Proton-Nucleus Elastic Cross Sections
Using Two-Body In-Medium Scattering Amplitudes

R. K. Tripathi and John W. Wilson
Langley Research Center, Hampton, Virginia

Francis A. Cucinotta
Lyndon B. Johnson Space Center, Houston, Texas

August 2001
The NASA STI Program Office . . . in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the lead center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- **TECHNICAL PUBLICATION.** Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA counterpart of peer-reviewed formal professional papers, but having less stringent limitations on manuscript length and extent of graphic presentations.

- **TECHNICAL MEMORANDUM.** Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- **CONTRACTOR REPORT.** Scientific and technical findings by NASA-sponsored contractors and grantees.

- **CONFERENCE PUBLICATION.** Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or co-sponsored by NASA.

- **SPECIAL PUBLICATION.** Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- **TECHNICAL TRANSLATION.** English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized databases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at http://www.sti.nasa.gov

- Email your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA STI Help Desk at (301) 621-0134

- Telephone the NASA STI Help Desk at (301) 621-0390

- Write to:
  NASA STI Help Desk
  NASA Center for AeroSpace Information
  7121 Standard Drive
  Hanover, MD 21076-1320
Proton-Nucleus Elastic Cross Sections Using Two-Body In-Medium Scattering Amplitudes

R. K. Tripathi and John W. Wilson
Langley Research Center, Hampton, Virginia

Francis A. Cucinotta
Lyndon B. Johnson Space Center, Houston, Texas

National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23681-2199

August 2001
Abstract

Recently, a method was developed of extracting nucleon-nucleon (NN) cross sections in the medium directly from experiment. The in-medium NN cross sections form the basic ingredients of several heavy-ion scattering approaches including the coupled-channel approach developed at the Langley Research Center. The ratio of the real to the imaginary part of the two-body scattering amplitude in the medium was investigated. These ratios are used in combination with the in-medium NN cross sections to calculate elastic proton-nucleus cross sections. The agreement is excellent with the available experimental data. These cross sections are needed for the radiation risk assessment of space missions.

Introduction

The transportation of energetic ions in bulk matter is of direct interest in several areas including shielding against ions originating from either space radiations or terrestrial accelerators, cosmic ray propagation studies in galactic medium, or radiobiological effects resulting from the work place or clinical exposures. For carcinogenesis, terrestrial radiation therapy, and radiobiological research, knowledge of beam composition and interactions is necessary to properly evaluate the effects on human and animal tissues. For the proper assessment of radiation exposures both reliable transport codes and accurate input parameters are needed. One such important input is elastic cross sections. The motivation of the work is to develop a method for calculating accurate cross sections. These elastic cross sections are needed in transport methods both deterministic and Monte Carlo.

Nucleon-nucleon (NN) cross sections are the basic ingredients of many approaches (refs. 1 to 10) to heavy-ion scattering problem. Most information about these NN cross sections comes from the free two-body scattering. These cross sections are significantly modified in a nucleus, due to the presence of other nucleons, which is affected through the Pauli exclusion principle and modification of meson field coupling constants. (See ref. 11.) Our theoretical approach is based on the coupled-channel method used at the Langley Research Center. (See refs. 1 to 6.) This method solves the Schrödinger equation with an eikonal approximation. The method needs modifications at low and medium energies. In an earlier work (refs. 12 and 13), we developed a unique method of extracting medium modified NN cross sections from experiments and found that the renormalization of the free NN cross sections is significant at lower and medium energies. These modified in-medium NN cross sections, in combination with the newly developed ratio of the real to the imaginary part of the two-body scattering amplitudes in the medium, were used to calculate the total cross sections for proton-nucleus collisions (refs. 14 and 15). The blend of the renormalized NN cross sections, the in-medium ratio of the real to the imaginary part of the two-body amplitude, and the coupled-channel method gave reliable approach to the total cross sections. The purpose of the current paper is threefold:

1. To put in place a reliable method for calculating elastic cross sections for collisions of protons with ions
2. To use our previously developed NN cross sections in the medium and modified two-body amplitudes to calculate elastic cross sections for proton-nucleus collisions
3. To validate and compare the calculated results with the available experimental data
4. To provide theoretical results where data are not available (due to nonexistence of experimental facilities and/or difficulty in experimental data analysis)

**Method**

For completeness, the essentials of the coupled-channel method are briefly sketched. (See refs. 1 to 6 for details.) In this approach, the matrix for elastic scattering amplitude is given by

\[
f(q) = \frac{-ik}{2\pi} \int \exp(-iq.b) \{\exp[i\chi(b)] - 1\} \ d^2b
\]  

(1)

where

- \( f \) and \( \chi \) matrices
- \( k \) projectile momentum relative to center of mass
- \( b \) projectile impact parameter vector
- \( q \) momentum transfer
- \( \chi(b) \) eikonal phase matrix

The total cross section \( \sigma_{tot} \) is found from the elastic scattering amplitude by using the optical theorem as follows:

\[
\sigma_{tot} = \frac{4\pi}{k} \operatorname{Im}[f(q=0)]
\]

(2)

Equations (1) and (2) give

\[
\sigma_{tot} = 4\pi \int_{0}^{\infty} \{1 - \exp[-\operatorname{Im}(\chi)] \cos [\operatorname{Re}(\chi)]\} b \ db
\]

(3)

The absorption cross section \( \sigma_{abs} \) is given by (refs. 12 and 13)

\[
\sigma_{abs} = 2\pi \int_{0}^{\infty} \{1 - \exp[-2 \operatorname{Im}(\chi)]\} b \ db
\]

(4)

Having calculated the total and absorption cross sections for many nuclei, elastic cross section is the difference of these quantities:

\[
\sigma_{el} = \sigma_{tot} - \sigma_{abs}
\]

(5)

The eikonal phase matrix \( \chi \) (see refs. 1 to 6 for details) is given by

\[
\chi(b) = \chi_{dir}(b) - \chi_{ex}(b)
\]

(6)
The direct and exchange terms are calculated by using the following expressions (refs. 1 to 6):

\[
\chi_{\text{dir}}(b) = \frac{A_P A_T}{2\pi k_{NN}} \int \exp(iq.b) F^{(1)}(-q) G^{(1)}(q) f_{NN}(q) \, d^2q
\]

\[
\chi_{\text{ex}}(b) = \frac{A_P A_T}{2\pi k_{NN}} \int \exp(iq.b) F^{(1)}(-q) G^{(1)}(q) \, d^2q \times \frac{1}{(2\pi)^2} \int \exp(iq'.b) f_{NN}(q+q') C(q') \, d^2q'
\]

where

\( F^{(1)} \) and \( G^{(1)} \) projectile and target ground-state one-body form factors, respectively

\( k_{NN} \) relative wave number in two-body center-of-mass system

\( C \) correlation function (ref. 6)

\( A_P \) and \( A_T \) mass numbers of projectile and target nuclei, respectively

The two-body amplitude \( f_{NN} \) is parameterized as

\[
f_{NN} = \frac{\sigma(\alpha + i)}{4\pi} k_{NN} \exp\left( -\frac{Bq^2}{2} \right)
\]

where

\( \sigma \) two-body cross section

\( B \) slope parameter

\( \alpha \) ratio of real part to imaginary part of forward, two-body amplitude

It is well-known that the absorption cross section depends on the imaginary part of the eikonal phase matrix. This leads us to write the two-body amplitude in the medium \( f_{NN,m} \) as

\[
f_{NN,m} = f_m f_{NN}
\]

where \( f_{NN} \) is the free NN amplitude and \( f_m \) is the system- and energy-dependent medium multiplier function. (See refs. 12 and 13.) Then the nucleon-nucleon cross sections in the medium (\( \sigma_{NN,m} \)) can be written as

\[
\sigma_{NN,m} = f_m \sigma_{NN}
\]
where $\sigma_{NN}$ is the nucleon-nucleon cross section in free space, and the medium multiplier is given by

$$f_m = 0.1 \exp\left(-\frac{E}{12}\right) + \left[1 - \left(\frac{\rho_{av}}{0.14}\right)^{1/3} \exp\left(-\frac{E}{D}\right)\right]$$

(12)

where $E$ is the laboratory energy in units of $A$ MeV, $D$ is a parameter in units of MeV, as defined subsequently in equation (13). The numbers 12 and 0.14 are in units of MeV and fm$^{-3}$, respectively. For $A_T \leq 56$ (mass number for iron ion representing heavy elements considered in our transport phenomena),

$$D = 46.72 + 2.21A_T - (2.25 \times 10^{-2})A_T^2$$

(13)

and for $A_T > 57$,

$$D = 100 \text{ MeV}$$

(14)

In equation (12), $\rho_{av}$ refers to the average density of the colliding system and is

$$\rho_{av} = \frac{1}{2}\left(\rho_{Ap} + \rho_{AT}\right)$$

(15)

where the density of a nucleus $A_i (i = P, T)$ is calculated in the hard sphere model and is given by

$$\rho_{A_i} = \frac{A_i}{(4\pi/3) r_i^3}$$

(16)

where the radius of the nucleus $r_i$ is defined by

$$r_i = 1.29 (r_i)_{rms}$$

(17)

The root-mean-square radius $(r_i)_{rms}$ is obtained directly from experiment (ref. 16) after “subtraction” of the nucleon charge form factor (ref. 2).

Note from equation (3) that total cross section depends on real component of eikonal phase matrix, and hence (eqs. (5), (6), and (7)), on the product of $\sigma\alpha$ in two-body amplitude. Since the modification of the cross sections in the medium has been determined and tested thoroughly (refs. 12 and 13), the modification of $\alpha$—ratio of real to imaginary part of the two-body amplitude—is studied in the medium to calculate the total cross sections (refs. 14 and 15). Some data for total cross sections are available for a few systems at high energies. Unfortunately, no data are available for total cross sections at low and medium energy range; there are some data for $p + \text{Pb}$ in the 100 A MeV range. Therefore, values of the medium-modified $\alpha$ have been tested for higher energies. At low and medium energies, our theoretical results, which incorporate the in-medium two-body amplitudes, can be validated, if and when experimental data become available.
A best estimate of medium-modified $\alpha$ takes into account the enhancement of the cross sections (ref. 17) and stability and is given by

$$\alpha_m = 3 \exp\left[-\frac{(E-13A^{1/3})^2}{5000}\right] + \frac{K}{1+\exp\left[\frac{(10-E)/75}{50-0-K}\right]}$$

where

$$K = 0.35 + 0.65 \exp\left[-\frac{2}{3}(N-Z)\right]$$

with $N$ being the neutron number of the nucleus and $Z$ its charge number.

Equation (3) has also been modified to account for the Coulomb force in the proton-nucleus cross sections. This modification has significant effects at low energies and becomes less important as the energy increases and practically disappears for energies around 50 A MeV and higher.

For nucleus-nucleus collisions, the Coulomb energy is given by

$$V_B = \frac{1.44 Z_p Z_T}{R}$$

where the constant 1.44 is in units of MeV-fm, $Z_p$ and $Z_T$ are charge numbers for the projectile and target, respectively, and $R$, the radial distance between their centers, is given by

$$R = r_p + r_T + 1.2 \frac{A_p^{1/3} + A_T^{1/3}}{E_{cm}^{1/3}}$$

The number 1.2 in equation (21) is in units of fm-MeV$^{1/3}$. In our earlier work (refs. 12 and 13), these expressions were used also for the proton-nucleus collisions to have a unified picture of any colliding system. However, as shown in the references, equation (21) overestimates the radial distance between proton-nucleus collisions, and hence, equation (20) underestimates the Coulomb energy between them. To compensate for this, we multiplied equation (20) by the following factor (refs. 12 and 13), which gives the Coulomb multiplier to equation (3):

$$X_{\text{Coul}} = \left(1 + \frac{C_1}{E_{cm}}\right) \left(1 - \frac{C_2 V_B}{E_{cm}}\right)$$

For $A_T \leq 56$ (mass number for iron),

$$C_1 = 6.81 - 0.17 A_T (1.88 \times 10^{-3}) A_T^2$$

$$C_2 = 6.57 - 0.30 A_T (3.6 \times 10^{-3}) A_T^2$$
The constant $C_1$ is in units of MeV. For $A_T > 57$, 

$$
C_1 = 3.0 \text{ MeV} \\
C_2 = 0.8
$$

For the nucleus-nucleus collisions, $C_1 = 0 \text{ MeV}$ and $C_2 = 1$. This form of Coulomb energy was found to work well for the proton-nucleus absorption cross sections (ref. 12). Equation (5) is the main equation. The total cross section (eq. (3)) and absorption cross section (eq. (4)) are multiplied by equation (22) to get the total and absorption cross sections in the medium and then these are used in equation (5) to get the results shown in figures 1 to 6. For clarity, the final expressions used for calculating the total and absorption cross sections in the medium are given as 

$$
\sigma_{\text{tot}} = 4\pi \left( \frac{1+C_1}{E_{cm}} \right) \left( \frac{1-C_2 V_B}{E_{cm}} \right) \int_0^\infty \{ 1 - \exp[\text{Im}(\chi_m)] \} \cos[\text{Re}(\chi_m)] \, b \, db 
$$

and 

$$
\sigma_{\text{abs}} = 2\pi \left( \frac{1+C_1}{E_{cm}} \right) \left( \frac{1-C_2 V_B}{E_{cm}} \right) \int_0^\infty \{ 1 - \exp[-2 \text{Im}(\chi_m)] \} \, b \, db
$$

The constants $C_1$ and $C_2$ are given by equations (23) and (24) and the in-medium eikonal phase matrix $\chi_m$ takes the form:

$$
\chi_m(b) = \chi_{m,\text{dir}}(b) - \chi_{m,\text{ex}}(b)
$$

The direct and exchange terms in the medium are calculated using the following expressions:

$$
\chi_{m,\text{dir}}(b) = \frac{A p A_T}{2\pi k_{NN}} \int \exp(iq.b) \, F^{(1)}(-q) \, G^{(1)}(q) \, f_{NN,m}(q) \, d^2q
$$

$$
\chi_{m,\text{ex}}(b) = \frac{A p A_T}{2\pi k_{NN}} \int \exp(iq.b) \, F^{(1)}(-q) \, G^{(1)}(q) \, d^2q \\
\times \frac{1}{(2\pi)^2} \int \exp(iq'.b) \, f_{NN,m}(q+q') \, C(q') \, d^2q'
$$

The two-body amplitude in the medium $f_{NN,m}$ includes the in-medium nucleon-nucleon cross section ($\sigma_m$)

$$
f_{NN,m} = \frac{\sigma_m(i\alpha_m + i)}{4\pi} k_{NN} \exp \left( -\frac{Bq^2}{2} \right)
$$

Equations (25) to (30) are used to calculate the elastic cross sections shown in figures 1 to 6. The procedure is as follows: Using the in-medium NN cross sections, calculate the two-body amplitude in the medium by equation (30). Then use equations (28) and (29) to calculate direct and exchange part of the eikonal phase matrix and equation (27) for the full eikonal matrix, which in turn is used in equations (25) and (26) to calculate the total and absorption cross sections in the medium. And finally, equation (5) gives the results shown in figures 1 to 6.
Results and Conclusions

Figures 1 to 6 show the results of our calculations for the elastic cross sections for proton on beryllium, carbon, aluminum, iron, lead, and uranium targets, respectively. The experimental data have been taken from the compilation of references 18 and 19. There is paucity of data at lower and intermediate energies where the medium modifications play a significant role. For the energy ranges considered, where the data are unavailable, our results provide good theoretical values of total cross sections because many renormalization effects due to medium, which play an important role in cross sections, have been incorporated in the formalism. The reason for the enhancement in elastic cross sections in the intermediate energy range is mainly because the real part of the two-body scattering amplitude is more dominant compared with the imaginary of the two-body scattering amplitude in this energy range.

We find very good agreement with the experimental results for all the systems at higher energies where some data are available. The in-medium cross sections derived earlier in combination with the modified ratio of the real to the imaginary part of the amplitude discussed provide good results for the proton-nucleus elastic cross sections. It is gratifying to note that the present method gives a consistent basic approach for the total reaction and the total cross sections, hence, the total elastic cross sections for the entire energy range for all the systems studied.

The in-medium two-body amplitudes developed in our approach can be used with ease in other nuclear processes such as the collision of ions with nuclei as well. The next step is to examine effects on specified reaction channels.
References


Figure 1. Elastic cross sections for proton-beryllium collision as function of energy.

Figure 2. Elastic cross sections for proton-carbon collision as function of energy.
Figure 3. Elastic cross sections for proton-aluminum collision as function of energy.

Figure 4. Elastic cross sections for proton-iron collision as function of energy.
Figure 5. Elastic cross sections for proton-lead collision as function of energy.

Figure 6. Elastic cross sections for proton-uranium collision as function of energy.
Recently, a method was developed of extracting nucleon-nucleon (NN) cross sections in the medium directly from experiment. The in-medium NN cross sections form the basic ingredients of several heavy-ion scattering approaches including the coupled-channel approach developed at the Langley Research Center. The ratio of the real to the imaginary part of the two-body scattering amplitude in the medium was investigated. These ratios are used in combination with the in-medium NN cross sections to calculate elastic proton-nucleus cross sections. The agreement is excellent with the available experimental data. These cross sections are needed for the radiation risk assessment of space missions.