Effect of Speed (Centrifugal Load) on Gear Crack Propagation Direction

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ABSTRACT

The effect of rotational speed (centrifugal force) on gear crack propagation direction was explored. Gears were analyzed using finite element analysis and linear elastic fracture mechanics. The analysis was validated with crack propagation experiments performed in a spur gear fatigue rig. The effects of speed, rim thickness, and initial crack location on gear crack propagation direction were investigated. Crack paths from the finite element method correlated well with those deduced from gear experiments. For the test gear with a backup ratio (rim thickness divided by tooth height) of \( n_0 = 0.5 \), cracks initiating in the tooth fillet propagated to rim fractures when run at a speed of 10,000 rpm and became tooth fractures for speeds slower than 10,000 rpm for both the experiments and analysis. From additional analysis, speed had little effect on crack propagation direction except when initial crack locations were near the tooth/rim fracture transition point for a given backup ratio. When at that point, higher speeds tended to promote rim fracture while lower speeds (or neglecting centrifugal force) produced tooth fractures.

INTRODUCTION

Effective gear designs balance strength, durability, reliability, size, weight, and cost. However, unexpected gear failures may occur even with adequate gear tooth design [1]. In order to design an extremely safe system, the designer must ask and address the question “what happens when a failure occurs”. With regard to gear tooth bending fatigue, tooth or rim fractures may occur. For aircraft, a crack which propagates through a rim would be catastrophic, leading to disengagement of a rotor or propelled, loss of an aircraft, and possible fatalities [2]. This failure mode should be avoided. A crack which propagates through a tooth itself may or may not be catastrophic, depending on the design and operating conditions. Also, early warning of this failure mode may be possible due to advances in modern diagnostic systems.

Fracture mechanics has developed into a useful discipline for predicting strength and life of cracked structures. Linear elastic fracture mechanics applied to gear teeth has become increasingly popular. The stress intensity factors are the key parameters to estimate the characteristics of a crack. Analytical methods (such as weight function techniques) as well as numerical methods have been used to estimate gear tooth stress intensity factors [3-4]. Based on stress intensity factors, fatigue crack growth and gear life predictions have been investigated [5-6]. In addition, gear crack trajectory predictions have been addressed in a few studies [7-10]. In high speed components, centrifugal force can significantly contribute to the applied loading of the structure. This effect on crack propagation has been investigated for turbine blades [11] but has not been looked at with respect to gears.

The objective of the current study is to determine the effect of centrifugal force on gear crack propagation. The goal is to prevent catastrophic rim fracture failure modes when considering gear tooth bending fatigue. Analysis was performed using the finite element method with principles of linear elastic fracture mechanics. Crack propagation paths were predicted for a variety of gear tooth and rim configurations. Experiments were performed in a gear fatigue apparatus to validate crack predictions. The effects of speed (centrifugal force), rim thickness, and initial crack location were considered. Crack trajectories are presented for the variety of cases studied indicating gear tooth or gear rim fracture modes.

ANALYSIS

Basic gear tooth geometry data was input to a tooth coordinate generation computer program. The tooth coordinate generator program used the method of [12] to determine the tooth coordinates. The output was tooth coordinate and rim coordinate data which defined a single-tooth sector of a gear. This output was used by a commercially available pre- and post-processing finite element analysis software package [13]. This package created the finite element mesh of the complete gear. The mesh was then imported to the FRANC (Fracture Analysis Code) computer program.

FRANC is a general purpose finite element code for the static analysis of cracked structures [14]. The program is designed for two-dimensional problems, uses principles of linear elastic fracture mechanics, and is capable of analyzing plane strain, plane stress, or axisymmetric problems. Eight-node quadrilateral or six-node triangular elements can be used. Point forces can be applied at nodes to simulate gear tooth loading. Also, body forces due to angular rotation can be used to account for centrifugal force speed effects. The deflections, stresses, and stress intensity factors from the point forces and body forces can then be added together using superposition.

Among the variety of capabilities, a unique feature of the program is the ability to model a crack in a structure. The program uses a method called “delete and fill” to accomplish this. To illustrate, consider a finite element mesh of an uncracked structure. The user would first define an initial crack by identifying the node of the crack mouth and coordinates of the crack tip (Fig. 1a). FRANC would then delete the elements in the vicinity of the crack tip, insert a rosette of quarter-point, six-node triangular elements around the crack tip to model the inverse square-root stress singularity, then fill the remaining area between the rosette and original mesh with conventional six-node triangular elements (Fig. 1b). The user would then run the finite element equation solver to determine nodal displacements, forces, stresses, and strains. Mode I and mode II stress intensity factors, \( K_I \) and \( K_{II} \), respectively, can be calculated using a variety of methods. (Mode I loading refers to loads applied normal to the crack plane which tend to open the crack. Mode II refers to in-plane shear loading. In addition, the crack propagation angle is a function of \( K_{II} / K_I \).) The stress intensity factors quantify the state of stress in the region near the crack tip. In the program, the stress intensity factors can also be used to predict the crack propagation trajectory angles, again using a variety of methods.

A further unique feature of FRANC is the automatic crack propagation capability. After an initial crack is inserted in a mesh, the program simulates crack propagation as a number of straight line segments. For each segment (or step), the program solves the
finite element equations, calculates the stress intensity factors, and calculates the crack propagation angle. The program then places the new crack tip at the calculated angle and at a user-defined crack increment length. The model is then re-meshed using the "delete and fill" method described above. The procedure is repeated a number of times as specified by the user. Fig. 1c shows the predicted crack propagation path of a gear tooth. In this example, the predicted crack trajectory was after 29 steps, i.e., the crack trajectory was approximated by 29 line segments.

A typical finite element gear model used in the current study is shown in Fig. 2. The mesh shown is for an uncracked gear. The gear design is the baseline used in the current study. The design parameters are: 28 teeth, 3.175-mm module, 44.45-mm pitch radius, 20° pressure angle, and a 6.25-mm face width. The model had 3255 plane stress, 8-node, quadrilateral elements and 7122 nodes. For improved accuracy, the mesh was refined in the upper portion of the model (this is the region where cracks will be inserted). The tooth load was placed at the highest point of single tooth contact (HPSTC), normal to the surface. Although the tooth load changes in magnitude and direction in actual gear operations, a static analysis with the load at the HPSTC has given accurate results with respect to crack propagation analysis [9]. Four hub nodes at the gear inner diameter were fixed to ground to constrain the model. The material used was steel. In addition, slots were incorporated in the model to vary the rim thickness. The model shown has a backup ratio, \( m_b = 1.0 \). The backup ratio is defined as the rim thickness, \( h \), divided by the tooth height, \( b \) (Fig. 3). As stated before, crack propagation angles are determined from the calculated stress intensity factors. In the current study, the stress intensity factors were determined from the finite element method nodal displacements and forces using the J-integral method [15]. In addition, the crack propagation angles were determined from the stress intensity factors using the maximum tangential stress theory [16].

EXPERIMENTS

Crack propagation experiments were performed in the NASA Glenn Spur Gear Fatigue Rig (Fig. 4). The test stand operated on a torque-regenerative principle in which torque was circulated in a loop of test gears and slave gears. Oil pressure was supplied to load vanes in one slave gear which displaced the gear with respect to its shaft. This produced a torque on the test gears, slave gears, and connecting shafts proportional to the amount of applied oil pressure. An 18.6-kW, variable-speed motor provided speed to the drive shaft using a belt and pulley. Note that in a torque-regenerative system, the required input drive power needs only to overcome the losses in the system.

Separate lubrication systems were provided for the tests gears and the main gearbox. The test gears were lubricated using a single oil jet at the in-to-mesh location. The main gearbox lubrication system provided oil to the loading vanes using a high-pressure pump. Also, the main gearbox lubrication system provided oil to
the slave gears and support bearings. The test gear and main
gearbox lubrication systems were separated by labyrinth seals on
the gear shafts pressurized with nitrogen gas. Even though two
separate systems existed, a common oil was used for both since
some leakage occurred between the two. The lubricant used was a
synthetic paraffin oil. In addition, the test gear lubricant was
filtered through a 5-micron fiberglass filter.

The spur gear fatigue rig was primarily developed for surface
pitting fatigue life investigations. For surface pitting fatigue tests,
the test gears are run offset to increase the tooth contact stress and
promote surface fatigue. For the current crack propagation studies,
however, the desired failure mode was tooth bending fatigue.
Therefore, the gears were run with full tooth width contact. Not
offset.

The test gear geometry data are given in Table I. The gears
were external spur gears. The teeth had involute profiles with
linear tip relief starting at the HPSTC and ending at the tooth tip.
The maximum amount of tip relief was 0.013 mm at the tooth tip.
The test gear material was consumable-electrode vacuum-melted
AISI 9310 steel. The gears were case-carburized and ground. The
teeth were hardened to a case hardness of $R_c$ 61 and a core hardness
of $R_c$ 38. The effective case depth (depth at a hardness of $R_c$ 50)
was 0.81 mm. Previous studies on these test gears showed that
speed effects on crack propagation direction would be most
prevalent for a backup ratio of $m_b = 0.5$ [7]. Thus, slots were
machined in the gears to give a backup ratio of $m_b = 0.5$ (Fig. 5a).

<table>
<thead>
<tr>
<th>Test</th>
<th>Notch mouth, $R_o$</th>
<th>Notch length, $l$</th>
<th>Notch angle, $\gamma$</th>
<th>Test speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40.50</td>
<td>0.36</td>
<td>93.8</td>
<td>2500</td>
</tr>
<tr>
<td>2</td>
<td>40.60</td>
<td>0.32</td>
<td>108.7</td>
<td>4920</td>
</tr>
<tr>
<td>3</td>
<td>40.42</td>
<td>0.50</td>
<td>96.3</td>
<td>7500</td>
</tr>
<tr>
<td>4</td>
<td>40.50</td>
<td>0.26</td>
<td>118.5</td>
<td>10'000</td>
</tr>
</tbody>
</table>

Fig. 5. Test gear. $m_b = 0.5$. (a) full view, (b) magnified
view of notch in tooth fillet.

It was believed that tooth bending fatigue cracks would be
difficult to initiate based on the load capacity of the test rig. Due
to this limitation, notches were fabricated in the fillet region (loaded
side) on one tooth of each of the test gears to promote crack
initiation. Notch specifications were as follows: 0.25 to 0.50-mm
notch length, 40.5-mm notch mouth radius (location of greatest
tensile stress of an uncracked gear), notch angle normal to fillet
surface, notch machined along entire tooth face width. Fig. 5b
shows a magnified view of a typical notched tooth. The notches
were fabricated using electrodischarge machining (EDM) with a
0.10-mm diameter wire electrode. A total of four test gears had
notches installed. Table I gives the measured notch characteristics.
Fig. 6 gives the nomenclature for the notch parameters of Table II.

The objective of the tests was to determine the effect of speed
on gear crack propagation direction. Four tests were performed.
Test 1 was run at 2500 rpm, test 2 at 4920 rpm, test 3 at 7500 rpm,
and test 4 at 10,000 rpm (Table II). Note that test 2 was originally
specified for 5000 rpm, but was reduced to 4920 rpm due to facility
resonance conditions. At the start of each test, the gears were run
for a short break-in period of 20 to 123 N-m torque. After break-in,
the gears were run at a steady load condition of 123 N-m. If fracture
did not occur after a reasonable period of time, the load was
incrementally increased and the gears were run until fracture
occurred. The actual run loads ranged from 123 to 152 N-m. Test
gear oil inlet temperature was set at 46°C. After occurrence of a
failure (tooth or rim breakage), the gears were removed from the rig,
cleaned, and photographed.

RESULTS AND DISCUSSION

Rotating Disk Model

Before analyzing gears, the FRANC finite element solution
using the body force loading option was compared to a closed-form
theoretical solution of a rotating disk. This was done to validate the
solution using the body force load option of the program. A thin
disk of uniform thickness rotating with a constant angular velocity $\omega$ was considered. The finite element model of the disk is shown in
Fig. 7a. The inner and outer radii of the disk were $a = 14.73$ mm and $b = 48.26$ mm, respectively. The disk thickness was 25.40 mm.
These dimensions roughly matched those of the test gears used in
the study. However, no slots were used such as with the test gears.
Material properties for steel were used (Young's modulus: $E = 207$ GPa, Poisson's ratio: $\nu = 0.3$, mass density: $\rho = 7700$ kg/m$^3$).
The model had 768 8-node, plane stress, quadrilateral elements and
2432 nodes. Only body force due to the angular velocity was used.

Fig. 6. Measured notch characteristics nomenclature.
as applied loading. Four nodes of the inner radius were fixed to prevent rigid body motion but allow the disk to grow radially. The stresses and displacements from the finite element analysis are shown in Fig. 7b and 7c for an angular velocity of \( \omega = 1047.2 \) rad/s. These compared rather well to those calculated from the closed-form elasticity solution of [17].

**Comparison of Predicted and Experimental Gear Crack Paths**

The results of the crack propagation experiments performed in the gear fatigue rig are shown in Fig. 8. As stated before, four experiments were performed. Test 1 was run at 2500 rpm (Fig. 8a), test 2 at 4920 rpm (Fig. 8b), test 3 at 7500 rpm (Fig. 8c), and test 4 at 10,000 rpm (Fig. 8d). Again, the measured notch dimensions of the four test gears are shown in Table II. From the experiments, all cracks originated from the notch tip. For tests from 2500 to 7500 rpm, tooth fractures occurred. For the test at 10,000 rpm, rim fracture occurred. For the test at 2500 rpm, tooth fracture occurred at an applied torque of 152 Nm. For all other tests, fracture occurred at an applied torque of 123 Nm.

The predicted crack paths from the finite element analysis are compared to the actual crack paths from the experiments in Fig. 8. Separate finite element models were analyzed for each test gear configuration. Again, the models incorporated slots to give a backup ratio of \( m_b = 0.5 \). Initial cracks were inserted in the models to correlate with the corresponding notch size, location, and orientation of the test gears from Table II. Tooth loads were placed at the HPSTC and the magnitudes corresponded to the applied torque at fracture of the given test. Also, body loads were applied corresponding to the gear speed of the given test. Crack propagation was numerically simulated using a crack increment size of 0.25 mm. This took 24 to 27 steps to reach the tooth or rim boundaries. For models at speeds of 2500 to 7500 rpm, tooth fracture occurred. For the model at 10,000 rpm, rim fracture occurred. This correlated very nicely with the experiments (Fig. 8).

Further analysis was performed on the finite element models described above. Each of the models of the test gears in Fig. 8 were analyzed at gear speeds of 2500, 5000, 7000, and 10,000 rpm. For the models of Figs. 8a (test gear 1) and 8b (test gear 2), tooth fractures were predicted for all speeds. For the model of test gear 3, the notch mouth radius was slightly lower than the other cases (Table II, test gear 3, 7500 rpm). For the model of test gear 4, the notch orientation angle was slightly larger than the other cases (Table II, test gear 4, 7500 rpm). These later two conditions tended to start the initial crack more toward the tooth root, and thus, promoted rim fractures at the higher speeds.

It should also be noted that the effect of speed on predicted crack path is dependent on the magnitude of the applied tooth load as well as the magnitude of the gear speed. This is due to the fact that the crack propagation angle is a function of \( K_u / K_c \) and the stress intensity factors are determined from the superposition of the applied tooth load and body force. Crack propagation paths due to only the applied tooth load can lead to tooth or rim fracture, depending on the tooth design, backup ratio, and location of the initial crack. Crack propagation paths due to the speed-related body force lead to rim fracture due to the hoop stress component. Thus, the crack path due to the superposition of the tooth load and body load is dependent on the relative contributions due to each.
As an example of this, the model of the test gear in Fig. 8c at 7500 rpm and 123 N·m torque predicted tooth fracture while the same model at 7500 rpm and 61 N·m predicted rim fracture.

**Parametric Studies**

Parametric studies were performed using the FRANC finite element crack propagation code. The effects of speed, backup ratio, and initial crack location on gear crack propagation direction were investigated. Backup ratios of $m_b=0.3$, 0.5, 0.7, 1.0, and 1.3 were studied. Gear speeds of 0, 5000, 10,000, and 15,000 rpm were used. Initial crack locations from those slightly below the base circle on the tooth fillet to those on the tooth root centerline were investigated. For all cases, the applied tooth load was placed at the HPSTC and corresponded to an applied torque of 123 N·m.

Fig. 9 shows the analysis for a backup ratio of $m_b=0.3$. For an initial crack mouth location of $R_0=41.33$ mm (slightly below the base circle), rim fracture occurred for a speed of 15,000 rpm and tooth fracture occurred for all other speeds (Fig. 9a). For initial cracks mouth locations lower than $R_0=41.33$ mm, rim fractures occurred for all speeds ($R_0=40.81$ mm shown in Fig. 9b). Fig. 10 shows the analysis for a backup ratio of $m_b=0.5$. Here, the transition point was at $R_0=40.35$ mm. At $R_0=40.35$ mm, tooth fractures occurred for 0 and 5000 rpm while rim fractures occurred at 10,000 and 15,000 rpm (Fig. 10b). For $R_0>40.35$ mm, tooth fractures occurred for all speeds (Fig. 10a). For $R_0<40.35$ mm, rim fractures occurred for all speeds (Fig. 10c). Fig. 11 shows the analysis for a backup ratio of $m_b=0.7$. Here, the tooth/rim fracture transition point was at $R_0=41.33$ mm. Fig. 12 shows the analysis for a backup ratio of $m_b=1.0$ and Fig. 13 shows the analysis for a backup ratio of $m_b=1.3$. For both of these cases, the tooth/rim fracture transition point was at the root centerline.

As seen from these studies, speed had little effect on crack propagation direction except when the initial crack location was near the tooth/rim fracture transition point for the given backup ratio. When at that point, higher speeds tended to promote rim fracture while lower speeds (or neglecting centrifugal force) produced tooth fractures.

**CONCLUSIONS**

The effect of speed (centrifugal force) on gear crack propagation direction was explored. Gears were analyzed using finite element analysis and linear elastic fracture mechanics. The analysis was validated from crack propagation experiments performed in a spur gear fatigue rig. The effects of speed, backup ratio, and initial crack location on gear crack propagation direction were investigated. The following conclusions were derived:

1) Stresses and deflections from the finite element method correlated well with a closed-form elasticity solution for a rotating disk example problem. Crack paths from the finite element method correlated well with those deduced from gear experiments.

2) For the test gear with a backup ratio of $m_b=0.5$, cracks initiating in the tooth fillet propagated to rim fractures when run at a speed of 10,000 rpm and tooth fractures for speeds slower than 10,000 rpm for both the experiments and analysis.

3) Speed had little effect on crack propagation direction except when initial crack locations were near the tooth/rim fracture transition point for a given backup ratio. When at that point, higher speeds tended to promote rim fracture while lower speeds (or neglecting centrifugal force) produced tooth fractures.
REFERENCES

The effect of rotational speed (centrifugal force) on gear crack propagation direction was explored. Gears were analyzed using finite element analysis and linear elastic fracture mechanics. The analysis was validated with crack propagation experiments performed in a spur gear fatigue rig. The effects of speed, rim thickness, and initial crack location on gear crack propagation direction were investigated. Crack paths from the finite element method correlated well with those deduced from gear experiments. For the test gear with a backup ratio (rim thickness divided by tooth height) of $m_b = 0.5$, cracks initiating in the tooth fillet propagated to rim fractures when run at a speed of 10,000 rpm and became tooth fractures for speeds slower than 10,000 rpm for both the experiments and analysis. From additional analysis, speed had little effect on crack propagation direction except when initial crack locations were near the tooth/rim fracture transition point for a given backup ratio. When at that point, higher speeds tended to promote rim fracture while lower speeds (or neglecting centrifugal force) produced tooth fractures.