A Probabilistic Approach to Model Update

Lucas G. Horta and Mercedes C. Reaves
Langley Research Center, Hampton, Virginia

David F. Voracek
Dryden Flight Research Center, Edwards, California

June 2001
Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA’s mission.

Specialized services that complement the STI Program Office’s diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results ... even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at http://www.sti.nasa.gov

- E-mail your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA STI Help Desk at (301) 621-0134

- Phone the NASA STI Help Desk at (301) 621-0390

- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7121 Standard Drive
  Hanover, MD 21076-1320
A Probabilistic Approach to Model Update

Lucas G. Horta and Mercedes C. Reaves
Langley Research Center, Hampton, Virginia

David F. Voracek
Dryden Flight Research Center, Edwards, California
A Probabilistic Approach to Model Update

Lucas G. Horta*, Mercedes C. Reaves**, and David F. Voracek+

First Annual Probabilistic Methods Conference
June 18-19, 2001
Newport Beach, CA

Abstract

Finite element models are often developed for load validation, structural certification, response predictions, and to study alternate design concepts. In rare occasions, models developed with a nominal set of parameters agree with experimental data without the need to update parameter values. Today, model updating is generally heuristic and often performed by a skilled analyst with in-depth understanding of the model assumptions. Parameter uncertainties play a key role in understanding the model update problem and therefore probabilistic analysis tools, developed for reliability and risk analysis, may be used to incorporate uncertainty in the analysis. In this work, probability analysis (PA) tools are used to aid the parameter update task using experimental data and some basic knowledge of potential error sources. Discussed here is the first application of PA tools to update parameters of a finite element model for a composite wing structure. Static deflection data at six locations are used to update five parameters. It is shown that while prediction of individual response values may not be matched identically, the system response is significantly improved with moderate changes in parameter values.

Introduction

Numerous algorithms have been proposed to reconcile differences between behavior prediction from dynamic models and experimental data. Although this has been a very prolific area of research, no single technique has been universally accepted. One common flaw in many of the approaches is attempting to find a non-existent “single solution” to a problem. Often sensitivity information is provided to judge the relative importance of parameters and to assist in making model changes. These tools, in the hands of experienced engineers, provide heuristic approaches to model updating that work very well for problems with a small number of parameters. Published work in this area is quite extensive, and some of the most promising of the recent contributions are in Refs. [1-4]. Hasselman in Ref. 1, discussed propagation of parameter uncertainty in frequency response calculations and presented various approaches to handle variability of response values near dynamic resonant conditions. Herendeen and co-workers Ref. 2 discussed a mathematical procedure using optimization to conduct analysis/test correlation studies of frequency response data. Their technique sought to adjust finite

* Assistant Branch Head Structural Dynamics Branch, NASA Langley Research Center
** Aerospace Technologist, Structural Dynamics Branch, NASA Langley Research Center
+ Test Engineer, NASA Dryden Flight Research Center
element model parameters to correlate with observed frequencies and mode shapes. An error localization approach was used to detect trouble areas in the model. Changes in the model were made in such a way that parameter changes were minimized. Alvin in Ref. 3 extended a procedure developed by Farhat in Ref. 4 to improve convergence of Farhat’s approach and to incorporate uncertainty information into the estimation process. Results for the sample case showed excellent correlation of frequencies and modal assurance criteria even for cases with a relatively large number of modes.

Although some newly developed techniques incorporate response or parameter uncertainty to some extent, one aspect of the problem that is often overlooked is the fact that engineers can provide uncertainty information about the parameters used in the model. This information resides with the modeler and is seldom used to guide the analysis. Parameter uncertainty is precisely what probabilistic analysis, Ref. 5-6, captures well and when used in conjunction with conventional finite element analysis can guide the analyst to determine modeling errors. A commercially available computer program called UNIPASS, discussed in Ref. 7, will be used for all numerical results discussed later. Tools like this one are there to help the analyst but he/she still needs to select which parameters are likely to be in error. PA provides results in terms of distribution functions, adding a new dimension to the solution. A fundamental difference between PA approaches and those presented in Refs. [1-4] is that in PA techniques, the objective function maximizes the probability that a certain condition will occur as opposed to minimizing the prediction error. Both of these objective functions are important and should be addressed in any model update problem but the means to address each one would dictate how the parameters are updated.

The work described in this paper uses probabilistic analysis to determine parameter changes that are most likely to cause predictions to agree with experimental results. To demonstrate the procedure using models created with finite element tools, a test-case of the NASA Dryden Flight Center Aerostructure Test Wing (ATW) will be discussed. This model had several areas where model parameters were uncertain and predictions from the finite element model were in error. Analysis of the ATW allows incorporation of static test data into the update process. A brief description of the application of probabilistic analysis for model updating is presented in the next section.

**Application of Probabilistic Analysis Tools to Model Update**

Although PA tool have been used extensively in manufacturing, it has not been used for parameter updates. Before using these tools for parameter updates, one needs to understand their meaning within the reliability framework to be able to interpret results for the model update problem. To clarify this point, Fig. 1 depicts a fictitious reliability problem in terms of two variables \(x\) and \(y\), with joint probability density function contours \(f(x,y) = \text{const.}\), and a limit state function \(g(x,y) = x - y = 0\). Probability density contours are monotonically decreasing away from the center, which corresponds to the maximum value. For reliability analysis, any pair \((x_0, y_0)\) such that \(g(x_0, y_0) \leq 0\) represents a failed condition in the failure domain. Among all the points corresponding to fail conditions the one with the highest failure probability is denoted as the most probable point (MPP). This MPP provides not only the probability of failure but also the parameter values that are most likely to cause the failure. In this simple example, the failure probability is given in a mathematical form as,
\[ p_f = p(g(x, y) \leq 0) = \int_{\mathbb{R}^2 : g(x,y) \leq 0} g(x,y) f(x,y) dx dy \]  \hspace{1cm} (1)

where the integration is over the failure domain.

For model update, the freedom in defining the limit state function is used to evaluate the probability that certain parameters are responsible for erroneous predictions in the analysis. To explain how this is accomplished for a dynamic problem, assume that a limit state function is defined as the difference between a measured and a predicted natural frequency. Furthermore, the analyst suspects that a certain thickness parameter is in error and is able to provide a general description of the probability distribution function for the thickness. PA computes the MPP thickness value that would cause test and analysis to agree. Also computed is the probability of this condition being true given the original distribution for the parameter in question. If the thickness parameter is not responsible for the frequency errors, the corresponding probability value should be small. The converse statement is not necessarily true, that is if the probability is high, this could indicate that this is one of many possible causes but not necessarily the only cause. At this point the analyst would need to examine the required changes in parameters to determine if the changes are reasonable and rule out parameters that are not affecting the solution.

Description of Aerostructure Test Wing (ATW)

NASA Dryden Flight Research Center formulated an approach to predict stability margins in flutter analysis called the flutterometer. This tool represents a departure from traditional approaches in that it uses both analytical models and flight test data simultaneously. In order to validate the flutterometer, a flight experiment called the Aerostructure Test Wing (ATW) was initiated. The ATW, shown in Fig. 2a, consist of a NACA 65A004 airfoil, a fiberglass cloth skin, and internal spar web. During flight tests, the ATW will be mounted on the NASA Dryden F-15B Flight Test Fixture. This fixture is located under the F-15B fuselage, as shown in Fig. 2. The flight test program will fly the ATW to flutter at about Mach 0.8 at 10,000 Feet.

As shown in Fig. 2b the ATW has a wing area of 197 in\(^2\) and an aspect ratio of 3.28. Internally, the spar at 30% chord line is one ply 0.005” thick of graphite/epoxy M55J-24K at the tip and 10 plies 0.05” thick at the root. The wing core is made of rigid foam. The wing has a semi-span of 18 inches, a root chord of 13.2 inches, and tip chord of 8.7 inches. The total weight of the wing is 2.66 pounds. A 1-inch diameter boom made of graphite/epoxy is attached to the wing.

ATW Finite Element Model

A Finite Element Model (FEM) of the ATW was constructed using MSC/NASTRAN. The FEM, shown in Fig. 3, has 265 grid points with 1590 degrees of freedom. Quadrilateral CQUAD4 elements are used to model the composite wing skin, spar cap, spar web and ribs. The wing foam core is modeled using CHEXA and CPENTA solid elements. Bar elements are used to model the boom. Concentrated masses are modeled using CONM2 elements including the forward, middle and aft-boom mass, wing mounting fixture mass and wing leading edge balancing masses. PCOMP cards in MSC/NASTRAN are used to specify the properties of the composite lay-ups. The wing
root is fully constrained at 16 grid points with RBE2 rigid elements and 6 CELAS1 spring elements located at the spar centerline (30% cord). Isotropic material properties are included in MAT8 cards for the spar cap, spar web, and the skin. MAT1 linear isotropic material properties are used for the rest of the structural components. RBE2 rigid elements connect the aileron to the wing.

For the PA studies, uniform distribution functions were used to describe the Young’s modulus for the spar, skin, foam core and the root bending spring. Other parameters such as skin thickness and ply angles were ruled out early in the analysis because of their small influence on the overall stiffness. The following section describes results from experimental tests that form the basis of our initial study.

**ATW vibration and proof load tests**

Prior to certifying the ATW configuration for flight, several static and dynamic tests were performed. Load certification up to 125% of design limit loads were conducted both in bending and torsion. Dynamic tests were also conducted to correlate frequencies and mode shapes with test data, those results will be discussed elsewhere. Detailed test procedures and configuration were documented in an internal NASA Dryden test plan, Ref. 7, but some of the conditions related to the model update effort are presented herein.

To verify the stiffness predictions with the finite element model, two test configurations were used. For bending, discrete point loads were applied at 19 different locations, as shown in Fig. 4, with corresponding pressure values given in Table 1. Figure 5 is a side view photograph of the bending test set-up. For torsion, a 10 lb weight was suspended from the front of the boom (front view shown in Fig. 6). Displacement data was measured using linear variable differential transducers (LVDT) at 8 locations on the wing for both load cases. Figure 4 also shows sensor locations labeled with numbers within squares ranging from 1 to 110. Table 2 shows the measured displacement mean and standard deviation for the 25% design load bending test and a 10 lb torsion test. Data for sensors 100 and 110 were not used for subsequent analysis because they show a large coefficient of variation. Results for the PA analysis are discussed next.

**Probabilistic Analysis of the ATW Wing**

The first step in any PA study is to examine all the parameters involved in the problem and select those that are most uncertain. Often this step is somewhat subjective because uncertainty data is a matter of engineering judgment. In our study, Young’s modulus was selected for analysis because for composite materials it varies significantly from one reference to another and also because of their large impact on the system response. Also, boundary conditions are critical. In the finite element model, several springs are used to represent the boundary stiffness, for the update study only the root bending stiffness is used. Table 3 shows the parameters used, uniform distribution bounds, and the nominal values. Since the distribution is uniform, all values within the bounds are equally likely. A uniform distribution conveys our lack of knowledge and confidence in the nominal values. Assumed distribution functions should incorporate information on parameter uncertainties gained through experience, previously observed tests, or simply good engineering judgment.
To conduct the study, UNIPASS (Ref. 6) First/Second Order Reliability Methods were used to evaluate the MPP solution for both bending and torsion. UNIPASS replaces the variables defined as random variables in the NASTRAN input file and executes the analysis to compute the probability value in Eq. 2 and to determine the MPP solution.

To take advantage of the algorithms in the UNIPASS program, a limit state function was defined in terms of the predicted displacement vector \( \mathbf{x} = [x_1, x_{11}, x_{56}, x_{66}, x_{67}, x_{77}] \) and the measured test vector in Table 2 \( \mathbf{y} = [y_1, y_{11}, y_{56}, y_{66}, y_{67}, y_{77}] \) as:

\[
g(E_{11}, E_{11s}, E_{22s}, E_c, k_b) = \mathbf{y}^T \mathbf{x} - \mathbf{y}^T \mathbf{y}
\]

When these two vectors are identical the limit state function is zero. To account for the uncertainty in the measured displacement data, each component of the test vector \( \mathbf{y} \) was replaced with \( \tilde{\mathbf{y}} = Z \sigma + \mu \) where \( Z \) is a random variable from a normalized distribution function, \( \sigma \) is the measured standard deviation shown in Table 2, and \( \mu \) is the corresponding mean value.

Table 4 shows the MPP solution obtained for each static load case, the probability of observing those parameter values, and the corresponding sensitivity information. Note that the bending solution, shown in the third column, has a probability of 1% compared to the torsion solution probability of 63%. Although the bending solution is the most probable solution when using bending test data, the corresponding probability is low, therefore it is unlikely that the parameter values reported are the correct ones. In contrast, the solution obtained when using torsion data has a higher probability value indicating a more probable solution. Sensitivity values are provided to assess relative importance of the various parameters. To properly compare values, the sensitivity needs to be normalized by multiplying by the mean of the parameter values. If one were to use the nominal parameter values to normalize the sensitivity, the highest sensitivity corresponds to \( E_{11} \) followed by \( k_b \). Table 5 shows the predicted and measured displacement using both sets of MPP values. Fig. 7 shows two plots for the cases shown in Table 5 comparing test, updated model displacement predictions, and displacement prediction when using the nominal set of parameters; Fig. 7a depicts results using the MPP values for bending and 7b is the solution for torsion. When comparing the numerical values from tests and the updated solution, individual errors can be as high as 40%, however, when comparing them to the prediction using the nominal values the model has been improved significantly. Considering the torsion solution, parameter changes up to 24% are required to reconcile test with analysis. When examining the bending and torsion solutions it is apparent that the parameter values are moving in opposite directions but the most probable parameter values are those obtained for torsion.

Solutions for this class of problems are heavily dependant upon the initial distribution functions. Subsequent work to improve results needs to use a Bayesian approach to allow for the initial parameter distribution functions to be updated based on experimental data. Although this has not been emphasized, it is important to remember the probability and MPP calculation relies heavily on the assumed distributions. To simplify the analysis, the analyst can select simple distributions functions to gain understanding of the problem but it is imperative that these distribution functions be
updated when more information becomes available. This is a direction that would be
addressed in the near future.

Concluding Remarks

An initial study to evaluate the use of probabilistic approaches for the problem of
model update was presented. The goal was to include information on parameter
uncertainties known by the modelers to aid guiding the process of reconciling differences
between test results and analysis. A fundamental premise in this approach is that the
solution to the model update problem has an infinite number solutions, hence, seeking a
unique solution hides information about model errors that might be very important for a
certain class of problems. Rather than a single point solution, the engineer should have
probability distribution functions describing the full range of variations for an observed
quantity.

Using the UNIPASS probabilistic analysis code and data obtained from static
tests conducted by NASA Dryden in support of the Aerostructure Test Wing flutter
experiment, updates to parameters in the finite element model were obtained to reconcile
static test data with predictions from analysis. In the example, five parameters were
updated from their initial values. Also computed was the probability of those parameters
being responsible for the observed test results. Although results for individual sensor
values are not in full agreement, the overall system response was improved significantly
after parameter updates.

Acknowledgments

The authors would like to thanks Dr. Mohammad Khalessi and Dr. Hong-Zong
Lin from UNIPASS Technologies for the numerous discussions and useful suggestions
about the use of the UNIPASS probabilistic analysis code.

References

1- Hasselman, T.K., Chrostowski, J.D., and Ross, T.J.: Propagation of Modeling
ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials
Conference, Paper No. AIAA 94-1316, April 18-20, 1994, SC.
2- Herendeen, D.L., Woo, L., Hassleman, T.K., and Zimmerman, D.C.: Analysis-
Test Correlation and Model Updating of Dynamic Systems Using MDO Software
Tools. Proceedings of the 7th AIAA/ USAF/NASA/SSMO Symposium on
Multidisciplinary Analysis and Optimization, Paper No. 98-4730,St. Louis, MO,
3- Alvin, K.F.: Finite Element Model Update via Bayesian Estimation and
4- Farhat, C. and Hemez, F.M.: Updating Finite Element Dynamic Models Using an
Element-by-Element Sensitivity Methodology. AIAA Journal, Vol. 31, No. 9,
1993, pp. 1702-1711.
Probabilistic Analysis Software Package, FEBREL-MSC/NASTRAN.


<table>
<thead>
<tr>
<th>Location</th>
<th>Pressure (lb/in²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.116</td>
</tr>
<tr>
<td>2</td>
<td>0.277</td>
</tr>
<tr>
<td>3</td>
<td>0.146</td>
</tr>
<tr>
<td>4</td>
<td>0.014</td>
</tr>
<tr>
<td>5</td>
<td>0.049</td>
</tr>
<tr>
<td>6</td>
<td>0.129</td>
</tr>
<tr>
<td>7</td>
<td>0.021</td>
</tr>
<tr>
<td>8</td>
<td>0.045</td>
</tr>
<tr>
<td>9</td>
<td>0.092</td>
</tr>
<tr>
<td>10</td>
<td>0.007</td>
</tr>
<tr>
<td>11</td>
<td>0.026</td>
</tr>
<tr>
<td>12</td>
<td>0.038</td>
</tr>
<tr>
<td>13</td>
<td>0.042</td>
</tr>
<tr>
<td>14</td>
<td>0.069</td>
</tr>
<tr>
<td>15</td>
<td>0.011</td>
</tr>
<tr>
<td>16</td>
<td>0.027</td>
</tr>
<tr>
<td>17</td>
<td>0.034</td>
</tr>
<tr>
<td>18</td>
<td>0.027</td>
</tr>
<tr>
<td>19</td>
<td>0.040</td>
</tr>
</tbody>
</table>

Table 2. ATW measured displacement

<table>
<thead>
<tr>
<th>Location</th>
<th>Bending Disp. (in)- (Std. Dev.)</th>
<th>10 lbs Torsion Disp. (in)- (Std. Dev.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.224-(1.42e-4)</td>
<td>0.437-(5.14e-4)</td>
</tr>
<tr>
<td>11</td>
<td>0.290-(1.23e-4)</td>
<td>0.220-(1.45e-4)</td>
</tr>
<tr>
<td>56</td>
<td>0.053-(2.21e-4)</td>
<td>0.102-(2.33e-4)</td>
</tr>
<tr>
<td>66</td>
<td>0.129-(1.97e-4)</td>
<td>0.095-(1.95e-4)</td>
</tr>
<tr>
<td>67</td>
<td>0.032-(1.92e-4)</td>
<td>0.064-(2.11e-4)</td>
</tr>
<tr>
<td>77</td>
<td>0.090-(1.94e-4)</td>
<td>0.058-(2.03e-4)</td>
</tr>
<tr>
<td>100</td>
<td>0.001-(2.00e-4)</td>
<td>0.002-(1.90e-4)</td>
</tr>
<tr>
<td>110</td>
<td>0.017-(1.92e-4)</td>
<td>0.003-(1.99e-4)</td>
</tr>
</tbody>
</table>

Table 3. Parameter Uncertainty definitions

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameters</th>
<th>Uniform Dist. Bounds [Lower, Upper]</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar</td>
<td>$E_{11}$ (lb/in²)</td>
<td>0.79×10⁷, 2.37×10⁷</td>
<td>1.58×10⁷</td>
</tr>
<tr>
<td>Skin</td>
<td>$E_{11r}$ (lb/in²)</td>
<td>0.74×10⁶, 3.90×10⁶</td>
<td>2.67×10⁶</td>
</tr>
<tr>
<td>Skin</td>
<td>$E_{22s}$ (lb/in²)</td>
<td>0.74×10⁶, 3.90×10⁶</td>
<td>2.67×10⁶</td>
</tr>
<tr>
<td>Core</td>
<td>$E_c$ (lb/in²)</td>
<td>4.8×10³, 15×10³</td>
<td>13.1×10³</td>
</tr>
<tr>
<td>Root</td>
<td>$k_b$ (lbs/in)</td>
<td>2×10⁴, 3×10⁴</td>
<td>2.2×10⁴</td>
</tr>
</tbody>
</table>
Table 4. MPP solution for static case

<table>
<thead>
<tr>
<th>Component</th>
<th>Parameters</th>
<th>MPP Bending Pf=0.01</th>
<th>Bending Sensitivity ( \frac{\partial g}{\partial x} )</th>
<th>MPP Torsion Pf=0.63</th>
<th>Torsion Sensitivity ( \frac{\partial g}{\partial x} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spar</td>
<td>( E_{11} ) (lbs/in(^2))</td>
<td>( 8.42 \times 10^6 )</td>
<td>( 9.54 \times 10^{-9} )</td>
<td>( 1.77 \times 10^7 )</td>
<td>( 7.14 \times 10^{-9} )</td>
</tr>
<tr>
<td>Skin</td>
<td>( E_{11s} ) (lbs/in(^2))</td>
<td>( 2.16 \times 10^6 )</td>
<td>( 3.03 \times 10^{-9} )</td>
<td>( 2.64 \times 10^6 )</td>
<td>( 3.66 \times 10^{-9} )</td>
</tr>
<tr>
<td>Skin</td>
<td>( E_{22s} ) (lbs/in(^2))</td>
<td>( 2.16 \times 10^6 )</td>
<td>( 3.03 \times 10^{-9} )</td>
<td>( 2.64 \times 10^6 )</td>
<td>( 3.66 \times 10^{-9} )</td>
</tr>
<tr>
<td>Core</td>
<td>( E_s ) (lbs/in(^2))</td>
<td>( 7.70 \times 10^3 )</td>
<td>( 1.01 \times 10^{-6} )</td>
<td>( 9.97 \times 10^3 )</td>
<td>( 7.28 \times 10^{-7} )</td>
</tr>
<tr>
<td>Root</td>
<td>( k_b ) (lbs/in)</td>
<td>( 2.19 \times 10^4 )</td>
<td>( 1.95 \times 10^{-6} )</td>
<td>( 2.54 \times 10^4 )</td>
<td>( 3.99 \times 10^{-6} )</td>
</tr>
</tbody>
</table>

Table 5. Comparison of displacement data for updated model

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.224-(1.42e-4)</td>
<td>0.173</td>
<td>0.250</td>
<td>0.437-(5.14e-4)</td>
<td>0.442</td>
<td>0.425</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.290-(1.23e-4)</td>
<td>0.217</td>
<td>0.318</td>
<td>0.220-(1.45e-4)</td>
<td>0.295</td>
<td>0.261</td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.053-(2.21e-4)</td>
<td>0.054</td>
<td>0.072</td>
<td>0.102-(2.33e-4)</td>
<td>0.131</td>
<td>0.129</td>
<td></td>
</tr>
<tr>
<td>66</td>
<td>0.129-(1.97e-4)</td>
<td>0.096</td>
<td>0.136</td>
<td>0.095-(1.95e-4)</td>
<td>0.107</td>
<td>0.098</td>
<td></td>
</tr>
<tr>
<td>67</td>
<td>0.032-(1.92e-4)</td>
<td>0.037</td>
<td>0.048</td>
<td>0.064-(2.11e-4)</td>
<td>0.090</td>
<td>0.090</td>
<td></td>
</tr>
<tr>
<td>77</td>
<td>0.090-(1.94e-4)</td>
<td>0.073</td>
<td>0.103</td>
<td>0.058-(2.03e-4)</td>
<td>0.076</td>
<td>0.069</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>0.001-(2.00e-4)</td>
<td>0.006</td>
<td>0.007</td>
<td>0.002-(1.90e-4)</td>
<td>0.013</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>110</td>
<td>0.017-(1.92e-4)</td>
<td>0.015</td>
<td>0.019</td>
<td>0.003-(1.99e-4)</td>
<td>0.007</td>
<td>0.006</td>
<td></td>
</tr>
</tbody>
</table>
\[ g(x, y) = x - y = 0 \]

**Fig. 1 Reliability problem**

- **Failure Domain** 
  \( g(x, y) < 0 \)
- **Limit State Function**
- **Prob. Density** 
  \( f(x, y) = \text{const.} \)
- **Safe Domain** 
  \( g(x, y) > 0 \)

---

**ATW**

**a) Overall configuration**

**b) wing details**

**Fig. 2 Aerostructure Test Wing (ATW)**
Fig. 3 Description of the ATW finite element model

Fig. 4 Pressure load numbering and sensor locations
Fig. 5 ATW Proof test loading 25% design load

Fig. 6 Wing twist test at forward boom- loading 10 lbs
Fig. 7 Test and analysis comparison of MPP displacement solutions

(a) Bending comparison

(b) Torsion comparison
**Abstract**

Finite element models are often developed for load validation, structural certification, response predictions, and to study alternate design concepts. In rare occasions, models developed with a nominal set of parameters agree with experimental data without the need to update parameter values. Today, model updating is generally heuristic and often performed by a skilled analyst with in-depth understanding of the model assumptions. Parameter uncertainties play a key role in understanding the model update problem and therefore probabilistic analysis tools, developed for reliability and risk analysis, may be used to incorporate uncertainty in the analysis. In this work, probability analysis (PA) tools are used to aid the parameter update task using experimental data and some basic knowledge of potential error sources. Discussed here is the first application of PA tools to update parameters of a finite element model for a composite wing structure. Static deflection data at six locations are used to update five parameters. It is shown that while prediction of individual response values may not be matched identically, the system response is significantly improved with moderate changes in parameter values.

**Subject Terms**

- Parameter updates; Probabilistic analysis; Finite element; Model correlation

**Security Classification**

- Report: Unclassified
- OF THIS PAGE: Unclassified
- ABSTRACT: Unclassified

**Report Type and Dates Covered**

Technical Memorandum, June 2001

**Funding Numbers**

706-32-31-04

**Performing Organization**

NASA Langley Research Center
Hampton, VA 23681-2199

**Sponsoring/Monitoring Agency**

National Aeronautics and Space Administration
Washington, DC 20546-0001

**Distribution/Availability Statement**

Unclassified-Unlimited
Subject Category 05 Distribution: Nonstandard
Availability: NASA CASI (301) 621-0390

**Security Classification of Report**

Unclassified

**Security Classification of This Page**

Unclassified

**Security Classification of Abstract**

Unclassified

**Number of Pages**

17

**Price Code**

A03

UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-88)

Prescribed by ANSI Std. Z39-18

298-102