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Short-Term Dynamical Interactions Among Extrasolar Planets

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ABSTRACT

We show that short-term perturbations among massive planets in multiple planet systems can result in radial velocity variations of the central star which differ substantially from velocity variations derived assuming the planets are executing independent Keplerian motions. We discuss two alternate fitting methods which can lead to an improved dynamical description of multiple planet systems. In the first method, the osculating orbital elements are determined via a Levenberg-Marquardt minimization scheme driving an N-body integrator. The second method is an improved analytic model in which orbital elements such as the periods and longitudes of periastron are allowed to vary according to a simple model for resonant interactions between the planets. Both of these methods can potentially determine the true masses for the planets by eliminating the $\sin i$ degeneracy inherent in fits that assume independent Keplerian motions. As more radial velocity data is accumulated from stars such as GJ 876, these methods should allow for unambiguous determination of the planetary masses and relative inclinations.

Subject headings: stars: planetary systems

1. Introduction

Several thousand nearby stars are now being surveyed for periodic radial velocity variations which indicate the presence of extrasolar planets (Marcy, Cochran & Mayor 2000). In the past year, the pace of discovery has increased, and there are now nearly sixty extrasolar planets known.

Recently, systems with more than one planet have been found, and four (υ Andromedae,

GJ 876, HD 83443, and HD 168443) are now known. GJ 876 (Marcy et al 2001) provides an especially interesting case. In this system, a combined, two-Keplerian fit to the radial velocity data (see Table 1), suggests that the star is accompanied by two planets on orbits having a nearly commensurate 2:1 period ratio. The amplitudes of the star's radial velocity variations suggest minimum masses of 0.56 $M_{\rm JUP}$ for the inner planet, and 1.89 $M_{\rm JUP}$ for the outer planet. GJ 876, an M dwarf star with an estimated mass of $0.32 \pm 0.05 M_{\odot}$ (Marcy et al 2001), is the lowest mass star known to harbor planets.

For these orbital parameters, the mutual perturbations of the two planets in the system are considerable. Using a Bulirsch-Stoer integrator (with a timestep accuracy of $\Delta E/E = 1.0e-15$) we have computed radial velocity curves using the Table 1 orbital elements given in Marcy et al (2000) as an initial condition. For co-planar Keplerian orbits, the orbital elements $P_{1,2}$, $e_{1,2}$, $T_{1,2}$, $\omega_{1,2}$, and $K_{1,2}$, are constant, and these, along with the mass of the star, serve to completely determine the positions and velocities of the planets at any time. When the interactions between the planets are non-negligible, however, the orbital elements change continuously, and so one must also specify an initial epoch in order to determine the motion at future times. Every starting epoch corresponds to a different resultant three-body motion. In figure 1, we demonstrate this effect by mapping the elements reported by Marcy et al. (2001) onto initial conditions corresponding to JD 2450106.2 (the reported time of perihelion passage for the outer planet). The red line shows the radial velocity curve of the star which results from the superposition of the two Keplerian reflex motions. The black line shows the radial velocity curve resulting from the full three-body integration. After three orbits of the outer planet, the motion begins to deviate noticeably from the dual-Keplerian approximation. After several years, the motions have diverged completely.

The rest of this paper is organized as follows: in §2 we show that self-consistent radial

velocity curves are required for systems such as GJ 876. In §3, we derive improved, fully self-consistent dynamical fits to the observed radial velocities of the GJ 876 system. In §4, we show how dual-Keplerian fits can be improved using an approximate analytic model for the interactions between two massive planets in resonant systems. Further applications are discussed in §5, which also serves as a conclusion.

2. Dual Keplerian vs. Self-Consistent Fits

The orbital elements given in Table 1 (taken from Marcy et al. 2001) were derived under the assumption that they are constants of the motion. However, for a system such as GJ 876, where the mutual planetary interactions are strong, the elements will change quite rapidly on observable timescales. We can therefore regard the parameters in Table 1 as a set of osculating elements, which, given a particular starting epoch, correspond to a uniquely determined initial condition.

Even with the assumption that $\sin i = 1$ for both planets, the variety of motion corresponding to the starting conditions given in Table 1 is very broad. For some starting epochs, the planets are not in the 2:1 resonance, and the system experiences severe dynamical instabilities within five years. For other starting epochs, the planets undergo librations about the resonance, and the system is stable over timescales of at least 70 million years (Marcy et al 2001). One can thus ask the question: are there any starting epochs for which the osculating elements in Table 1 generate an evolution which is consistent with the observed reflex velocity of the star?

We have computed synthetic radial velocity curves resulting from these osculating elements using 10,000 different initial starting epochs spaced one day apart. In each case, a uniform velocity offset was applied to the synthetic radial velocity curve in order to match

the first radial velocity point obtained by Marcy et al (2001) with the Keck telescope (t=JD 2450602.1, $v = 343.72 \mathrm{ms}^{-1}$). We then compared each synthetic curve to the remaining 53 radial velocity observations obtained at Keck, and computed a reduced χ^2 statistic for the fit. The best fit occurred for an integration corresponding to a starting epoch of JD 2450671.98. The reduced χ^2 value for this fit is 17.27, and the rms scatter of the velocities about the curve is 83 ms^{-1}. The best fit curve, along with the observed data, is shown in Figure 2. Given that the observational errors lie in the range 3–5 ms^{-1}, this degree of scatter indicates that the best dual-Keplerian fit is a poor match to the observed velocities when mutual planetary perturbations are taken into account.

3. A Self-consistent N-body Minimization Scheme

The experiment described above demonstrates that it is essential to include mutual planetary perturbations when making fits to velocity observations of planetary systems resembling GJ 876. One way to do this is to attempt a fully self-consistent fit which employs N-body integrations to produce a synthetic reflex velocity curve for the central star. Starting with the best dual-Keplerian fit to the Keck data, we have used a Levenberg-Marquardt algorithm (Press et al 1992) to iterate an improvement to the osculating orbital elements reported in Table 1. Our implementation of the algorithm examines how the χ^2 value of the fit depends on variations of all 10 orbital elements, and attempts to find a set of elements for which the reduced χ^2 fit is at a minimum. The N-body integrations were done using the Bulirsch-Stoer integration package developed by Laughlin & Adams (1999). A preliminary investigation of systems in which $\sin i = 1$ for both planets has shown that there are many isolated minima within the ten-dimensional parameter space associated with every starting epoch.

Using the Levenberg-Marquardt algorithm, we have found a self-consistent model for

the radial velocity data which has a reduced χ^2 value of 1.93 and an rms scatter of 12.0 ms^{-1}. This fit is shown in Figure 3, and represents a large improvement over the fit shown in Figure 2. The osculating elements are given in Table 2. We would like to stress that this fit was the result of a preliminary investigation, and that better fits can almost certainly be found (even for $\sin i$=1). The fact that the best dual-Keplerian fit of Table 1 has a similarly low reduced χ^2 value, despite diverging substantially from true three-body motion, indicates that the present data set can be well-modeled by a variety of functions. It is likely that more observations will be required in order to secure all 14 orbital elements (allowing for mutually inclined planetary orbits). It seems clear, however, that as more data are acquired, the true masses of the planets will be revealed. The mutual perturbations between the planets will provide additional information which overcomes the degeneracy which previously made it impossible to determine the inclinations (and thus the true masses) of the planets.

4. Improved Analytic Approximations

The foregoing Levenberg-Marquardt minimization method is a potentially powerful technique for determinining all of the orbital parameters of the system in a completely self-consistent fashion. However, in the absence of a good initial model for iteration, it is difficult to locate the global minimum for the system. As we have shown above, the best dual-Keplerian fit provides a very reasonable starting model for the Keck data alone. However, the best dual-Keplerian fit for the combined Keck and Lick data sets of Marcy et al. (2001) has a much larger (22 ms^{-1}) rms velocity scatter, and thus provides a less attractive starting point. This has motivated us to find a better analytic model for interacting systems which can bridge the gap between the dual-Keplerian approximation and the full three-body motion.

We begin with a dual-Keplerian model using Jacobi coordinates. In this model, the

inner planet moves on an orbit about the star and the outer planet moves on an orbit around the center of mass of the inner two bodies. We then make the assumption that the two planets are undergoing librations about the 2:1 mean motion resonance. Marcy et al. (2001) have found that systems locked in this resonance can be stable on timescales of at least 70 million years, while other systems tend to be unstable. In our improved analytic model, the semi-major axes a of the two planets undergo sinusoidal oscillations about the exact resonant value in antiphase to each other. The period $P_{\rm res}$, amplitude Δ, and initial phase of these oscillations are treated as model parameters, in addition to the mean semi-major axis $\bar{a}_2$ of the outer planet. The average mean motions $\bar{n}_1$ and $\bar{n}_2$ of the two planets are related by the condition that the rate of change of the resonance critical argument is zero at exact resonance. Hence

$$\bar{n}_1 - 2\bar{n}_2 + 2(\dot{\pi}_2 - \dot{\pi}_1) = 0 \tag{1}$$

where π_1 and π_2 are the longitudes of periastron of each planet. The average semi-major axes of the planets are related to their average mean motions by

$$\begin{aligned} \bar{a}_1 &= \left[\frac{G(m_0+m_1)}{\bar{n}_1^2}\right]^{1/3} \\ \bar{a}_2 &= \left[\frac{G(m_0+m_1+m_2)}{\bar{n}_2^2}\right]^{1/3} \end{aligned} \tag{2}$$

where m_0, m_1 and m_2 are the masses of the star, the inner and outer planets respectively. At a time t, the semi-major axes are given by

$$\begin{aligned} a_1 &= \bar{a}_1[1 + \Delta_1 \cos n_{\rm res}(t - t_{\rm off})] \\ a_2 &= \bar{a}_2[1 - \Delta_2 \cos n_{\rm res}(t - t_{\rm off})] \end{aligned} \tag{3}$$

where $n_{\rm res} = 2\pi/P_{\rm res}$ and $t_{\rm off}$ determines the initial phase of the resonant oscillations. In addition, conservation of energy requires that

$$\Delta_2 = \Delta_1 \left(\frac{m_0 m_1 \bar{a}_2}{(m_0+m_1)m_2\bar{a}_1}\right) \tag{4}$$

The initial mean anomaly $M_2(0)$ of the outer planet is treated as a model parameter, and the initial mean anomaly of the inner planet $M_1(0)$ is then given by the critical argument for the resonance

$$\sigma = M_1 - 2M_2 + 2(\pi_1 - \pi_2) \tag{5}$$

where $\sigma = 0$ at $t = t_{\rm off}$. At time t, the mean anomaly of body i is given by

$$M_i = M_i(0) + \int_{t_{\rm off}}^{t} n_i dt \tag{6}$$

The integrals are straightforward to evaluate since for each planet a, and hence n, is an analytic function of time.

In this system, the mutual planetary perturbations are sufficiently strong that the longitudes of periastron will precess rapidly. We model this by allowing each periastron longitude to vary linearly with time, where the rates of change of the two angles represent additional free parameters. In principle, these parameters, and the resonance libration period $P_{\rm res}$, can be used to test the accuracy of the analytic model by comparing the precession rates with those from a full N-body integration. The orbits of the planets are assumed to be coplanar in our model, but the inclination i of this plane to the line of sight is included as a parameter.

We used the analytic model to generate synthetic radial velocities for the central star and compared these with the observations from the Keck and Lick telescopes given in Marcy et al. (2001). We initially generated a randomized population of sets of model parameters and then used a genetic algorithm to evolve promising sets towards an improved description of the system. At each generation, the genetic algorithm evaluates the degree of fit for each parameter set, and cross breeds the best members of the population to produce a new generation.

Figure 4 shows a model fit generated by the genetic algorithm for the Keck data alone.

This fit has a rms scatter of 8.1 ms^{-1}, which is comparable to the best dual-Keplerian fit or the preliminary fit obtained by the Levenberg-Marquardt N-body technique. Figure 5 shows a model fit for the combined Keck and Lick data set. The rms scatter in this case is 11.9 ms^{-1}, which represents a substantial improvement on the best dual-Keplerian model in Table 1. The apocentric orbital elements for the fits generated by the analytic model are given in Table 3. We again stress that other solutions with low rms scatter are likely to exist, and these orbital parameters, while being suggestive, do not necessarily represent the true dynamics of the system.

5. Discussion

The most important benefit of self-consistent dynamical fitting techniques for multi-planet systems is the ability to break the $\sin i$ degeneracy and determine the true masses of the extrasolar planets. The true masses can occasionally be found in cases where the planet transits the parent star (e.g. Charbonneau et al 2000, Henry et al 2000), but such cases are unusual, and will be confined largely to planets with short periods. The foregoing techniques can in principle be applied to any system containing more than one planet, given a sufficient baseline of observation. Fischer et al 2000 have shown that roughly half of the planetary systems uncovered in the Lick Radial survey show evidence of a second companion. Thus we expect that numerous additional multi-planet systems will be forthcoming. Systems having massive short-period planets are especially amenable to this technique. As a specific example, an N-body integration of the Upsilon Andromedae system indicates that the planetary interactions are already producing observable deviations from the multiple Keplerian fit.

Wolszczan (1994) has used a roughly similar analysis to the one described here to determine the true masses and inclinations of the planets orbiting pulsar PSR B1257+12.

However, in that case, a superposition of Keplerian fits provides a very good approximation to the observed reflex velocity of the pulsar. This stands in marked contrast to cases such as GJ 876, where the planetary interactions are an integral component of the overall motion of the star, and an analysis based on small perturbations to Keplerian motions may not necessarily succeed.

There are several avenues for immediate additional improvement of the dynamical description of the emerging multi-planet exosolar systems. For cases where the radial velocity data is inadequate to cleanly delineate the planetary masses and orbital parameters, numerical integrations can reveal dynamical instabilities which can eliminate large portions of parameter space from consideration. The semi-analytic model of §4 can also be substantially improved through inclusion of realistic expressions for the secular evolution of the orbital elements (see e.g. Dermott & Murray 1999).

We envision a four-stage procedure for determining the dynamical characteristics of multiple-planet systems in general. (1) As outlined by Butler et al 1996, periodograms (e.g. Lomb, 1976; Scargle 1982) can first be used to determine the fundamental periods within the system. (2) The radial velocity data can then be fit with fixed Keplerian ellipses (e.g. Butler et al 1999). (3) These multiple-Keplerian fits can then be further refined by versions of the semi-analytic scheme described in §4., and then (4) given a final self-consistent polish using the Levenberg-Marquardt scheme driving full N-body integrations.

5.1. Acknowledgements

Just prior to submitting this paper we became aware that Eugenio Rivera and Jack Lissauer are developing a scheme similar to the Levenberg-Marquardt procedure outlined above in order to model the effects of mutual planetary perturbations on the Doppler

velocity variations of GJ 876.

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Figure Captions

Fig. 1.— Synthetic radial velocity variations for GJ 876 assuming a superposition of 2 fixed Keplerian motions for planets with elements given in Table 1 (red line), and an N-body integration using the same elements (black line).

Fig. 2.— Synthetic radial velocity variations produced by an N-body integration using the osculating elements from Table 1 (Keck fit) and a best fit epoch of JD 2450671.98.

Fig. 3.— Synthetic radial velocity variations for a fit derived using the Levenberg-Marquardt procedure described in §3, using the Keck radial velocities.

Fig. 4.— Synthetic radial velocity variations for a fit using the semi-analytic model described in §4, using the Keck radial velocities (solid circles).

Fig. 5.— Synthetic radial velocity variations for a fit using the semi-analytic model described in §4, using the combined Keck (solid circles) & Lick (open circles) radial velocities.

Parameter	Inner (Keck)	Outer (Keck)	Inner (Keck & Lick)	Outer (Keck & Lick)
Period (day)	30.1	61.0	30.12	61.02
K (ms^{-1})	81	211	81	210
Eccentricity	0.11	0.29	0.27	0.10
ω (deg)	328	329	330	333
Periastron Time (JD)			2450031.4	2450106.2

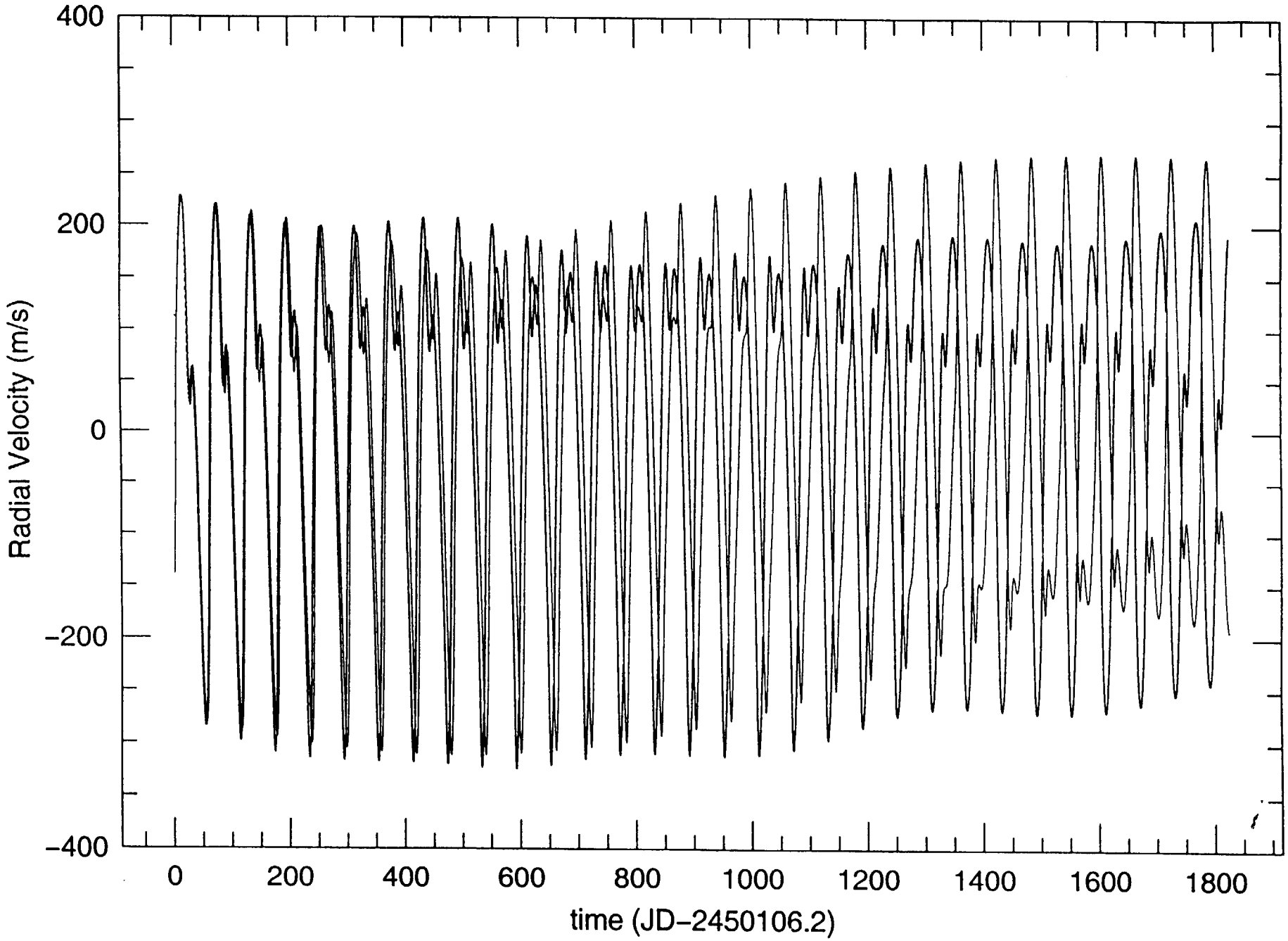
Table 1: **Best-fit dual-Keplerian elements for GJ876 (from Marcy et al. 2001)**

Parameter	Inner	Outer
Period (day)	30.13	61.58
K (ms^{-1})	80.9	203.6
Eccentricity	0.226	0.025
ω (deg)	156	70
Mean anomaly (deg)	277	31
a (AU)	0.1297	0.2092
Epoch (JD)	2450602.0931	

Table 2: **Osculating elements derived by Levenberg-Marquardt N-body integration scheme.**

Parameter	Inner	Outer	Inner	Outer
	(Keck)		(Keck & Lick)	
Mass (M_J)	0.740	6.73	1.927	5.81
a (AU)	0.1302	0.2077	0.1298	0.2082
Eccentricity	0.429	0.240	0.229	0.006
ω (deg)	207	179	204	97
Mean anomaly (deg)	186	301	136	354
Epoch (JD)	2449995.0		2449990.0	
$\sin i$	3.34		3.14	

Table 3: **Osculating apocentric elements derived by the analytic model plus genetic algorithm scheme.**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200
0
−200
−400
Radial Velocity (m/s)
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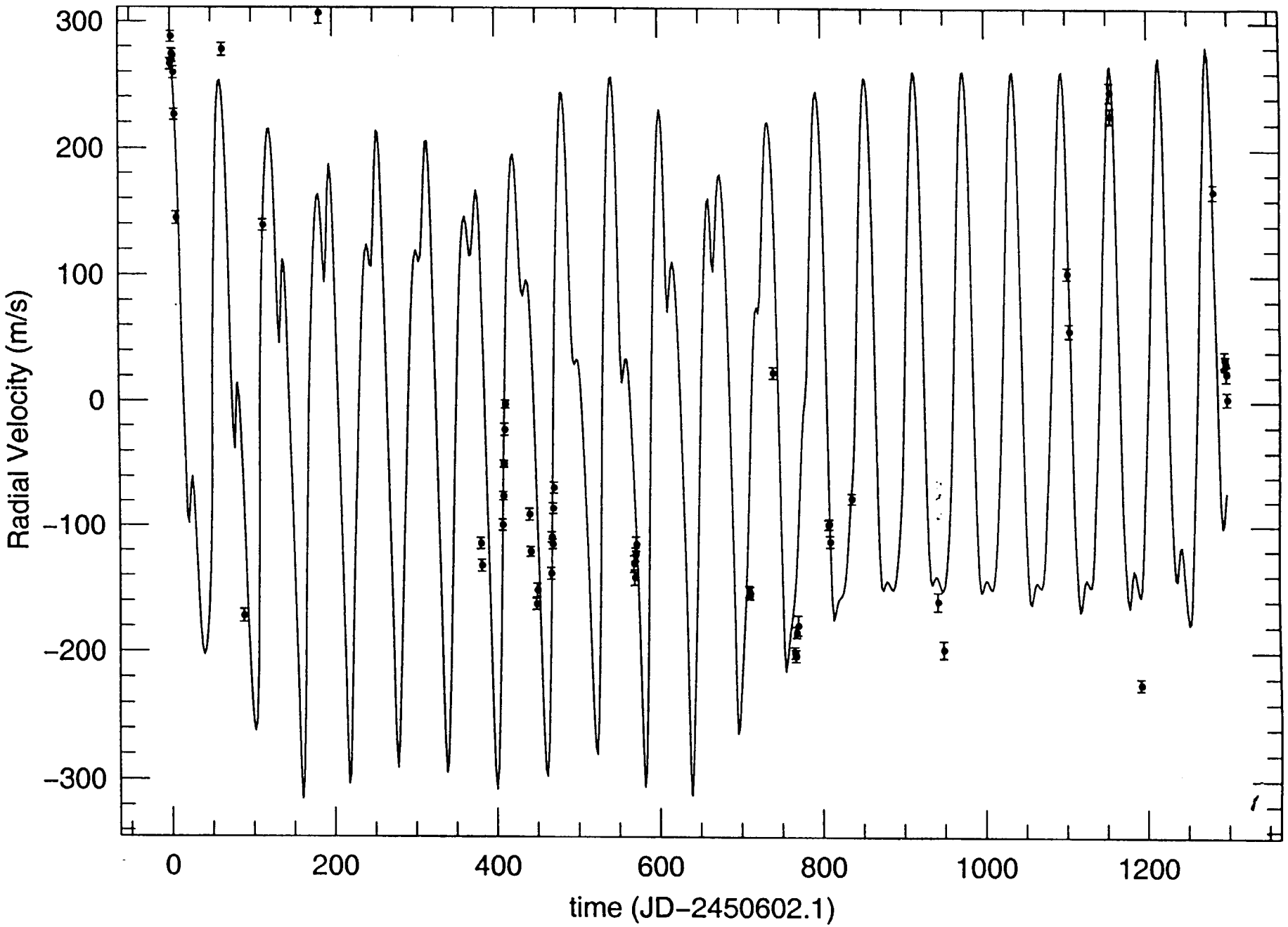
400
600
800
1000
1200
1400
1600
1800
time (JD−2450106.2)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0
−100
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Radial Velocity (m/s)
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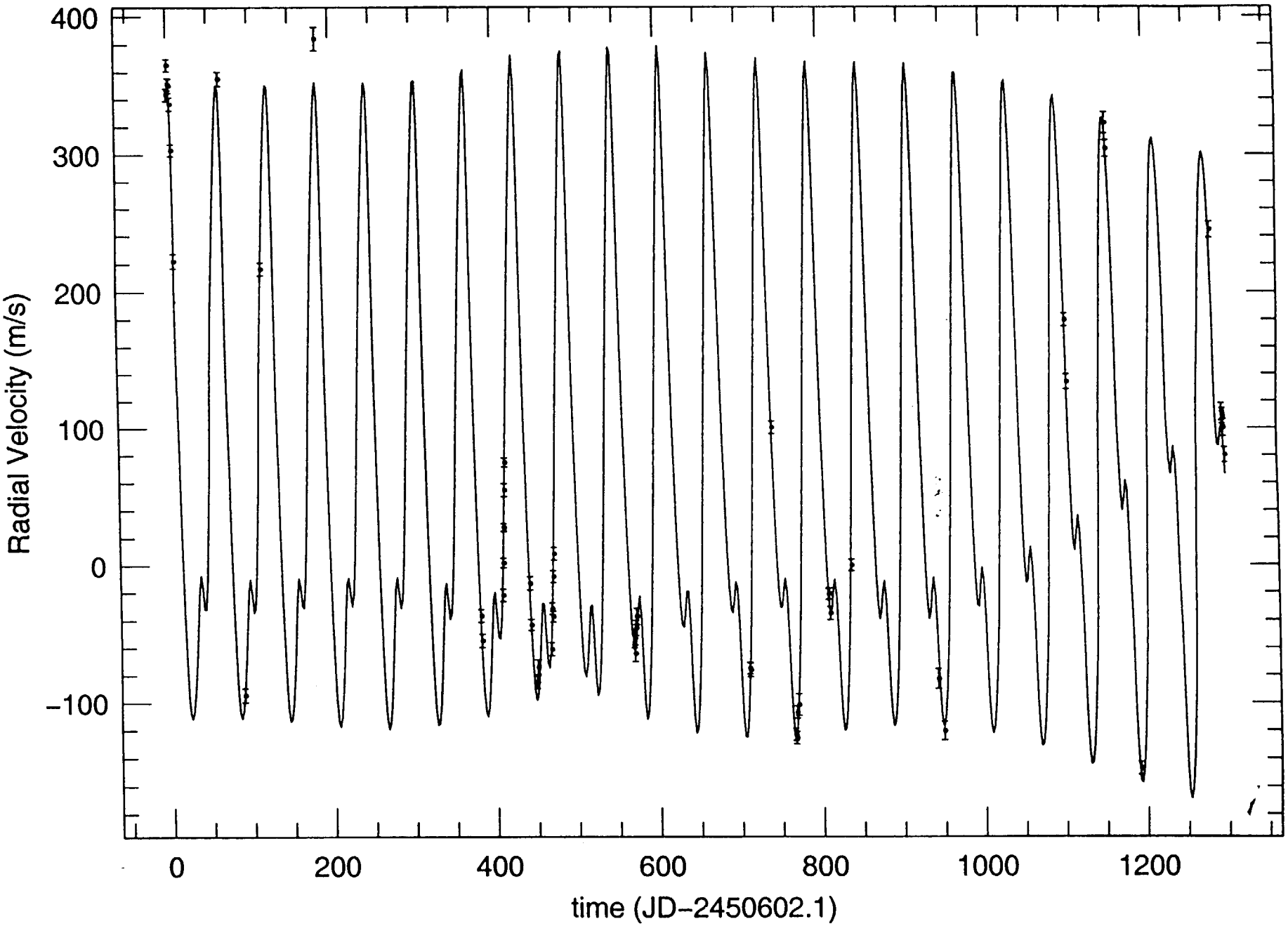
time (JD−2450602.1)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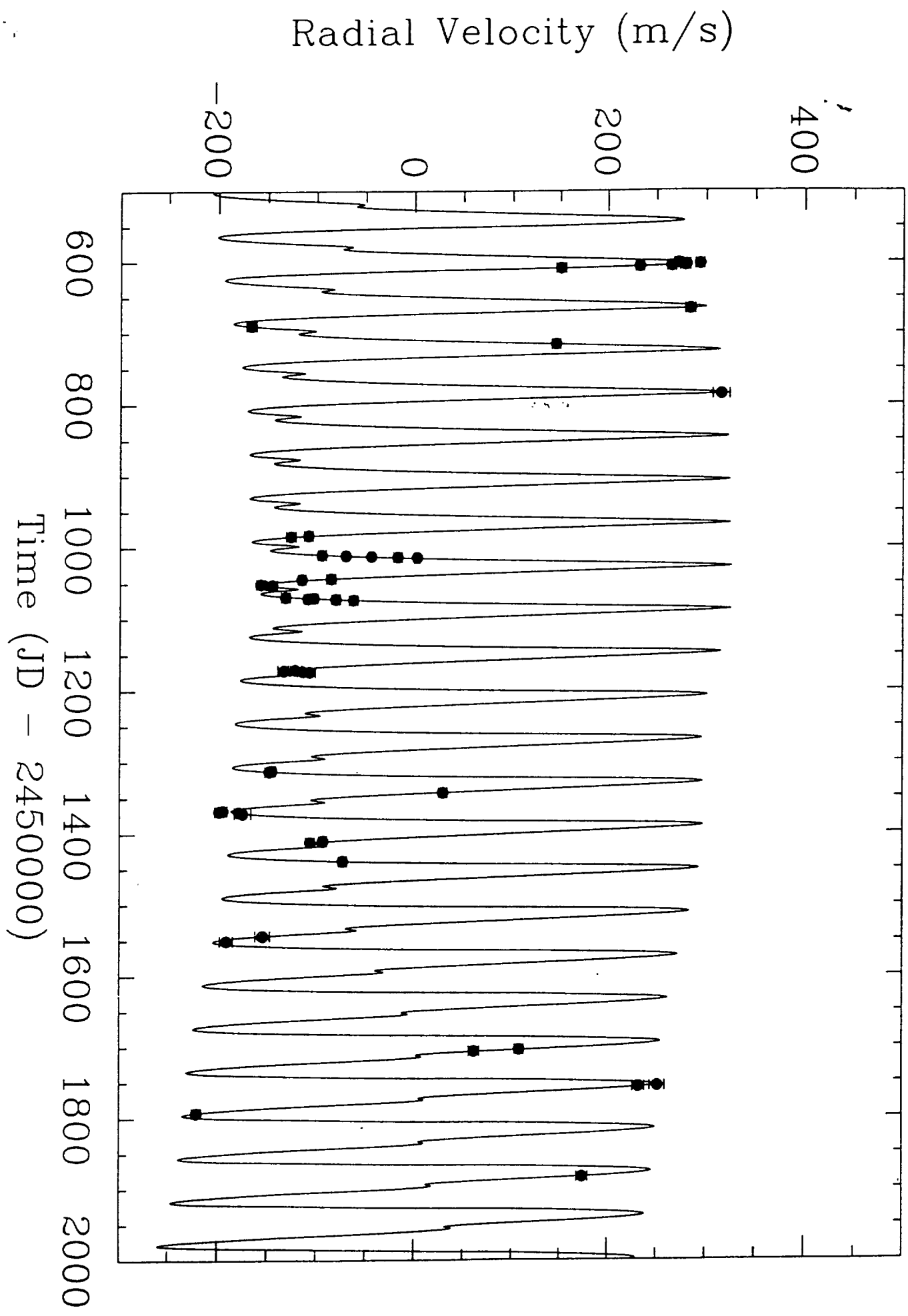
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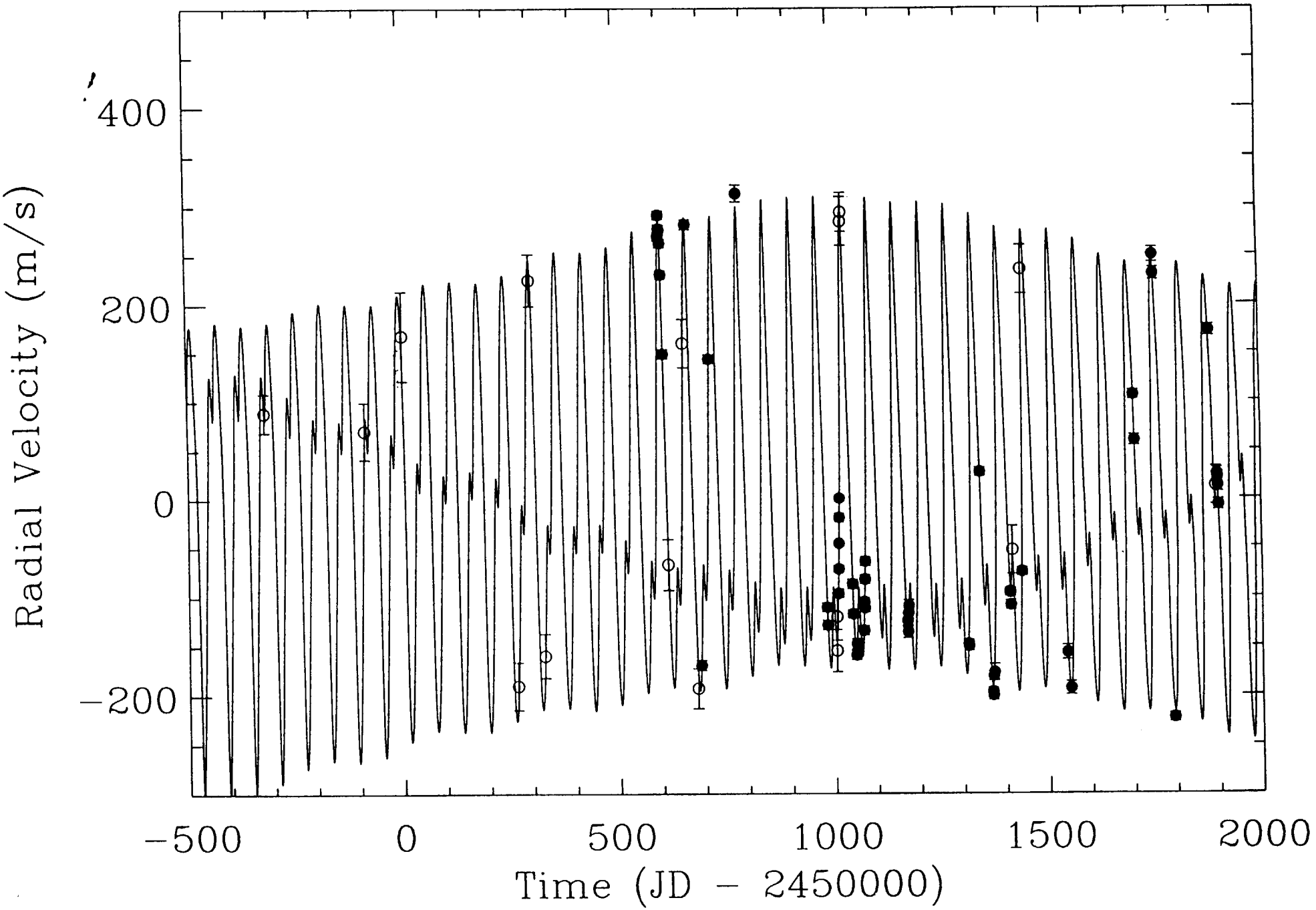
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Radial Velocity (m/s)
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2000
Time (JD − 2450000)