Introduction

This paper presents a design study for a pressure based Flush airdata system (FADS) on the Hypersonic Air Launched Option (HALO) Vehicle. The analysis will demonstrate the feasibility of using a pressure based airdata system for the HALO and provide measurement uncertainty estimates along a candidate trajectory. The HALO is conceived as a man-rated vehicle to be air launched from an SR-71 platform and is proposed as a testbed for airbreathing hydrogen scramjet. The vehicle is presently designed to provide 2 minutes of scramjet flight tests at Mach 10 and 1500 qbar. At these test conditions, it is possible to ignite and achieve positive thrust from a scramjet. Altitude and angle-of-attack as a function of Mach number for the HALO trajectory are presented in figures 1a and 1b.

A feasibility study has been performed and indicates that the proposed trajectory is possible with minimal modifications to the existing SR71 vehicle. The mission consists of launching the HALO off the top of an SR-71 at Mach 3 and 80,000 ft. A rocket motor is then used to accelerate the vehicle to the test condition. After the scramjet test is completed, the vehicle will glide to a lakebed runway landing. This option provides reusability of the vehicle and scramjet engine. The HALO design will also allow for various scramjet engine and flowpath designs to be
flight tested. Presently, the HALO is conceived to be 47 ft long with a 17 ft wingspan and weighs approximately 14000 pounds fully fueled. The baseline vehicle to be analysed, pictured in figure 2, has a blunted "shovel Nose" with a $5^\circ$ wedge half-angle near the upper surface leading edge and a $6^\circ$ wedge half-angle near the lower surface leading edge. The longitudinal leading edge radius at the nose is 0.1", and the lateral radius of the nose is approximately 20".

For the HALO flights measurements of freestream airdata are considered to be a mission critical to perform gain scheduling and trajectory optimization. Additionally, interpretation of in-flight results will depend on the accuracy of aerodynamic state parameters such as Mach number, flow incidence angles, and dynamic pressure. Because aerodynamic heating limits the ability to use probes which extend into the airstream, accurate on-board measurement of these parameters is more difficult than in the case of conventional aircraft. Moreover, the ability to repeat maneuvers for local aerodynamic calibration of the probes is generally restricted both by reduced flight time and mission rate.

One approach taken to obtaining airdata involves measurement of external atmospheric winds, temperature, pressure and density which can subsequently be combined with vehicle trajectory measurements of space position, velocity, attitude angles, angular rates, and acceleration, etc. to estimate the airdata quantities. Accuracy of the resulting computed airdata quantities depends on the response and accuracy of the measurements used relative to the specific flight research data requirements.

This study takes an alternate approach. Here the feasibility of obtaining airdata using a pressure-based flush airdata system (FADS) methods is assessed. The analysis, although it is performed using the HALO configuration and trajectory, is generally applicable to other hypersonic vehicles. The method to be presented offers the distinct advantage of inferring total pressure, Mach number, and flow incidence angles, without stagnating the freestream flow. This approach allows for airdata measurements to be made using blunt surfaces and significantly diminishes the heating load at the sensor. In the FADS concept a matrix of flush ports is placed in the vicinity of the aircraft nose, and the airdata are inferred indirectly from the measured pressures.
Aerodynamic Modeling

For this analysis, a matrix of flush-surface pressure ports will be distributed along the vehicle nose, and the measured pressure data will be related to airdata state parameters via an aerodynamic model. In order for the regression approach to work well, it is important that the locations of the pressure ports selected yield measurements that are 1) well modeled analytically, and 2) give good sensitivity to the desired airdata parameters. Since this is an inverse-flow problem, in which pressure measurements downstream of the bow shockwave are used to infer freestream parameters ahead of the shock wave, simple, invertible aerodynamic models must used to relate pressure observations to airdata. Furthermore, to be implemented on high-fidelity vehicle simulations, the surface pressure aero models must be fast enough to be performed in real time -- thus the selected pressure ports must be located in aerodynamically "simple" regions.

At hypersonic Mach numbers the shock wave at the leading edge of the vehicle is detached, and in the vicinity of the stagnation point, the pressure coefficient is accurately described by Lee's Modified Newtonian flow theory (ref. 2).

\[ \frac{P_\theta - P_\infty}{\overline{q}_\infty} = C_{P\theta} = C_{P\text{Max}} \cos^2(\theta) \]

where, \( C_{P\text{max}} \) is a function of Mach number, and \( \theta \) --the flow incidence angle-- is a function of angle of attack, angle of sideslip, and the surface coordinate angles. Away from the stagnation point Modified Newtonian flow is accurate only as Mach number approaches infinity (Ref. 1), but for this analysis Modified Newtonian Flow can be "calibrated" for Mach number inaccuracies by writing the Newtonian model as

\[ C_{P\theta_i} = C_{P\text{Max}} \left[ \cos^2(\theta_i) + \epsilon \sin^2(\theta_i) \right] \]

and \( \epsilon \) is evaluated by comparison to time-marching Euler Solutions at zero and two degrees angle of attack for a series of Mach numbers (figs. 3a and 3b).

Aft of the blunt leading edge, the geometry is two-dimensional and Modified Newtonian flow is not particularly accurate here. Thus, on the 2-D wedge
surfaces, the pressure distribution is analyzed using exact oblique shock wave theory (ref. 2). On the 2-D surface, the shock wave is inclined at an angle $\Gamma$ to the freestream flow and is an implicit function of the freestream Mach number and the surface inclination angle, $\delta$. For this analysis the surface inclination angle is determined by the wedge angle and the angle of attack. For curved ramp surfaces, conditions are modeled using the "tangent wedge" method (ref. 1).

Across the normal shock ia thermally perfect gas is assumed and the the ratio of specific heats, $\gamma$, is assumed to vary solely as a function of local static temperature. The shock wave equations are re-derived from momentum, continuity, and energy with variable gas specific heats across the discontinuity. Gas dissociation is ignored for this analysis. The normal shock equations are solved using a one sided "creeping" iteration to avoid numerical limit cycle which occurs at high Mach numbers (figure 4). The Effect of a variable $\gamma$ across the oblique shock is negligible.

**Port Layout**

Subject to the "simple" flow and nose geometry constraints, a sensitivity analysis was performed to optimize the port locations. Sample results of this optimization study for angle of attack are presented in figure 5a. The port layout is presented in figure 5b. A total of 9 ports will be used, 5 along the normal axis of symmetry, and 4 distributed on the 2-D wedge section. The ports along the normal axis of symmetry provide primary information for estimating Mach number, angle of sideslip, and static pressure (pressure altitude). The ports on the 2-D wedge provide primary information for angle-of-attack estimation. Because the wedge ports provide most of the meaningful angle-of-attack information, two sets of ports (one redundant pair) will be located on the ramp surfaces. This matrix of 9 ports is considered the minimum number which can be used to estimate the airdata parameters and still provide limited redundancy. Additional pressures observations will enhance the fidelity, accuracy, and redundancy of the final airdata system.
Airdata Estimation Algorithm

Given the measured array of pressures, inverse modeling must be performed to solve for the airdata parameters. This section develops the estimation algorithm which will be used to extract the airdata estimates from the measured pressure data. For this analysis the *airdata state vector* will be described in terms of 4 parameters: Mach number-$M_\infty$, angle-of-attack-$\alpha$, angle-of-sideslip-$\beta$, and static pressure-$P_\infty$. Using these four basic airdata parameters, other airdata quantities of interest may be directly calculated. For a given pressure observation, the form of the model is

$$P(\phi_i, \lambda_i) = \mathbf{F} \begin{bmatrix} M_\infty \\ P_\infty \\ \alpha \\ \beta \end{bmatrix}_{i,k} + \eta_i$$

The specific form of the non-linear function $\mathbf{F} \ldots$, depends on whether the port is located on a three-dimensional surface or a 2-dimensional surface. Taken together, the matrix of ports form an over determined non-linear model, and may be solved regressively (ref. 3) to determine estimates of the airdata states. The model is linearized about a starting value for each port and the perturbations between the measured data and the model predictions are evaluated. For each of the pressure port, $i=1, \ldots N$,

$$\delta P_{i}^{j+1} = P_i \cdot F_i^j(\alpha, \beta, M_\infty, P_\infty, \lambda_i, \phi_i, \gamma) =$$

$$\nabla \begin{bmatrix} M_\infty \\ P_\infty \\ \alpha \\ \beta \end{bmatrix} \times \begin{bmatrix} \delta M_\infty^{j+1} \\ \delta P_\infty^{j+1} \\ \delta \alpha^{j+1} \\ \delta \beta^{j+1} \end{bmatrix} + \ldots$$

Defining
the updated state vector is solved using recursive least squares

\[
\begin{bmatrix}
M_{\infty}^{j+1} \\
P_{\infty}^{j+1} \\
\alpha^{j+1} \\
\beta^{j+1}
\end{bmatrix}
= \begin{bmatrix}
M_{\infty}^{j} \\
P_{\infty}^{j} \\
\alpha^{j} \\
\beta^{j}
\end{bmatrix}
+ \begin{bmatrix}
\delta M_{\infty}^{j+1} \\
\delta P_{\infty}^{j+1} \\
\delta \alpha^{j+1} \\
\delta \beta^{j+1}
\end{bmatrix}
\]

For each data frame the iteration cycle is repeated until algorithm convergence is reached. For time-recursive implementation, at the beginning of each data frame, the system of equations is linearized about the result of the previous data frame.

**Error Analysis**

The performance of the proposed system along the HALO trajectory is analysed by Monte-Carlo Simulation. Here potential error sources for the pressure measurements will be considered, and error propagation models, both random and systematic, will be developed. For this simulation the HALO trajectory is used to compute the expected surface pressures at each data frame,

\[
P(i) = F(\alpha, \beta, M_{\infty}, P_{\infty}, \lambda, \phi, \gamma)
\]

and error models are then used to superimpose the random and systematic errors onto the predicted pressure data. Detailed error models for the following error sources are considered:

1) Transduction and Resolution Error,
2) Thermal Transpiration in The Pressure Tubing, (Ref. 4),
3) Port Misalignment,
4) Calibration (e) error,
5) Pneumatic Lag and attenuation, (Ref. 5).
In the pneumatic lag and attenuation model error model, the effects of various transducer plumbing arrangements will be analysed in detail.

Using the corrupted pressure data the state estimates are evaluated via the FADS estimation algorithm and the airdata estimation errors are evaluated by computing the residuals between the estimated and "true" (prescribed) airdata values. The estimation sequence is repeated a number of times, and ensemble averages of the squared residuals are computed. The resulting ensemble averages are representative of the state variances. (figure 6) Sample standard errors along the trajectory are plotted as a function of true Mach number in figures 7a and 7b.

Concluding Remarks

This paper presents a design study for a FADS system for the HALO configuration. The proposed HALO trajectory is presented. A simplified, invertible model which relates the measured pressure values to the desired airdata states is developed. Real gas effects caused by the variation of $\gamma$ across the normal shock wave are considered. Factors such as gas dissociation, or reactive chemistry will not be considered. A sensitivity analysis is performed, and a port layout presented. The regression algorithm to be used in estimating the airdata from the measured pressures is developed. Effects of measurements error sources-- 1) Transduction and Resolution Error, 2) Thermal Transpiration, 3) Port Misalignment, 4) Calibration error, and 5) Pneumatic Lag and attenuation are considered and their influence on the accuracy of the airdata estimates is analysed using a Monte-Carlo simulation.

All results presented indicate that the measurement system is feasible, and that (without extraordinary measures) it is possible to obtain very accurate airdata results. Analyses presented indicate that angle-of-attack can be evaluated with a steady accuracy approaching 0.1 deg. at Mach 10, and Mach number can be evaluated with an accuracy better than 0.005.
References


Figure 1: HALO Trajectory

a) Altitude vs. Mach number

b) Alpha vs. Mach

Altitude ~ r.

Mach number

Alpha ~ deg.

Mach number
Figure 2: Schematic of the Hypersonic Launch Option Configuration.
Figure 3: "Calibration" of Modified Newtonian Flow for Mach number Errors

a) Time Marching Euler Solutions

b) Epsilon Parameter vs. Mach
Figure 4: "Creeping" Solution for Variable Ratio of Specific Heats Across Normal Shock
Figure 5: Port Matrix Optimization

a) Normalized Alpha Pressure Error Due to Angle of Attack Error

- \( \delta p / \rho_\text{in} \)
- Pressure ~ psi
- Freestream Mach Number
- Various angles: \( \delta \alpha = 2.0 \) deg, \( \delta \alpha = 1.5 \) deg, \( \delta \alpha = 1.0 \) deg, \( \delta \alpha = 0.75 \) deg, \( \delta \alpha = 0.5 \) deg, \( \delta \alpha = 0.1 \) deg

- Coordinate axes: 0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 0.01, 0.001
### Port Coordinates

<table>
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<th>Port I.D.</th>
<th>$\lambda$ Clock Ang.</th>
<th>$\phi$ Cone Ang.</th>
<th>X (In.) (F.S)</th>
<th>Y (In.) (B.L.)</th>
<th>Z (In.) (W.L.)</th>
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Figure 6: Error Analysis via Monte Carlo Simulation
Figure 7: Example HALO Airdata Uncertainty Estimates

(a) Mach number Error Estimate

(b) Stagnation Pressure (BEHIND SHOCK) Error Estimate