fractal dimension was negatively correlated with perceived roughness.

INTRODUCTION

The perception of texture is an important means by which humans identify surfaces around them. In field geology, scientists use texture to help identify rocks and determine their history (e.g., the amount of weathering they have been exposed to). The work in this paper is part of an effort to allow field geologists to explore remote planetary surfaces, displayed as virtual surfaces in an immersive haptic and visual display. An important component of the perceived texture of rock surfaces is their roughness, and the focus of this paper is the display of roughness using models that produce surface profiles similar to those found in nature.

In particular, we have identified two parameters, RMS amplitude and fractal dimension, that are useful for generating synthetic surfaces similar to those measured using an optical profilometer on rock surfaces. In this paper we report on experiments to determine how human subjects relate perceived roughness to variations in these two parameters when interacting with the surfaces via a haptic display.

PREVIOUS WORK

Earlier surface roughness perception experiments had subjects touch metal gratings consisting of rectangular grooves [10][12]. These studies showed that the dominant factor in determining perceived roughness was the groove width. More recent work by Lederman and Klatzky has argued for the viability of using a probe to encode vibration information to discriminate roughness [8][9]. During these experiments subjects used a probe to explore surfaces made up of a pseudo-random pattern of raised dots. It was found that the perceived roughness of the patterns when using a probe or a finger increased with increased interelement spacing until reaching a peak and then decreased [8].

A common feature of these experiments is that they use artificially constructed surfaces of predetermined heights. While the gratings are more deterministic than the spatially randomized raised dots, the dot patterns still have only two heights: the height of the dot and the flat surface it is resting on.

Other work has examined the use of stochastic methods for texture display [16][17]. In the present case, we are interested in the perceived roughness of irregular virtual surfaces whose geometric properties (e.g., peak heights and spatial intensity) match those found on rocks.

We selected a fractal technique to simulate the surface profiles. Previously we had found this technique to favorably mimic rock surface profiles when compared in terms of standard metrics from surface metrology, e.g., mean amplitude departure from a mean reference line, RMS amplitude deviation from a mean reference line, peak density, and kurtosis [5].
Characterizing Surface Profile Roughness with Fractal Dimension and RMS Amplitude

Before synthesizing any profiles, the metrics used to characterize the surfaces must quickly be discussed. We use two parameters to describe the surfaces: the fractal dimension from fractal geometry, and the RMS amplitude from traditional surface metrology.

Fractals have been used to describe irregular shapes that do not lend themselves to description by Euclidean geometry. Natural structures such as mountains, coastlines, clouds and snowflakes, in addition to recursive, self-similar structures such as the Koch snowflake curve are examples of shapes that exhibit a fractal nature [13]. Each of these objects has a non-integer fractal dimension, measuring how much space they occupy. For example a straight line has a topological dimension of one, and a square has a dimension of two. The higher the dimension, the more space the curve occupies. Details of measuring the fractal dimension can be found in *The Science of Fractal Images* [13]. Several attempts have been made to use the fractal dimension to characterize the roughness of rock surface profiles [6][7]. A fractal analysis of a surface can yield a fractal dimension and an amplitude coefficient [13][3].

Amplitude measures have been a popular way in surface metrology to describe the surface roughness of a profile [5]. The amplitude measure that we use is \( R_q \), the root mean square deviation from the mean line. In continuous form \( R_q \) is defined as [5]

\[
R_q = \frac{1}{L} \int_0^L x^2 \, dx
\]

where \( L \) is the length of the profile. When measuring fractal properties of real surfaces we were able to compute the RMS deviation from the fractal amplitude coefficient. With this framework we can begin to synthesize rough surfaces.

Synthesizing a Surface: Fractal Motion using Fourier Filtering

For our experiment we desired to generate surfaces mimicking those found in nature, while being able to control their frequency and amplitude characteristics. Fractal geometry has been used in computer graphics to generate mountainous landscapes, plants, and water surfaces. Using an appropriate algorithm we can vary the fractal dimension and the root mean square amplitude of our generated surfaces. The first algorithm we tried was based on the "Fractal Motion using Fourier Filtering method" [13]. Fourier series has been used before in a similar algorithm to generate surfaces [18]. The Fourier Filtering method differs by imposing the power spectral density condition of fractal Brownian motion, \( P(f) \propto (1/f)^D \), onto the coefficients of the discrete Fourier transform. However this condition uses only the fractal dimension. This does not allow adjusting of the amplitudes of the generated surfaces.

Ganti uses a similar \( 1/f^\beta \) power spectral density function to generate fractal surfaces but also includes an amplitude parameter \( C \) that scales the amplitudes of frequencies [3]

\[
P(f_k) = \frac{C_1 K}{(2\pi f_0)^{5-2D}} \\
C_1 = C \tau^{2D-3} \\
K = \frac{\Gamma(\beta) \sin\left(\pi(2-D)\right)}{2\pi} \\
\beta = -2D + 5 \\
f_0 = \frac{1}{L}
\]

(1)

where \( P \) is the power spectral density at \( k \)th frequency \( f \). \( D \) is the fractal dimension, \( C \) is the amplitude coefficient, \( L \) is the length of the profile, and \( \eta \) is the instrument measurement resolution of the profile. For synthesizing profiles we used the encoder resolution for \( \eta \).

When measuring profiles, the amplitude coefficient \( C \) can be related to the RMS amplitude of the heights by

\[
R_q(x-x_0) = C \eta^{-D+1/2} \tau^{-2D}
\]

(2)

where \( \tau \) is the sampling resolution [3]. For synthesizing profiles we took \( \tau \) and \( \eta \) to be same. This equation enables us to synthesize a surface with any desired root mean square amplitude.

To synthesize a surface the conditions of equation (1) are imposed onto the coefficients of the discrete Fourier transform [13][3]

\[
X(t) = \sum_{k=0}^{N-1} H_k e^{2\pi i k t}
\]

The real and imaginary parts of the Fourier coefficients \( H_k \) are computed from

\[
\text{Re}H_k = \text{random sign} \cdot \text{rand} \left( \frac{C_1 K}{\sqrt{L(2\pi)^{5-2D}}} \right)
\]

\[
\text{Im}H_k = \text{random sign} \cdot \text{rand} \left( \frac{C_1 K}{\sqrt{L(2\pi)^{5-2D} - (\text{Re}H_k)^2}} \right)
\]

\[
k = 0, 1, \ldots N/2
\]
Reikk = Imll
Imll = Reikk
k = N/2, N/2 + 1, ..., N - 1

for a desired profile of N points, random sign is randomly ±1 and rand is a random Gaussian number from 0 to 1. We then form Hk using the complex conjugate operator 

\[ H'_k = \frac{1}{\sqrt{2}} (H_k + H^*_k) \]

The conditions of the power law are imposed onto \( H'_k \) by

\[ H'_k = \frac{1}{\sqrt{(k+1)_{5/2}}^{D}} H'_k, 0 \leq k \leq \frac{N}{2} \]

\[ H'_k = \frac{1}{\sqrt{(k+1-N/2)_{5/2}}^{D}} H'_k, \frac{N}{2} \leq k \leq N - 1 \]

where \( f \) is the fundamental frequency, \( 1/L \), the inverse of the desired length of the profile. We then compute our synthesized function, \( x(t) \), using our random coefficients and the discrete inverse Fourier transform [13]

\[ X(t) = \sum_{k=1}^{N/2} (Re\,H'_k \cos(kt) + Im\,H'_k \sin(kt)) \]

One problem with using this method is that the generated surface repeats itself every \( 2\pi \) intervals. To circumnavigate this problem we divide the surface we would like to generate into 3.9552 mm fragments of 128 equally spaced points, generating a new set of Fourier coefficient pairs for each fragment. The newly generated fragment has a constant added to it so that it is attached to end of the last fragment. This process continues until the new surface is complete.

In practice we also found that when generating profiles with amplitude coefficients based on equation (2), the profiles had \( R_q \)'s less than predicted. We increased the amplitude coefficient \( C \) until a surface was generated with the desired RMS amplitude. During our previous experiments when we analyzed actual surfaces, the measured amplitude coefficient and fractal dimension did predict the measured \( R_q \).

A real profile of 100 grit painter’s sandpaper and two fractal simulations based on its measured fractal parameters, \( D \) and \( C \), are shown in Fig. 1. The middle profile exhibits the algorithm’s problem with producing surfaces with amplitudes smaller than that predicted by equation (2). The last profile had an increased amplitude coefficient \( C \) to increase its \( R_q \) to approximately to the same \( R_q \) as the painter’s sandpaper.

**EXPERIMENTAL SETUP**

Fractally simulated surface profiles were haptically rendered on an Immersion Impulse Engine 2000 force feedback joystick. Fourteen subjects, eleven male and 3 female, from ages 21 to 34, participated in the experiment. Subjects were naive to the purpose of the experiment. All subjects had at least minimal experience using haptic devices. Subjects were asked to use the joystick to explore pairs of surfaces and report to the experimenter which one they felt was rougher, surface one or surface two. One surface was displayed at a time. Subjects controlled which surface was displayed by clicking the joystick trigger. Subjects were allowed to switch between surfaces as often as they desired and to explore surfaces as long as they desired. Sixteen pairs of surfaces were given to subjects in random order. Assignment of surfaces as surface one or two was also randomized. Typically subjects spent fifteen to twenty minutes exploring surfaces, with the shortest time being approximately ten minutes and the longest being approximately thirty-five minutes.

Two different reference surfaces were compared to eight surfaces each to form the sixteen trial pairs. Each reference surface was compared to surfaces with a combination of lesser, equal, and greater fractal dimension and RMS amplitude. Figure 2 demonstrates the surface group pairings. Reference surface one is compared against each of the surfaces of group one while reference surface two is compared against the group two surfaces.

The surfaces used in the experiment were generated in 20mm lengths and patched together to form 80mm lengths. Each length was offset to meet that last one so that no discontinuity between lengths could be detected. Surface profiles were
resampled to a resolution of 0.1545mm/sample, five times the encoder resolution of the joystick. This saved memory without compromising the feel of the surface.

Resampling the surface also sets its spatial resolution to just above the mechanical bandwidth of the force feedback joystick. The joystick can display forces of up to 8.9 N with a bandwidth of 120Hz [14]. With a typical minimum user lateral exploration speed of approximately 20 mm/s, the achievable spatial resolution considering the bandwidth is about 0.17 cycles/mm.

**Contact Model**

Subjects explore the virtual surfaces through the force feedback joystick. Using a dynamic model of stylus to surface contact, the forces are displayed to a user resulting from interaction with the surface profile data. Previously we had begun with an initial model similar to the Sandpaper system [15] that used tangential force feedback based on the change in heights with a vertical restoring force based on the penetration depth into the profile. In a previous project we found this model to be unsatisfactory when traversing surface profiles taken from rocks [1]. We therefore implemented a dynamic model (Fig. 3) that represents the normal and tangential forces at the stylus-surface contact and accounts for the possibility of users breaking contact and bouncing or skipping over the valleys of the surface. This model was used to display rock surfaces collected by a laser displacement sensor to planetary geologists during a NASA-Ames field experiment in February 1999 [1]. For the experiment described in this paper we used this model to haptically display the synthesized fractal profiles.

First a height profile as a function of position, \( y_p(x) \), is generated by our fractal algorithm. The stylus is then modeled as a mass, spring, damper system connected to the vertical input of the user’s haptic interface device, \( y \). The horizontal position of the stylus is directly coupled to the horizontal position of the interface. While in contact with the surface the stylus dynamics are computed by

\[
M \ddot{y} + C \dot{y} + K y = F_n + F_t
\]

For our application the parameters were tuned for feel to give our virtual stylus parameters of \( K = 1.28 \text{ N/mm} \) and negligible \( M \) and \( C \).

The contact point of the stylus against the surface is modeled as a frictionless contact point. As illustrated by the magnified portion of Fig. 3, the normal force \( F_n \) is perpendicular to the tangent of the contact point. It is this reaction force that is displayed to the user. The tangent is computed by taking the derivative of the surface with respect to the x coordinate system, \( dy/dx \). In our application this derivative is pre-computed, as we know the surface profile a priori.

The normal force is the sum of x and y component forces,

\[
F_n = \hat{F}_x + \hat{F}_y
\]

The y component represents the vertical reaction force. The x component represents the horizontal reaction force

\[
\hat{F}_x = M \frac{d\dot{y}}{dt} - c \dot{y} + k y
\]

If the user’s position \( y \) is ever greater than the profile height \( y_p \), contact is broken with the surface and contact forces are set to zero. This allows the user to bounce off and back onto the surface, especially while moving fast.

Because the subject can explore the surface freely, exact force levels displayed to the subject are dependent on how they chose to explore the surface, e.g. depth of penetration into the virtual surface, and speed of exploration.

It has been reported that surfaces modeled without friction feel glassy [2]. While we did not use friction in this model, during the experiment, none of the subjects commented that surface felt glassy or slippery. There is a small amount of friction inherent to the force feedback device we used. The Impulse Engine 2000 specifications report a maximum backdrive friction of 0.14 N [14]. It’s possible that a combination of factors including the forces displayed due to the irregularity of the surface, the backdrive friction, and device inertia eliminates any slippery feel by creating an illusion of friction.

Qualitatively interactions with fractal surfaces with smaller RMS amplitudes (.2mm-.27mm) using this model feels like using a stylus to stroke sandstone, while interacting with fractal surfaces with larger RMS amplitudes (> .5mm) feels like
Figure 3. Haptic Model Diagram

stroking a weathered conglomerate rock or broken concrete surface.

DATA AND ANALYSIS

Figures 4 and 5 show the perceived roughness of a surface when compared to a reference surface. Data points are plotted along the x and y axis by their fractal dimension and RMS amplitude. The center spot labeled ref represents the surface that was compared to the other eight surfaces surrounding it. The percentage of subjects who reported a surface as rougher than the center point surface is plotted along the z-axis. For example in Fig. 4, 75% of subjects reported that the surface of fractal dimension 1.35, RMS amplitude 0.3502mm. was rougher than the reference surface of dimension 1.275 and RMS amplitude 0.2781mm. Likewise the other data points in the bar graph are of the percentage perceived rougher when compared to the center surface of fractal dimension 1.275, RMS amplitude 0.2781mm. The reference surface response is set to 50%. This would represent the case of comparing the reference surface to itself, although for fatigue considerations this case was not actually presented to subjects.

Looking at Fig. 4 and 5 across the RMS amplitude axis, both cases show that surfaces with a higher RMS amplitude are perceived as rougher. For all cases of different RMS amplitudes between 71.4% to 100% of subjects reported the surface with the higher amplitude to be rougher. The mean percentage of subjects reporting that a surface with a higher amplitude to be rougher was 91.7% with a standard deviation of ±8.5% for both cases.

It is much more difficult to discern any trends in roughness perception due to fractal dimension by only examining the data plots. To check for the statistical significance of the dimension and amplitude parameters we performed a logistic regression analysis. A logistical regression analysis was chosen because subject responses are binary. After a subject indicates which surface they believe is more rough, the result is recorded as positive or negative depending if they chose the test surface (positive) or the reference surface (negative).

For each case we fitted the subject responses to a multiple logistic regression model of the form [4]

Figure 5. Perceived Roughness vs. Surface
D=1.425, RMS Amplitude=.75

Figure 4. Perceived Roughness vs. Surface D=1.275,
RMS Amplitude=.2781
\[ y(x) = 0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p \]

\[ P(Y = 1|x) = \frac{e^{\beta x}}{1 + e^{\beta x}} \]

using a software statistical analysis package [11]. Each coefficient \( \beta_i \) of the model represents a different experimental variable: fractal dimension, RMS amplitude, or a subject variable. After computing the coefficients and their respective estimated standard errors, we performed the Wald test for logistic regression models to check for statistical significance [4]. The Wald test is conducted by first computing the univariate Wald statistic \( W = \beta_i / (SE(\beta_i)) \), where \( SE(\beta_i) \) is the standard error of the \( i \)th coefficient. Next, the two-tailed p-value of the Wald statistic is computed from the Chi-square on one degree of freedom distribution with a significance of \( \alpha = 0.05 \). For any p-value less than .05, we consider the variable to be statistically significant.

The p-values for the coefficients are presented in Table 1 and Table 2 on page 6. As expected for both cases, the p-value of the RMS amplitude is well below 0.05, indicating that this variable is statistically significant. As the Wald statistic is positive in both cases, the likelihood of perceiving one surface rougher than the reference surface increases with increasing amplitude. For the second case of higher fractal dimensions, 1.35 to 1.5, the p-value for fractal dimension is 0.446. This indicates that this variable was not a significant factor in perceiving roughness when comparing surfaces of fractal dimension greater than 1.35. Interestingly in the first case when comparing surfaces of lower fractal dimension, 1.2 to 1.35, the fractal dimension parameter has a p-value of .0128 and a negative Wald statistic. With this p-value, this variable does meet our test for statistical significance. This suggests that when comparing fractal surfaces with dimensions between 1.2 and 1.35, the surface with the lower fractal dimension contributes to it being perceived as rougher.

This dichotomy between the higher and lower fractal dimension cases might exist because as the fractal dimension decreases, the surface becomes more coarse, enforcing the perception of roughness. The difference between two surfaces with higher fractal dimensions may not be noticeable to a person. This is supported by Fig. 5. Examining the two trials where the RMS amplitude is the same, the percentage of subjects perceiving the surface of dimension 1.5 and the surface of dimension 1.35 as more rough than the reference surface of dimension 1.425, is 80% and 57%, respectively. This indicates a random preference. Subjects reported that these two cases were difficult to distinguish.

**CONCLUSIONS**

From examining both the data plots and the p-values, the most important factor effecting the perceived roughness of fractal surfaces is the RMS amplitude. The logistic regression indicated that when comparing fractal surfaces with dimension 1.2-1.35, lower fractal dimension contributes to the perception of roughness. This agrees with previous work that reported roughness perception increased with increasing groove width.
and increasing interelement spacing [10][12][8]. The changes in those surface parameters would have decreased the fractal dimension as well.

This effect does not hold when comparing surfaces of higher fractal dimensions - those over 1.35. For these comparisons fractal dimension was statistically insignificant. When comparing surfaces of the same RMS amplitude in the set of surfaces with fractal dimension 1.35 to 1.5, the subjects' selection were apparently random.

In any case of fractal dimension, RMS amplitude is the overriding factor in determining surface roughness perception. When comparing surfaces of different RMS amplitudes, subjects selected the surface with the higher amplitude as rougher 71.4% to 100% of the time.

ACKNOWLEDGMENTS

The authors would like to thank Philip Beineke from the Statistical Consulting staff at Stanford University for his helpful advice on logistical regression and applying it to our data.

REFERENCES

\( g(x) = 0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \beta_p x_p \)

\[ P(Y = 1|x) = \frac{e^{g(x)}}{1 + e^{g(x)}} \]

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**Table 1: P-Values for Perceived Roughness when Comparing to Reference Surface 1: \( D=1.275 \), RMS Amplitude=2781mm**

<table>
<thead>
<tr>
<th>Coefficient Name</th>
<th>Wald Statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fractal Dimension</td>
<td>-2.48803</td>
<td>0.0128</td>
</tr>
<tr>
<td>RMS Amplitude</td>
<td>4.47037</td>
<td>7.8085e-6</td>
</tr>
<tr>
<td>Subject 1</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 2</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 3</td>
<td>1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 4</td>
<td>-1.5e-16</td>
<td>1.0000</td>
</tr>
<tr>
<td>Subject 5</td>
<td>-0.51</td>
<td>9.593</td>
</tr>
<tr>
<td>Subject 6</td>
<td>-1.815</td>
<td>0.0696</td>
</tr>
<tr>
<td>Subject 7</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 8</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 9</td>
<td>-1.815</td>
<td>0.0696</td>
</tr>
<tr>
<td>Subject 10</td>
<td>1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 11</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
<tr>
<td>Subject 12</td>
<td>-1.815</td>
<td>0.0696</td>
</tr>
<tr>
<td>Subject 13</td>
<td>-1.027</td>
<td>3.044</td>
</tr>
</tbody>
</table>

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