Probabilistic Study of Fluid Structure Interaction

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March 2002
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Prepared for the  
Turbo Expo 2002  
sponsored by the American Society of Mechanical Engineers  
and the International Gas Turbine Institute  
Amsterdam, The Netherlands, June 3–6, 2002

National Aeronautics and Space Administration

Glenn Research Center

March 2002
Acknowledgments

The authors are grateful to Dr. Christos Chamis of NASA Glenn Research Center for his technical guidance. The technical assistance given by N&R Engineering & Management Services is acknowledged.

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ABSTRACT

A combustor liner was computationally simulated and probabilistically evaluated in view of the several uncertainties in the aerodynamic, structural, material and thermal variables that govern the combustor liner. The interconnection between the computational fluid dynamics code and the finite element structural analysis codes was necessary to couple the thermal profiles with structural design. The stresses and their variations were evaluated at critical points on the liner. Cumulative distribution functions and sensitivity factors were computed for stress responses due to the aerodynamic, mechanical and thermal random variables. It was observed that the inlet and exit temperatures have a lot of influence on the hoop stress. For prescribed values of inlet and exit temperatures, the Reynolds number of the flow, coefficient of thermal expansion, gas emissivity and absorptivity and thermal conductivity of the material have about the same impact on the hoop stress. These results can be used to quickly identify the most critical design variables in order to optimize the design and make it cost effective.

INTRODUCTION

Predictive technologies based on a probabilistic method of problem solving are gaining a steady foothold as a method of finding answers to engineering problems. These can be used for design, sensitivity analysis, mathematical modeling of complex processes, uncertainty analysis, competitive analysis and process optimization. With the increase in gas turbine engine structural complexity and performance over the past 50 years, structural engineers have created an array of safety nets to ensure against component failures in turbine engines. In order to reduce what is now considered to be excessive conservatism and yet maintain the same adequate margins of safety, there is a pressing need to explore methods of incorporating probabilistic design procedures into engine development. Probabilistic methods combine and prioritize the statistical distributions of each design variable, generate an interactive distribution and offer the designer a quantified relationship between robustness, endurance and performance. The designer can therefore iterate between weight reduction, life increase, engine size reduction, speed increase, etc.

Fox [1] developed a design system that integrated the deterministic design methods with probabilistic design techniques. Here, two different approaches were used for estimating uncertainty. A Monte Carlo approach was used on design codes that were judged to run relatively quickly. For more computationally intensive design codes, a second order response surface model in conjunction with Box-Behnken design experiments was used and then a Monte Carlo simulation was executed. Lykins and Thompson [2] and Thompson and Fecke [3] developed a system for probabilistic design of gas turbine engines. They relied on direct numerical integration using closed form solutions by fast probability integration to establish risk estimates.

Several researchers at NASA Glenn Research Center have applied the probabilistic design approaches to turbine engines and related systems. Chamis [4] developed a Probabilistic Structural Analysis Method (PSAM) using different distributions such as the Weibull, normal, log-normal etc. to describe the uncertainties in the structural and load parameters or primitive variables. Nagpal, Rubinstein, and Chamis [5] presented a probabilistic study of turbopump blades of the Space Shuttle Main Engine (SSME). They found that random variations or uncertainties in geometry have statistically significant influence on the response variable and random variations in material properties have statistically insignificant effects. Chamis [6] summarized the usefulness and importance of the probabilistic approach, especially for turbopumps. Pai and Chamis [7] outlined the probabilistic evaluation of the buckling of truss structures for non-uniform thermal loads, other loads and moments using the NESSUS computer code. Their results indicated that the buckling loads and member axial forces are most sensitive to the uncertainties in geometry variables. The structural and thermal aspects of the probabilistic assessment of a combustor liner design were reported by Pai and Chamis [8].
Probabilistic CFD design is needed because the flow effects on structures will have to be described more accurately. To cost effectively accomplish the design task, we need to formally quantify the effect of uncertainties (variables) in the design. Probabilistic design is one effective method to formally quantify the effect of uncertainties. It is essential to strengthen the structural probabilistic analysis capability to include aerodynamic and heat transfer uncertainties. The objective is to establish a revolutionary new early design process, by developing non-deterministic physics-based probabilistic design tools, which will include all the life cycle processes. Breakthroughs will be sought in speed, accuracy, intelligence, usability of the system.

A new three-dimensional approach was developed to investigate the application of a parametric optimization method coupled with a CFD Navier-Stokes analysis code, NPARC [9] for the aerothermal design of a combustor liner. The general benefits of the proposed computational research will be improvements to both accuracy and efficiency of the present analysis techniques and will provide savings in computational simulation efforts as well as greater understanding of flow physics issues associated with turbomachinery design.

GOVERNING EQUATIONS AND COMPUTATIONAL FLUID DYNAMICS METHOD

The governing equations are the three-dimensional, unsteady, compressible Navier-Stokes equations coupled with the k-ω SST turbulence model and may be written as:

\[
\frac{\partial p}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0
\]

\[
\frac{\partial \rho u_j}{\partial t} + \frac{\partial (\rho u_ju_i)}{\partial x_i} = -\frac{\partial p}{\partial x_j} + \frac{\partial \tilde{\tau}_{ij}}{\partial x_j}
\]

\[
\frac{\partial \rho E}{\partial t} + \frac{\partial (\rho E u_j)}{\partial x_j} = -\frac{\partial p u_j}{\partial x_j} + \frac{\partial \left[u_i \tilde{\tau}_{ij} - q_j\right]}{\partial x_j}
\]

where

\[
\tilde{\tau}_{ij} = 2\mu \left( S_{ij} - \frac{S_{kk} \delta_{ij}}{3} \right) + \tau_{ij}
\]

\[
\tau_{ij} = 2\mu_T \left( S_{ij} - \frac{S_{kk} \delta_{ij}}{3} \right) - \frac{2\mu_T \delta_{ij}}{3}
\]

and where \( q_j \) is the total heat-flux rate defined as

\[
q_j = -\left( \frac{\gamma}{\gamma - 1} \right) \left( \frac{\mu}{Pr} + \frac{\mu_T}{Pr_T} \right) \frac{\partial T}{\partial x_j}
\]

The turbulent eddy viscosity is defined as

\[
\mu_T = 0.31 \rho k / \max(0.31 \omega; \Omega F_2)
\]

where \( F_1 \) and \( F_2 \) are turbulence functions and \( \Omega \) is the absolute value of vorticity. The idea is to retain the robust and accurate formulation of the Wilcox [11] k-ε model in the near wall region and to take advantage of the free stream independence of the k-ε model in the outer part of the boundary layer. To achieve this, the k-ε model is transformed into a k-ω formulation. The original model is then multiplied by a function \( F_1 \) and the transformed model by a function \((1-F_1)\) and both are added together. The function \( F_1 \) will be designed to be one in the near wall region and zero away from the surface. The blending will take place in the wake region of the boundary layer. Similarly, \( F_2 \) is used as a blending function for the eddy viscosity model. More details about the turbulence models can be found in Ref. [10]. The equation of state is introduced to complete the set of the governing equations as

\[
p = \rho(\gamma - 1) \left[ E - \frac{1}{2} u_i u_i - k \right]
\]

The governing equations are transformed in generalized coordinates and are solved with a finite volume method. With a backward Euler implicit method, the governing equations are discretized in time and linearized in delta form as

\[
\begin{align}
\left( \frac{I}{J\Delta t} + \left[ \frac{\partial R}{\partial Q} \right] \right) \Delta Q &= -R^n
\end{align}
\]

where \( J \) is the Jacobian of transformation, \( R \) is the residual of the steady-state flow equations, and \( Q \) is the six-element vector of conservative variables \((p, pu, pv, pe, pk, pw)^T\).

For the calculation of the residual, convective terms are upwind differenced based on Roe’s flux difference splitting (FDS) scheme [12], and viscous terms are central differenced. The third order of spatial accuracy is kept in all calculations.

The flow simulations were performed by obtaining a converged steady state flow solution. The steady state simulation to initialize the flow field was done with the approximate factorization algorithm using local time stepping. Typically, about 2500 time steps were required to reduce the \( L_2 \) norm to about 10^-7. For the time accurate computations, a five step Jameson algorithm second order accurate in time was selected. The time step throughout the grid block was set to a constant equal to the Courant-Friedrichs-Lewy (CFL) limit specified by the global time step (DTCAP) at the location of maximum change in the flow variables. This option provides the most rapid time-accurate simulation.
GRID GEOMETRY DESCRIPTION

Figure 1 shows a single grid block used for computations. The flow incidence angle was specified and the backpressure is adjusted until the average Mach number along the inflow boundary matches a specified value. Computational grids were generated using the GRIDGEN3D [13] grid generation program. Uniform grid spacing was used in each coordinate direction. A grid refinement study was completed to ensure that the computed results were independent of the grid density. The combustor liner was axisymmetric. A grid of 97 x 101 was selected for the combustor liner aerodynamic simulations.

THE PROBABILISTIC STRUCTURAL ANALYSIS CODE, NESTEM

NESTEM [14] is an enhanced version of NESSUS (Numerical Evaluation of Stochastic Structures Under Stress) developed by NASA Glenn Research Center. NESTEM maintains all the capabilities of NESSUS, including structural analysis using a finite element approach and adds three significant features, namely, heat transfer analysis, geometry generation and ceramic material property generation. The code combines state of the art probabilistic algorithms with general-purpose structural analysis methods to compute the probabilistic response and the reliability of engineering structures. Uncertainty in loading, material properties, geometry, boundary conditions and initial conditions can be simulated. The structural analysis methods include nonlinear finite element methods and boundary element methods. Several probabilistic algorithms are available such as the advanced mean value method and the adaptive importance sampling method. The application of the code includes probabilistic structural response, component and system reliability and risk analysis of structures considering cost of failure. The basic heat transfer variables can be included as random variables along with the mechanical random variables to quantify risk using probabilistic methods to perform sensitivity analysis.

In general, the finite element equations of motion may be written as:

\[
[M] \{\ddot{u}\} + [C] \{\dot{u}\} + [K] \{u\} = F(t) \tag{7}
\]

Here, \([M]\), \([C]\) and \([K]\) denote the mass, damping and stiffness matrices respectively. Further more, \(\{\ddot{u}\}\), \(\{\dot{u}\}\) and \(\{u\}\) are the acceleration, velocity and displacement vectors at each node, respectively. The forcing function \(F(t)\) is time independent at each node.

In this paper, the static case is considered by setting the mass and damping matrices to zero and considering the forcing function to be independent of time in Eq. (7) such that

\[
[K] \{u\} = F \tag{8}
\]

COMPUTATIONAL APPROACH BY THE COUPLING OF CFD AND NESTEM CODES

A thorough literature search has revealed that so far, no one has reported in the literature on the probabilistic study based upon the coupling of the computational fluid dynamics and structural finite element analysis. Therefore, the present study was undertaken in order to accomplish this task. The NPARC [9] program performs aerodynamic analyses for all mean and perturbed values of the random aerodynamic variables. The environmental temperature distribution along the length of the combustor liner was computed in each case. A typical plot of the environmental temperature is provided in Figure 2. The NESTEM program performs heat transfer analyses for all mean and perturbed heat transfer and mechanical random variables using the environmental temperature data. Once all the response analyses are completed, the program uses that data for a probabilistic analysis using the fast probability integration (FPI) module. This module determines the probabilistic distribution and sensitivity factors for the respective random variables.

The proposed methodology will be demonstrated by applying it to a combustor liner assuming uncertainties in the random variables. The liner is made of Haynes alloy, 9.5 inches long, 50 inches inner diameter and 0.1 inch thick. A finite element model was created using 1400 eight node brick elements and 2400 nodes. There were two elements through the thickness and 100 elements around the circumference. All nodes on the left end of the liner were held against axial translation. On this end, nodes on the inside surface and located at ninety degrees from each other were held tangentially. Thus, the liner was free to expand in both radial and axial directions.

The aerodynamic, mechanical and thermal random variables and their respective values used in this analysis are shown in Table 1. All the random variables were assumed to be independent. A scatter of ± 20% was specified for all the variables. This variation amounted to two standard deviations. Although the variations chosen may not be realistic, they can be used to illustrate the procedure used and validate the approach. Normal distribution was assumed for all random variable scatters. Figure 3 shows the various steps in the probabilistic CFD and structural analysis.

DISCUSSION OF RESULTS

Maximum hoop stress location was determined from a pre-analysis of the combustor liner. This location was used to evaluate the cumulative distribution functions (CDF) and the sensitivity factors for stress response. CDF for the hoop stress is shown in Figure 4. The sensitivity factors for hoop stress versus the random variables are plotted in Figures 5 to 9. Table 2 shows the details of the random variables. From Figures 5 to 9 we observe that the inlet and exit temperatures have a lot of influence on the hoop stress. For prescribed values of inlet and exit temperatures, the Reynolds number of the flow, coefficient of thermal expansion, gas emissivity and absorptivity and thermal conductivity of the material have a lot of impact on the hoop stress. These results can be used to further optimize the design for cost effectiveness.

CONCLUDING REMARKS

In this paper, a non-deterministic, non-traditional method has been developed to support reliability-based aerospace design. The revolutionary part of the proposed work is the probabilistic evaluation of the Computational Fluid Dynamics (CFD) methodology. The nontraditional part of the proposed
work is the identification of criteria for using different materials and computational accuracy. Probabilistic methods were applied to the aerothermodynamics of a combustor liner by developing novel concepts for lowering the computational cost. The interconnection between the CFD code and NESTEM codes was necessary to couple the thermal profiles with structural design. Stresses and their variations were evaluated at critical points on the liner using the random variables including the aerodynamic variables, material properties, pressure loading and basic heat transfer variables. Cumulative distribution functions and sensitivity factors were computed for stress responses due to the aerodynamic, mechanical and thermal random variables. Results show that the hoop stress will be less than 50 MPa (7500 Psi) with about 1% probability and will be greater than 200 MPa (30,000 Psi) with a probability of 99%. The deterministic value of hoop stress is given by 113 MPa (17,000 Psi). The inlet and exit temperatures primarily affect the results. Evaluating probability of risk and sensitivity factors will enable the identification of the most critical design variables in order to optimize the design and make it cost effective.

ACKNOWLEDGMENTS

The authors are grateful to Dr. Christos Chamis of NASA Glenn Research Center for his technical guidance. The technical assistance given by N&R Engineering & Management Services is acknowledged.

REFERENCES

Table 1.—Random variables

<table>
<thead>
<tr>
<th>Random Variable</th>
<th>Mean Value</th>
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<tbody>
<tr>
<td>Mach number</td>
<td>0.3</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>1.6591036E07</td>
</tr>
<tr>
<td>Turbulence intensity</td>
<td>0.05</td>
</tr>
<tr>
<td>Inlet Total Pressure</td>
<td>150 Psi</td>
</tr>
<tr>
<td>Inlet Total Temperature</td>
<td>3460 R</td>
</tr>
<tr>
<td>Exit Temperature</td>
<td>3460 R</td>
</tr>
<tr>
<td>Angle of attack</td>
<td>5 degrees</td>
</tr>
<tr>
<td>Coefficient of thermal expansion</td>
<td>9.5 E –06 per ºF</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
<td>23 E +06 Psi</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
<tr>
<td>Emissivity of surface</td>
<td>0.8</td>
</tr>
<tr>
<td>Gas Emissivity inside</td>
<td>0.71</td>
</tr>
<tr>
<td>Gas absorptivity inside</td>
<td>0.52</td>
</tr>
<tr>
<td>Conductivity axial</td>
<td>16.67 BTU/hr ft ºF</td>
</tr>
<tr>
<td>Conductivity tangential</td>
<td>16.67 BTU/hr ft ºF</td>
</tr>
<tr>
<td>Conductivity through thickness</td>
<td>16.67 BTU/hr ft ºF</td>
</tr>
</tbody>
</table>

Table 2.—Random variable labels

<table>
<thead>
<tr>
<th>Label</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANGLE</td>
<td>Angle of attack for flow</td>
</tr>
<tr>
<td>CKXX</td>
<td>Thermal conductivity in axial direction</td>
</tr>
<tr>
<td>CKYY</td>
<td>Thermal conductivity in tangential direction</td>
</tr>
<tr>
<td>CKZZ</td>
<td>Thermal conductivity in thickness direction</td>
</tr>
<tr>
<td>COEFF</td>
<td>Coefficient of thermal expansion</td>
</tr>
<tr>
<td>EMIS12</td>
<td>Surface emissivity</td>
</tr>
<tr>
<td>GABS12</td>
<td>Gas absorptivity</td>
</tr>
<tr>
<td>GEMIS12</td>
<td>Gas emissivity</td>
</tr>
<tr>
<td>MACHNO</td>
<td>Mach number</td>
</tr>
<tr>
<td>MODULUS</td>
<td>Modulus of elasticity</td>
</tr>
<tr>
<td>POISSON</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>PRESSIN</td>
<td>Inlet pressure</td>
</tr>
<tr>
<td>REYNOLD</td>
<td>Reynolds number</td>
</tr>
<tr>
<td>TEMPEX</td>
<td>Exit temperature</td>
</tr>
<tr>
<td>TEMPIN</td>
<td>Inlet temperature</td>
</tr>
<tr>
<td>TURBINT</td>
<td>Turbulent intensity</td>
</tr>
</tbody>
</table>
Figure 1.—Grid for combustor liner.
Figure 2.—Combustor liner environmental temperature.

Figure 3.—Probabilistic CFD and structural analysis.
Figure 4.—Cumulative probability of stress.

Figure 5.—Sensitivity factors versus random variables.
Figure 6.—Sensitivity factors versus random variables.

Figure 7.—Sensitivity factors versus random variables.
Figure 8.—Sensitivity factors versus random variables.

Figure 9.—Sensitivity factors versus random variables.
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