Charge Islands Through Tunneling

Daryl C. Robinson
Glenn Research Center, Cleveland, Ohio
The NASA STI Program Office ... in Profile

Since its founding, NASA has been dedicated to the advancement of aeronautics and space science. The NASA Scientific and Technical Information (STI) Program Office plays a key part in helping NASA maintain this important role.

The NASA STI Program Office is operated by Langley Research Center, the Lead Center for NASA's scientific and technical information. The NASA STI Program Office provides access to the NASA STI Database, the largest collection of aeronautical and space science STI in the world. The Program Office is also NASA's institutional mechanism for disseminating the results of its research and development activities. These results are published by NASA in the NASA STI Report Series, which includes the following report types:

- TECHNICAL PUBLICATION. Reports of completed research or a major significant phase of research that present the results of NASA programs and include extensive data or theoretical analysis. Includes compilations of significant scientific and technical data and information deemed to be of continuing reference value. NASA's counterpart of peer-reviewed formal professional papers but has less stringent limitations on manuscript length and extent of graphic presentations.

- TECHNICAL MEMORANDUM. Scientific and technical findings that are preliminary or of specialized interest, e.g., quick release reports, working papers, and bibliographies that contain minimal annotation. Does not contain extensive analysis.

- CONTRACTOR REPORT. Scientific and technical findings by NASA-sponsored contractors and grantees.

- CONFERENCE PUBLICATION. Collected papers from scientific and technical conferences, symposia, seminars, or other meetings sponsored or cosponsored by NASA.

- SPECIAL PUBLICATION. Scientific, technical, or historical information from NASA programs, projects, and missions, often concerned with subjects having substantial public interest.

- TECHNICAL TRANSLATION. English-language translations of foreign scientific and technical material pertinent to NASA's mission.

Specialized services that complement the STI Program Office's diverse offerings include creating custom thesauri, building customized data bases, organizing and publishing research results . . . even providing videos.

For more information about the NASA STI Program Office, see the following:

- Access the NASA STI Program Home Page at http://www.sti.nasa.gov

- E-mail your question via the Internet to help@sti.nasa.gov

- Fax your question to the NASA Access Help Desk at 301-621-0134

- Telephone the NASA Access Help Desk at 301-621-0390

- Write to:
  NASA Access Help Desk
  NASA Center for AeroSpace Information
  7121 Standard Drive
  Hanover, MD 21076
Charge Islands Through Tunneling

Daryl C. Robinson
Glenn Research Center, Cleveland, Ohio

Prepared for the
2002 International Conference on Computational Nanoscience
sponsored by the Applied Computational Research Society
San Juan, Puerto Rico, April 22–25, 2002

National Aeronautics and
Space Administration

Glenn Research Center

February 2002
This report contains preliminary findings, subject to revision as analysis proceeds.
ABSTRACT

It has been recently reported that the electrical charge in a semiconductive carbon nanotube is not evenly distributed, but rather it is divided into charge “islands” [1,2]. This paper links the aforementioned phenomenon to tunneling and provides further insight into the higher rate of tunneling processes, which makes tunneling devices attractive [3]. This paper also provides a basis for calculating the charge profile over the length of the tube so that nanoscale devices’ conductive properties may be fully exploited.

1 INTRODUCTION

Though it sounds futuristic, the idea of the warp tunnel is straightforward and sound, and it is essential for subsequent calculations in this paper. The concept of the warp tunnel is somewhat similar to Newton’s Cradle. The warp tunnel is introduced in Figure 1. When the black mass in Figure 1 is inserted into the warp tunnel, its representative exits almost immediately from the opposite end of the tunnel. If the warp tunnel is full, the effective distance traveled by the black mass in “its journey across the tube” is on the order of its diameter. The warp tunnel’s quantitative properties are derived here, based on Figure 2. m is the mass of the incoming or outgoing particle, and d its diameter. M is the mass of the average particle in the tube involved in the momentum transfer; D is the diameter of such a particle. The total mass of particles inside the tube is M (r/D). g’ is the acceleration due to an arbitrary force F acting on m and the particles inside the tube. So, F = M (r/D) g’, assuming m << M (r/D). The equations of uniform acceleration give d = g’ t^2/2, t = (2 d/g’) °'5. Pretending that the outgoing particle is identical to the incoming particle, we can define m’s velocity across the tunnel, v_T, as

\[ v_T = (r + d)t = (r + d) (F/(2 d M (r/D)))^{0.5} \]

Instead of F acting on a particle, we now assume it has a corresponding initial velocity v_i, then “it” will effectively traverse the tunnel with v_T. Now we calculate an input velocity v_i equivalent to F based on energy considerations:

\[ mv_i^2/2 = F d, \text{ so } v_i = (2 F d/m)^{0.5}. \]

As the particle is inserted into the warp tunnel, “it” simultaneously emerges from the other end, independent of the length of the tunnel.

The nearly simultaneous emergence of “the” particle as it is inserted is illusory. Actually, the outgoing particle emerges as a consequence of tunnel overflow and is not the selfsame particle inserted, but effectively represents it.

Figure 1: The warp tunnel.

Figure 2: Auxiliary diagram for transmission velocity derivation.

\[ v_T/v_i = (r + d)/(2 d M (r/D))^{0.5} (m/(2 d))^{0.5} \]
\[ = (r + d) (m D)^{0.5} (2 d)/M r)^{0.5} \]
\[ \approx 0.5 (r/d) (m/M)^{0.5} (D/r)^{0.5}, \text{ if } r >> d. \]

An important conclusion of this analysis is that the incoming particle appears to traverse the tunnel with a velocity v_T that is much greater than its input or initial velocity v_i. The longer the tube is, the larger the mass, and the smaller the diameter or width of the inserted particle, the greater the speedup. However, due to conservation of energy, v_o ≤ v_i. In this paper, we assume that a warp tunnel spans a potential barrier. This analysis then provides insight into the higher rate of tunneling processes.

We derive inspiration from the optical tunneling scenario in Figure 3(a). When the upper right prism is removed, a detector to the right of the lower left prism in-line with the incident beam detects nothing. However, when the upper right prism is brought into place, that same detector now detects. Feynman discusses this phenomenon in ref. [4]. Though no translating photons are in the gap, oscillating electrons on the lower left prism create an oscillating field, which shakes
electrons on the upper right prism, thereby producing the light captured by the detector.

We propose something similar for Figure 3(b). Electrons incident on the potential barrier do not actually enter the barrier, but rather produce photons free to enter the barrier instead. Through this action, electrons absorb energetic photons at the opposite end of the barrier. The energized electrons continue the motion once possessed by electrons on the other side of the barrier. In general, it is not the selfsame photon that enters the prism on the left that is detected beyond the prism on the right. Likewise, it is not the selfsame electron that impinges on the barrier in Figure 3(b) that emerges on the other side.

We regard the neutral gaps between charge islands as being spanned by a warp tunnel. These warp tunnels are full of photons, so full that the injection of a single photon at one end of the tunnel causes the ejection of another at the other end near another charge island. The ejected photon is absorbed by an electron; maintaining, by this mechanism, a discontinuous charge distribution.

---

**2 SPACING CALCULATION**

Since charge is segregated into islands, to model this structure, an appropriate circuit seems to be that of a series RC circuit. A capacitor plate represents a charge island; the spacing between the plates of the capacitor represents the neutral gaps between adjacent charge islands. The capacitors are modeled as having equal capacitance while R represents the resistance of everything in the circuit except the nanotube, since the nanotube is essentially treated as a capacitive element in this circuit (Figures 4 and 5). Ordinarily, we would not expect a capacitor to conduct a DC current, but if the plate spacing is sufficiently close, a tunneling current will flow in the same way that electrons tunnel through thin barriers. When tunneling is viewed in this new light, a tunneling current can readily transmit through 40 nm barriers.

Consider the warp tunnel in the top of Figure 1. Ignoring its coloring, let the particle entering the tunnel be a photon emitted by an electron, not displayed, to its left. Assume that the warp tunnel spans a barrier of width a. The particle emerging from the right side of the warp tunnel may be a photon that will be absorbed by an electron, not displayed, on the right side of the barrier. The outgoing photon thereby imparts that electron with the motion originally possessed by the other electron on the left side of the barrier. In these calculations, it is assumed that the photon has a width, denoted "It," directly proportional to its energy or frequency. The proportionality constant \( \alpha \) is derived below.

Assuming the warp tunnel of length a in Figure 1 is full, the entering photon need only move into the tunnel an amount It when a representative photon will move out of the tunnel. The electron, which originally emitted the photon, has effectively bridged the distance across the barrier by "traveling" only a distance \( \text{It} \ll a \). The effective length of the barrier is only \( \text{It}(f) \), the width or thickness of the atom of light of frequency \( f \) perpetuating the motion.

Figures 4 and 5, which depict a DC RC circuit, motivate the derivation of tunneling current formulae. While ordinarily only a transient current flows in response to the application of a constant DC voltage across the series RC couple, given a certain dielectric between the capacitor plates, Giaever showed in ref. [5] that if \( d_0 \) is small enough, a relatively constant tunneling current will flow, even for a DC power supply. We can obtain this result from the classical analysis of a DC RC circuit provided the capacitor plate separation is \( d(t) \), a certain decreasing function of time, and where this \( d \) is a distinctly different variable from prior usages. From a mathematical perspective, the only \( d(t) \) that allows for a

---

**Figure 3: Optical and quantum mechanical tunneling.**

---

**Figure 4: The nanotube is modeled as a resistor and capacitor in parallel. However, the resistance \( R' \) of the couple is very high so that it may treated as an open circuit. \( R \) is the internal resistance of the circuit. It essentially includes resistance of the leads. \( R \) will be an essential parameter.**

---

**Figure 5: Change islands modeled as charge on capacitor plates.**
constant current in classical DC RC circuits is 

\[ d(t) = d_0 e^{-s_3 t}, \]

where \( s_3 \) is an undetermined constant. The capacitance \( C \) of the capacitor increases according to 

\[ C = s/(d_0 e^{-s_3 t}) = s_2 e^{s_3 t}, \]

where \( s \) and \( s_2 \) are constants easily determined by the definition of capacitance. With that \( d(t) \), we can calculate a constant tunneling current through the capacitor, 

\[ I_c = (v/R) e^{-1/(R s_2 s_3)}, \]

which decreases exponentially with increasing initial barrier width \( d_0 \). This formula corresponds to a probability density of effective transit lengths \( L \) inside the plates (the originally fixed plates) that varies as \( 1/(L s_3) \), a distribution that obviously favors shorter effective lengths. We assume, based on the model of a full warp tunnel, that the effective length of the barrier is about the width, \( L_t \), of a photon, and that the more energetic or the higher the frequency of a photon, the thicker it is (\( L_t = \alpha \cdot \omega \)). For verification, the thermodynamic Planck Distribution Law for the distribution of radiation of various frequencies inside a cavity may then give us the density of effective lengths inside the barrier. Using the approximation \( e^x \approx 1 + x \) for small \( x \), the Planck Distribution [6] simplifies to:

\[ df = 1/(h \omega/(2 \pi \tau)), \]

where \( h \) is Planck’s constant and \( \tau \) is temperature.

So the density goes as \( 1/\omega = 1/[\ln(\alpha \omega)] \propto 1/L \). This result is highly convincing, almost as much as the numbers that will be derived from it. Since the required distribution function for \( d(t) \) is actually \( df = 1/(L s_3) \) [m⁻¹], if \( L = L_t = \alpha \cdot \omega \), thermodynamics gives

\[ df = df = 1/(h \omega/(2 \pi \tau)) = 1/(\alpha \cdot \omega \cdot s_3) \]

yielding \( s_3 = h/(2 \pi \cdot \alpha \cdot \tau) \) [s⁻¹], where \( \tau \) is in Joules.

To find \( \alpha \), now we derive an expression for the width of a photon as a function of its frequency. To derive an approximation for the size of a photon, we assume that its density is similar to that of a neutron, which essentially has the maximum known finite density in the observable universe.

We regard photons originating from both protons and electrons as having the density of a neutron. Since the proton is larger than the electron, the other charge quantum, it seems reasonable to approximate the upper size limit of one dimension of the photon by the radius of a proton [7].

As shown in Figure 6, if in its rest frame a mass is spherical, it will appear to observers in a rest frame viewing perpendicular to the uniform motion of the mass as having the profile of a convex lens or vertical ellipse, due to its length contraction in the direction of its motion. As the relative velocity between the perpendicular observers’ frame and the frame of the mass increases, the width of the mass will decrease greatly while its height remains unaffected. In the limit, as the relative velocity approaches the speed of light, the mass will nearly have the profile of a rectangle. If observers are in a rest frame looking in a direction parallel to the motion of the mass as it approaches (a dead center view), the mass will appear circular. So from the combined views of those observing perpendicular and parallel to the motion of the mass, the mass has a shape approximated by a hockey puck or a short, fat cylinder, which has both a rectangular and circular profile. This mass, the subject of this discussion, is a photon. The width of the rectangular profile or the short height of the hockey puck or photon is referred to here as the light thickness or \( L_t \). The mass, \( m_p \), of the photon is given by \( m_p \cdot c^2 = h \cdot f \), where \( f \) is here used to represent the frequency as opposed to \( v \). The volume of the photon, \( v_p \), is then the volume of a cylinder, \( v_p = L_t \pi r_p^2 = m_p/d^2 = h/(c^2 \cdot d^2) \).

\[ L_t(f) = (h f/(c^2 d^2))/\pi r_p^2. \]

This result says that less energetic photons are thinner. We convert \( f \) into angular frequency: \( f = \omega/(2 \pi) \).

\[ L_t(\omega) = 2.24 \times 10^{-40} \omega \] [m].

\[ \alpha = 2.24 \times 10^{-40} \text{ ms}^{-1}. \]

For the temperature variable, we use room temperature, 298 K times Bk, Boltzmann’s constant.

\[ \tau = 298 \text{ Bk} = 4.11 \times 10^{-21} \text{ J}. \]

\[ s_3 = h/(2 \pi \cdot \alpha \cdot \tau) = 1.15 \times 10^{26} \text{ s}^{-1}. \]

From ref. [8] we use the low-end dielectric constant for carbon powder, 2.5. We assume a diameter of 1 nm for the nanotube to obtain its circumference, \( c = 10^{-9} \pi \). We obtain 2.45 Å from refs. [9] and [10] as the parallel carbon bond separation or the contribution of each atom along the CNT’s cross-section towards the circumference (Figures 7 and 8). So the number of atoms, \( n_a \), along the CNT’s cross-section is

\[ n_a = c/circ = 2.45 \times 10^{-10} = 12.82. \]

The partial area, \( p_a \), an atom offers to its cross-section is obtained from refs. [9] and [10] as a double trapezoidal area with bases represented by the carbon bond length and the distance from opposite carbon atoms, and double height represented by the parallel carbon bond separation. So,

\[ p_a = ((2.83 + 1.42) \times 10^{-10}) \times (2.45 \times 10^{-10}/2) \] [m²]

\[ = 5.21 \times 10^{-20} \text{ m}^2. \]

The cross-sectional area, \( a \), of the circular wall of the CNT is

\[ a = n_a p_a = 6.68 \times 10^{-19} \text{ m}^2. \]

The permittivity of free space \( \varepsilon_0 \) is \( 8.85 \times 10^{-12} \text{ F/m} \).

\[ s = kd \varepsilon_0 a = 1.48 \times 10^{-29} \text{ F/m}. \]
value of $9.08 \times 10^{16}$ kg/m$^3$ for the neutron density, an incorrect value for the proton radius, and also a value of 5 instead of 2.5 for the CNT dielectric constant.

4 CONCLUSIONS

This model allows one to conclude that the effective length of the barrier may be given by the width, $l(t)$, of a photon within the barrier. To find the probability density function, $f(L)$, of the length of the barrier, we use the Planck distribution function, which gives the thermal average number of photons in a single mode of frequency. For the temperature variable, we use room temperature. The majority of the frequencies are smaller than that of visible light; hence, $f(L) \propto 1/L$, where $L$ is a function of frequency.

We return now to our classical RC circuit model. When the power is turned on in a classical DC RC circuit, we obtain a transient solution: a current that exponentially decays in time. However, if the capacitor plate spacing is not fixed, but is exponentially decreasing with increasing time, we find that the current through the capacitor is essentially constant and inversely exponentially depends on the original plate spacing. The probability distribution function, $g(L)$, of the length $L$ between the capacitor plates, or the plate spacing, must give $g(L) \propto 1/L$ to explain a plate spacing that exponentially decays with time. This remarkable and highly convincing result shows that harmony may be obtained between standard tunneling current formulae and that predicted by DC RC circuit analysis by assuming that the tunneled current follows a discrete trajectory [11] across the barrier.

REFERENCES

Charge Islands Through Tunneling

Daryl C. Robinson

National Aeronautics and Space Administration
John H. Glenn Research Center at Lewis Field
Cleveland, Ohio 44135-3191


It has been recently reported that the electrical charge in a semiconductive carbon nanotube is not evenly distributed, but rather it is divided into charge “islands.” This paper links the aforementioned phenomenon to tunneling and provides further insight into the higher rate of tunneling processes, which makes tunneling devices attractive. This paper also provides a basis for calculating the charge profile over the length of the tube so that nanoscale devices' conductive properties may be fully exploited.