In this presentation, I would like to review historical perspective on the program architecture used to build design optimization capabilities based on mathematical programming and other numerical search techniques.

It is rather straightforward to classify the program architecture in three categories as shown above. However, the relative importance of each of the three approaches has not been static, instead dynamically changing as the capabilities of available computational resource increases. For example, we considered that the direct coupling architecture would never be used for practical problems, but availability of such computer systems as multi-processor.

In this presentation, I would like to review the roles of three architecture from historical as well as current and future perspective. There may also be some possibility for emergence of hybrid architecture. I hope to provide some seeds for active discussion where we are heading to in the very dynamic environment for high speed computing and communication.
Up to early 1970's, direct coupling of numerical search program and analysis/simulation was the only method used for engineering design optimization. In late sixties, the method to compute sensitivity of responses obtained by linear structural analyses was found and provided significant improvement of this scheme.

However, by early 1970's, it became obvious that the size of the analysis and design models that could be handled by this scheme was very limited and unlikely to have practical impacts using the best computing facility such as CDC6600, Univac 1108, or IBM360. This was summarized as "insurmountable computational inefficiency" related to hundreds and even thousands of finite detailed analyses/simulations demanded by the optimization algorithms. This observation motivated some investigators to develop alternative scheme for design optimization as manifested by the development of the optimality criteria approaches headed by people at USAF Wright Patterson.

The best feature of the direct coupling, however, is the fact that no approximations are involved. For very complex behaviors such as transonic aerodynamics that exhibits many relative minima/maxima, it is difficult to create approximate models that represents such rugged responses, hence the direct coupling may be recognized as the only viable approach. Fortunately, with the advent of HPC, especially, availability of multi-processor computational facilities has given a hope to work with the direct coupling scheme by processing relatively large number of analyses/simulations in parallel and couple with such algorithms as GA or SA.
Disappointment directed to design optimization was prevalent at the Conference on Matrix Methods for Structural Analysis (WPAFB, Oct. 1971) and Conference on Structural Optimization (Swanson, UK, Jan. 1972) and the future of the applications of mathematical programming techniques was put in a difficult position.

During 1970's, various schemes to make better use of sensitivity data was developed. The simplest forms were expansions into direct and reciprocal variables. Furthermore, the concept of intermediate responses in creating better approximations improved the quality of approximate models. Rayleight quotient approximation of eigenvalues, internal force approximation for stresses, product form of frequency responses, approximation of reduced modal matrices, etc. These new types of approximation concepts made it possible to implement design optimization capabilities into commercial codes, such as NASTRAN\(s\) (MSC, CSA, UAI), ASTROS, GENESIS, etc. in 1980's to early 1990's.

This architecture took advantage of efficient sensitivity analysis capability for linear finite element structural analysis capability even with relatively large number of design variables. In auto industry, problems with hundreds of design variables associated with over millions of degrees of freedom are solved almost routine basis, thanks to the development of this architecture.
Variable Target Profile
Optimization of Fully Trimmed Vehicle for NVH Tuning

FE Model: 470,000 Structural DOFs + 3,600 Acoustic DOFs
Design Variables: 148 panel thickness
Objective: Response at driver's ear < specified value in range of 20 Hz to 120 Hz (2 Hz interval)
Modal analyses:
    Structure - up to 250 Hz  840 modes
    Acoustic - up to 500 Hz  60 modes
Formulation \( (t_1, t_2, \ldots, t_{NOV}, \beta) \)

Find \( (t_1, t_2, \ldots, t_{NOV}, \beta) \) so that \( \beta \) is minimized while satisfying:

\[
\frac{R(f_i)}{R_0(f_i)} - \beta + 1 \leq 1 \quad i = 1, 2, \ldots, N
\]
\[
W \leq W_{\text{limit}}
\]
\[
0.5 t_{i,\text{limit}} \leq t_i \leq 1.5 t_{i,\text{limit}} \quad i = 1, 2, \ldots, 148
\]

For propriety reasons, the model of this structure cannot be shown, but this is a fully trimmed vehicle. This problem is more than a year old, thus the model size is less than a half million, but currently this company is working with the models over 1 million DOF on routine basis for NVH problems.

The purpose of this design problem is to reduce the acoustic pressure at the driver head position over the excitation frequency range 20-120 Hz.
As shown in this figure, the pressure magnitude exceeds the target pressure over the frequency range 75 Hz to 116 Hz.

Our objective is to overcome this violation and, if possible, to push down the target line further as much as possible.
As shown in this figure, it was possible to reduce the FRF well under the target profile over the entire frequency range and eliminate any FRF that exceeded the target.
Multiple Objective Design of BIW Vehicle

Problems:
1. The baseline design is over-weight
2. The 1st bending frequency is 32.6 Hz, but must be at least 35 Hz
3. The 1st torsional frequency is 40.0 Hz, but must be at least 44 Hz

Analysis model size: 345,000 DOFs
Design variables: 114 panel thickness

Formulation

Find \((t_1, t_2, \cdots, t_{114}, \beta_1, \beta_2)\) so that a function \(F = \beta_1 + \beta_2 + 5W\) is minimized while satisfying:

\[
\frac{f_{bb}}{35.0} - \beta_1 + 1 \leq 1 \\
\frac{f_{bt}}{44.0} - \beta_2 + 1 \leq 1 \\
W \leq W_{\text{target}} = 0.302
\]

\(\beta_1 \leq 1.0 \quad \beta_2 \leq 1.0\)

This problem has three distinct objectives to be achieved, while satisfying prescribed constraints. The structure is a body-in-while of an automobile.

The design problem was formulated by multiple beta method described in Ref. 1. The objective function is the weighted sum of the values assigned to the structural mass (scaled) and \(\beta_1\) and \(\beta_2\). The weight factor was selected by experiments and was settled to use 5 for the mass, while 1.0 was assigned to both beta.

Ref. 1
As shown in this table, the mass was reduced by 11%, while all requirements on the bending and tortional natural frequencies are satisfied. Especially, the tortional frequency was raised far more than the given target. Therefore, we have succeeded to make everyone happy.
After successful implementation of approximation concepts with linear structural analysis programs, it was natural to look into the possibility for application of design optimization methods to nonlinear analyses. However, it has been difficult to develop efficient sensitivity analysis capabilities and approximate models based on sensitivity data.

In the past few years, the approach based on response surface approximations attracted attention of many investigators and developers. This is a scheme to build approximate models based on the results of multiple number of analyses without depending on the sensitivity data. "Multiple number of analyses" are carried out for perturbed designs and the results are stored in the database. For each of the responses that might have any effects on the design process, a quadratic approximate model is built using the least square method based on the results stored in the database.

The number of analyses required to build reliable approximations is closely related to the number of design variables. For this reason, the number of design variables is limited by computer resources available to carry out enough number of analyses/simulations. Again, this limitation is relieved if multiple number of analyses are processed in parallel on powerful multiprocessor computers.
This example is the result of application of optimization method based on the response surface approximation to improvement of the structural design of vehicle side impact (crash). Each analysis (61 ms of duration) takes about 83 hours of CPU time on SGI Origine2000 computer using 1 CPU per job.

The number of design variables is 9 and all are plate thicknesses. The objective was to reduce structural mass and to increase the absorption energy.

In this case, we did not produce a single optimal design. Instead, we produced a Pareto curve that presents the boundary of performance that can be achieved by modifying the prescribed 9 thicknesses. In the figure given above, the state represented below the blue line can be achieved by modifying 9 thicknesses, but any state above the blue line cannot be achieved unless basic design changes such as shape or topology changes are implemented.
3.0 I

Total Energy * 10E-7

2.5

2.0

1.5

1.0

0.5

Design Variables
- Thickness-48
- Thickness-49
- Thickness-147

Scaled Thickness

Total Energy * 10E-7

Mass

1.378 1.380 1.382 1.384