Models of planet formation and of the orbital stability of planetary systems are described and used to discuss possible characteristics of undiscovered planetary systems. Modern theories of star and planet formation, which are based upon observations of the Solar System and of young stars and their environments, predict that rocky planets should form in orbit about most single stars. It is uncertain whether or not gas giant planet formation is common, because most protoplanetary disks may dissipate before solid planetary cores can grow large enough to gravitationally trap substantial quantities of gas. A potential hazard to planetary systems is radial decay of planetary orbits resulting from interactions with material within the disk. Planets more massive than Earth have the potential to decay the fastest, and may be able to sweep up smaller planets in their path. The implications of the giant planets found in recent radial velocity searches for the abundances of habitable planets are discussed.

1 Introduction

What are the characteristics of planetary systems around stars other than the Sun? How many planets are typical? What are their masses and compositions? What are the orbital parameters of individual planets, and how are the paths of planets orbiting the same star related to one another? These questions are difficult to answer because planets are so faint that none have yet been directly observed over interstellar distances. Radial velocity surveys have demonstrated that planets with masses and orbits quite different from those within our own Solar System are present around main sequence stars in our region of the galaxy (Marcy, this volume, and references therein). All of the extrasolar planets thus far discovered induce variations in stellar reflex motion much larger than would a planetary system like our own, and surveys accomplished to date are consequently strongly biased against detecting low mass and long period planets. Our own Solar System may represent a biased sample of a different kind, because it contains a planet with conditions suitable for life to evolve to the point of being able to ask questions about other planetary systems (Wetherill 1994).

Extrapolating from the small and biased sample of planets that have been detected to a model of the variety of planetary systems which may be present elsewhere in the galaxy is a daunting challenge surely fraught with pitfalls. Detailed predictions are almost certain to be erroneous. However, the substantial progress made over the past few decades towards understanding the origins and dynamical stability of planetary systems makes it possible to assess hypothesized common attributes and scaling relations of planetary systems in a quantitative manner (Lissauer 1995).
The nearly planar and almost circular orbits of the planets in our Solar System argue strongly for planetary formation within flattened circumstellar disks. Astrophysical models suggest that such disks are a natural byproduct of star formation from the collapse of rotating molecular cloud cores (Cassen et al. 1985). Observational evidence for the presence of disks of Solar System dimensions around pre-main sequence stars has increased substantially in recent years (McCaughean and O'Dell 1996). Observations of infrared excesses in the spectra of young stars suggest that the lifetimes of protoplanetary disks span the range of $10^6 - 3 \times 10^7$ years (Strom et al. 1993).

The orbital spacing of planets is an important factor in determining how many planets are likely to exist within habitable zones. Although modern theories of planetary growth do not yield deterministic “Bode's Law” formulae for the orbits of planets (Wetherill 1990), characteristic orbital spacings do arise. These scalings suggest that spacings between planets grow roughly in proportion to the distance from the central star, but that separations also depend on the masses of the star and planets in the system and on quasi-random stochastic factors.

Terrestrial planets are believed to grow by pairwise accretion of solid bodies until the spacing of planetary orbits becomes large enough that the configuration is stable for the lifetime of the system (Safronov 1969, Lissauer 1995). The initial stages of the formation of gas giant planets probably also involved binary agglomeration of solid bodies, with accretion of significant quantities of gas initiated once the heavy-element core had acquired enough mass to gravitationally trap hydrogen and helium (Pollack et al. 1996), the two cosmically most abundant elements, which together are believed to make up over 98% of the mass entering protoplanetary disks. Issues involving the ultimate sizes and spacings of gas giant planets are complex and poorly understood (Lissauer et al. 1995), and provide a major source of uncertainty for modeling the potential diversity of planetary systems.

Theoretical models of planetary growth based upon observations of our Solar System are summarised in Section II (see Lissauer 1993 for a more detailed review). Section III discusses possible modifications to these models as a result of new data concerning extrasolar planetary systems. Stability criteria for planetary systems are reviewed in Section IV. Section V concludes this article with a brief discussion of the implications of these theoretical models for estimates of the abundance of habitable planets in our galaxy.

2 Planet Formation

Even a very slowly rotating molecular cloud core has far too much angular momentum to collapse down to an object of stellar dimensions, so a significant fraction of the material in a collapsing core falls onto a rotationally-supported disk orbiting the pressure-supported (proto)star. Such a disk has the same initial elemental composition as the growing
star. At sufficient distances from the central star, it is cool enough for \( \sim 1 - 2\% \) of this material to be in solid form, either remnant interstellar grains or condensates formed within the disk. During the infall stage, the disk is very active and probably highly turbulent, as a result of the mismatch of the specific angular momentum of the gas hitting the disk with that required to maintain keplerian rotation (Cassen and Moosman 1981). Gravitational instabilities and viscous and magnetic forces may add to this activity. When the infall slows substantially or stops, the disk becomes more quiescent. Interactions with the gaseous component of the disk affect the dynamics of small solid bodies, and the growth from micron-sized dust to kilometer-sized planetesimals remains poorly understood (Weidenschilling and Cuzzi 1993).

The dynamics of larger solid bodies within protoplanetary disks are better characterized. The primary perturbations on the keplerian orbits of kilometer-sized and larger planetesimals in protoplanetary disks are mutual gravitational interactions and physical collisions. These interactions lead to accretion (and in some cases erosion and fragmentation) of planetesimals. Gravitational encounters stir planetesimal random velocities up to the escape speed from the largest common planetesimals in the swarm (Safronov 1969). The most massive planets have the largest gravitationally-enhanced collision cross-sections, and accrete almost everything with which they collide. If the random velocities of most planetesimals remain much smaller than the escape speed from the largest bodies, then these “planetary embryos” grow extremely rapidly. The size distribution of solid bodies becomes quite skewed, with a few large bodies growing much faster than the rest of the swarm, in a process referred to as runaway accretion (Wetherill and Stewart 1989). Eventually, planetary embryos accrete most of the small bodies within their gravitational reach, and the runaway growth phase ends (Ida and Makino 1993).

Slower growth continues (at least for terrestrial-type planets) as the eccentricities of planetary embryos are pumped up by long-range mutual gravitational perturbations (Chambers et al. 1996). As planetary masses increase, they become more efficient at stirring random velocities of neighboring bodies. If sufficiently massive and dense planets exist far enough from the star that they are not too deep within its gravitational potential well, they can eject material into interstellar space. Oort Cloud comets are believed to be icy planetesimals which were sent outwards at very close to the Solar System escape velocity, and were perturbed into long-lived orbits by nearby stars, interstellar material or the galactic tidal field. The highly nonlinear nature of this process suggests that larger mass protoplanetary disks may produce only slightly more massive planetary systems; however, the complexities of gas accretion by planets and of disk dispersal time scales cast some doubt upon this conclusion.

Giant planet growth times predicted by current models (Pollack et al. 1996) are similar to estimates of the lifetime of the gaseous protoplanetary disk. At present, it is not even possible to confidently predict whether or not giant planets form in most protoplanetary disks (Wetherill 1994),
much less make quantitative estimates of their likely masses. Nonetheless, gas giants are more likely to form in high surface mass density, long-lived protoplanetary disks. However, planets which become massive while a substantial amount of gas remains in the disk may migrate into the star as a consequence of their gravitational interactions with the disk (Goldreich and Tremaine 1980, Ward 1986).

3 Models For The Formation Of Planets Observed To Orbit Main Sequence Stars Other Than The Sun

All planets thus far identified in radial velocity surveys share three characteristics, each of which act to increase their detectability (Butler and Marcy 1997): Their masses exceed that of Saturn, their orbital semimajor axes are less than roughly 2 AU, and they dominate the radial velocity variations of their parent stars over a broad range of timescales (thus, the most massive planet near these stars surpasses the second most massive planet by a factor larger than the ratio of the mass of Jupiter to that of Saturn).

Three of these planets, henceforth referred to as "vulcans", after the hypothetical planet once believed to travel about the Sun within the orbit of Mercury, have periods less than one week. Lin et al. (1996) suggested that these vulcans formed substantially farther from the star and subsequently migrated inwards to their current short-period orbits. Planetary orbital decay had been suggested prior to the discovery of extrasolar planets, but no one predicted giant planets near stars because migration speeds were expected to increase as the planet approached the star, so the chances that a planet moved substantially inwards and was not subsequently lost was believed to be small. Lin et al. suggested two possible mechanisms for stopping a planet less than one-tenth of an AU from the star: tidal torques from the star counteracting disk torques or a substantial reduction in disk torque when the planet was well within a nearly empty zone near the star. However, the discovery of giant planets with orbital periods ranging from 15 days to three years, which feel negligible tidal torque from their star and are unlikely to have entered a clear zone of the disk, cast doubt upon this model.

Some of the giant planets move on quite eccentric (0.2 < e < 0.7) orbits. These eccentric orbits may be the result of stochastic gravitational scatterings among massive planets (which have subsequently merged or been ejected to interstellar space, Weidenschilling and Masar 1996) by perturbations of a binary companion (Holman et al. 1997) or by past stellar companions (if the now single stars were once members of unstable multiple star systems). However, as neither scattering nor migration offer a good explanation for those planets with nearly circular orbits and periods from a few weeks to a few years, the possibility of giant planet formation quite close to stars should not be dismissed.
Each of the planets identified in radial velocity surveys orbits a different star, and each is much more massive \((M \sin i)\) than any other companions that the star may possess which have periods of a few years or less. This is in sharp contrast to our Solar System, where planets of comparable size have orbital periods within a factor of two or three of their neighbors. The systems thus appear to be more overstable than is our own (see Section IV), which may indicate different mechanisms important in the formation process.

The total number of known extrasolar planets is still small, and the sample contains strong biases. Most solar type stars could well have planetary systems which closely resemble our own. Nonetheless, if giant planets (even of relatively modest Uranus/Neptune masses) orbiting near or migrating through 1 AU are the norm, then terrestrial planets in habitable zones may be scarcer than they were previously believed to be. However, such giant planets could have large moons which themselves might be habitable.

4 Stability Of Orbital Configurations

Dynamical stability criteria provide some of the best constraints on the plausible range of planetary system properties. A system containing only two planets, both of which are on initially circular orbits, will never experience close approaches (i.e., is Hill stable) provided the ratio of the separation in orbital radii to the planets' semimajor axes, \(\Delta a\), satisfies

\[
\frac{|\Delta a|}{a} > 2.4 \left(\frac{M_1 + M_2}{M_*}\right)^{1/3}
\]

(1)

where \(a\) is the average semimajor axis of the planets and \(M_1, M_2\) and \(M_*\) are the masses each of the planets and that of the star, respectively (Gladman 1993). Additionally, deterministic chaos plays an important role in the stability of planetary orbits. Trajectories tend to be chaotic whenever resonances overlap. Strong resonances are more closely spaced near a planet, and resonances overlap over a range in semimajor axis centered on the perturbing planet. In order to avoid the resonance overlap region of a planet of mass \(M_p\), a test particle must be sufficiently distant that

\[
\frac{|\Delta a|}{a} > 1.5 \left(\frac{M_p}{M_*}\right)^{2/7}
\]

(2)

(Duncan et al. 1989). Note that for both Hill stability and stability against strong chaos, the minimum separation between two planets increases with planetary mass, but the dependence is much slower than linear. Thus, the merger of planets on adjacent orbits usually increases the stability (dynamical lifetime) of a planetary system.

Although separation in semimajor axis is very important in determining the stability of a planetary system, other factors are also involved. Resonances can either increase orbital stability, e.g., Neptune and Pluto, or lead to instability, such as at the 3:1 Kirkwood gap in the asteroid belt.
Eccentricities and inclinations also affect stability. As both eccentricity and inclination require excess energy, they give the systems more freedom of motion, and thus allow for greater instability.

Perturbations from several planets add in a complicated, nonlinear manner. Nonetheless, formulae based on generalisations of the Hill-Jacobi exclusion zone (Eq. 1) and the resonance overlap criterion of the restricted 3-body problem (cf. Eq. 2) appear to be quite useful for assessing the stability of our planetary system and the satellite systems of the giant planets (Lissauer 1995). However, as the root cause of most long-term orbital instabilities is resonances, these criteria provide at best rules of thumb for judging stability, not a precise formulation that always works. For planets of a given mass, the number of them which can lie within a given logarithmic distance interval increases slowly (roughly as the two-sevenths power) with stellar mass. Smaller planets may be more closely spaced. See Lissauer (1995) for a more detailed and quantitative discussion of proposed scaling rules for planetary systems.

The planets in our Solar System orbit close enough to each other that the final phases of planetary growth could have been the merger or ejection of planets on unstable orbits. However, the low eccentricities of the orbits of the outer planets imply that some damping process, such as accretion/ejection of numerous small planetesimals or interactions with residual gas within the protoplanetary disk, must also have been involved (cf. Lissauer et al. 1995).

5 Conclusions

Prior to the discovery of extrasolar planets, models of planetary growth suggested that most single solar-type stars possess planetary systems which are grossly similar to our Solar System. It was realised that stochastic factors are important in planetary growth, so that the number of terrestrial planets (as well as the presence or lack of an asteroid belt) would vary from star to star, even if their protoplanetary disks were initially very similar (Wetherill 1990). The difficulty in accreting giant planet atmospheres prior to dispersal of circumstellar gas suggested that many systems might lack gas giants (Wetherill 1994, Lissauer 1995). The low eccentricities of the giant planets in our Solar System (especially Neptune) are difficult to account for (Lissauer et al. 1995), so systems with planets on highly eccentric orbits were viewed as possibilities, although researchers did not hazard to estimate the detailed characteristics of such systems. A maximum planetary mass similar to that of Jupiter was suggested as a possibility if Jupiter's mass was determined by a balance between a planet's gap clearing ability and viscous inflows (Lin and Papaloizou 1979), although it was noted that the value of the viscosity could well vary from disk to disk. Orbital migration of some giant planets towards their parent star was also envisioned (Goldreich and Tremaine 1980), but since migration rates increased as the planet approached the star, such planets were expected to be accreted by their star (Ward 1986, Lissauer 1995), and the
existence of numerous giant planets with orbital periods ranging from a few days to several weeks was not predicted.

It must thus be admitted that theoretical models based upon observations within our Solar System failed to predict the types of planets thus far detected by radial velocity surveys. On the other hand, note that the surveys which prove that such planets exist also show that they are fairly rare, occurring in fewer than ten percent of the systems. The radial velocity surveys conducted thus far are quite biased in favor of detecting massive planets orbiting close to stars, and planets similar to those in our own Solar Systems would not yet have been detected. Thus, it is possible that the vast majority of single Sun-like stars possess planetary systems quite similar to our own. Alternatively, although theoretical considerations suggest that terrestrial planets are likely to grow around most Sun-like stars, they may typically be lost if most systems also contain giant planets which migrate into the central star.

We still do not know whether terrestrial planets on which liquid water flows are rare, are the norm for solar type stars or have intermediate abundances. Nonetheless, I personally believe that, even if planetary migration destroys some promising systems, planets qualifying as continuously habitable for long periods of time by this definition are sufficiently common that if we are the only advanced life form in our sector of the galaxy, biological and/or local planetary factors are much more likely to be the principal limiting factor than are astronomical causes.

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References

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