A Flow Solver for Three-Dimensional DRAGON Grids

Meng-Sing Liou
Glenn Research Center, Cleveland, Ohio

Yao Zheng
Taitech, Inc., Cleveland, Ohio

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Meng-Sing Liou  
National Aeronautics and Space Administration  
Glenn Research Center  
Cleveland, Ohio 44135  

Yao Zheng  
Taitech, Inc.  
Cleveland, Ohio 44135

Abstract

DRAGONFLOW is a flow solver devoted to the Computational Fluid Dynamics (CFD) simulation using the three-dimensional DRAGON grid technology. DRAGONFLOW is a program suite adopted and redesigned from two existing successful flow solvers OVERFLOW and USM3D. OVERFLOW is a Navier-Stokes code working with Chimera grid, and is used here to solve flow problems on the domain occupied by composite structured grids of a DRAGON grid. USM3D is characteristic-based scheme for steady solutions to the Navier-Stokes equations on unstructured tetrahedral grid, and is employed to pursue flow simulation over a collection of unstructured grids of a DRAGON grid.

DRAGON grid is constructed to be a combination of a Chimera grid and a collection of unstructured grids. In the DRAGONFLOW suite, both OVERFLOW and USM3D are presented in form of module libraries, and a master module controls the invoking of these individual modules. The alternating invoking of these solvers in each time step, and the immediate data exchange on the DRAGON grid interfaces, leads to a seamless coupling of these two solvers. This report includes essential aspects, programming structures, benchmark tests and numerical simulations.

1 Introduction

During the past decades, both structured and unstructured grid techniques have been extensively developed and applied to solution of various engineering problems. To deal with
situations in which complex geometry imposes considerable constraints and difficulties in generating grids, composite structured grid schemes and unstructured grid schemes currently are the two mainstream approaches.

In general, unstructured grid methods are considered to be more versatile and easier to adapt to complex geometries but less suitable for resolving viscous layers, while composite structured grid methods are considered to be more flexible to use efficient numerical algorithms and require less computer memory. Clearly, both methods complement each other on strengths and weaknesses. Hence, a method that properly employs a hybrid of structured and unstructured grids may prove to be fruitful. In fact some hybrid schemes have already appeared in the literature [7, 17, 18, 20] and have shown promising features.

The DRAGON grid [8, 13, 22], as a hybrid grid, is created by means of a Direct Replacement of Arbitrary Grid Overlapping by Nonstructured grid. The DRAGON grid scheme is an adaptation to the Chimera thinking. The Chimera grid is a composite structured grid, composing a set of overlapped structured grids, which are independently generated and body-fitted. The grid is of high quality and amenable for efficient solution schemes. However, the interpolation used in the overlapped region between grids introduces error, especially when a sharp-gradient region is encountered [10, 19]. The DRAGON grid scheme is capable of completely eliminating the interpolation and preserving the flux conservation property. It adapts and maximizes the advantages of both structured and unstructured grids, while at the same time it eliminates the weaknesses of the Chimera scheme and minimizes the memory requirement associated with the unstructured grid.

We have developed an extension of the DRAGON grid technology into three-dimensional space [15, 16, 21, 22]. The present work is to develop a flow solver devoted to the Computational Fluid Dynamics (CFD) simulation using DRAGON grid technology in the three-dimensional space [15, 16, 21]. The resulting program suite is named as DRAGONFLOW. It is the result of integrating works of other researchers, and it is adopted and redesigned from two existing successful flow solvers OVERFLOW [1] and USM3D [4, 5]. OVERFLOW is a Navier-Stokes code working with Chimera grid, and is used here to solve flow problems on the domain occupied by composite structured grids of a DRAGON grid. USM3D solves Navier-Stokes equations on an unstructured tetrahedral grid, and is employed to perform flow simulation over a collection of unstructured grids of a DRAGON grid.

The present work is written to define rules over the programming practice employed during the development of DRAGONFLOW. This introductory document includes essential aspects, programming structures, benchmark tests and numerical simulations.
2 Solution Scheme

2.1 Governing Equations

The time-dependent compressible Navier-Stokes equations given by Equation (1), in an integral form over an arbitrary control volume $\Omega$, are solved:

$$\int_{\Omega} \frac{\partial \mathbf{U}}{\partial t} dv + \int_{\partial \Omega} \mathbf{F} \cdot d\mathbf{S} = 0,$$

where the conservative variable vector $\mathbf{U} = (\rho, \rho u, \rho v, \rho e)^T$, with the specific total energy $e_t = e + \frac{|\mathbf{V}|^2}{2} = h_t - p/\rho$. The flux vector $\mathbf{F}$ includes both the inviscid and viscous fluxes, in which the turbulence variables are also included. We denote with an overhead arrow the vector quantities expressed in terms of Cartesian coordinates.

2.2 Flux Splitting

Based on the cell-centered finite volume method, the governing equations are semi-discretized. We use the flux scheme AUSM+, described in full detail in [11], to express the numerical inviscid flux at the cell faces. The basic idea of AUSM+ follows that of its predecessor, AUSM [14], but has substantial improvements. The AUSM+, incurring negligible additional cost over the earlier AUSM scheme, allows exact capture of a normal shock by using a suitably chosen interface speed of sound, yields a smoother solution by utilizing higher-order polynomials, and often results in a faster convergence rate.

The semi-discretized form, describing the time rate of change of $\mathbf{U}$ in $\Omega$ via balance of fluxes through all enclosing faces, $S_l$, $l = 1, \ldots, LX$, independent of whether they are in the structured or unstructured grid regions, can be cast as

$$\int_{\Omega} \frac{\partial \mathbf{U}}{\partial t} dv + \sum_{l=1}^{LX} \mathbf{F}_l \cdot \mathbf{n}_l = \int_{\Omega} \frac{\partial \mathbf{U}}{\partial t} dv + \sum_{l=1}^{LX} \mathbf{F}_{nl} |\mathbf{n}_l| = 0.$$  

(2)

Noting that the flux terms account for only normal components of the flux at the face, $\mathbf{F}_{nl} = \mathbf{F}_l \cdot \mathbf{n}_l$ where $\mathbf{n}_l$ is the unit normal vector of $S_l$. The task is then to represent the numerical flux at the cell interface $S_l$, which straddles cells denoted by subscripts "L" and "R". The AUSM+ scheme gives the numerical flux in the following expression.

$$\mathbf{F}_{nl} = M_l \frac{\alpha_l}{2} (\Phi_L + \Phi_R) - |M_l| \frac{\alpha_l}{2} \Delta \Phi + \mathbf{P}_{nl}.$$  

(3)

We remark that there is enough freedom for defining the interface speed of sound $\alpha_l$ so that certain criteria are met [11, 12]. An interesting notion is the so-called numerical speed of sound, which is introduced to handle low Mach number flows and later is extended to the multiphase flows [12]. The interface Mach number $M_l$ is an important variable and defined in the following steps.
1. Project velocity vectors at the cell centers, “L,” and “R”, to \( \tilde{S}_l \),

\[
V_L = \tilde{V}_L \cdot \tilde{n}_l, \quad V_R = \tilde{V}_R \cdot \tilde{n}_l.
\]  

(4)

2. Define the corresponding Mach numbers,

\[
M_L = \frac{V_L}{a_L}, \quad M_R = \frac{V_R}{a_L}.
\]  

(5)

3. Define the interface convective Mach number by writing

\[
M_I = M_{(4)}^{+} (M_L) + M_{(4)}^{-} (M_R),
\]  

(6)

where the formulas for \( M_{(4)}^{+} \) are expressed in terms of eigenvalues of the nonlinear waves, \( M + 1 \) and \( M - 1 \) [11].

\[
M_{(4)}^{\pm} (M) = \begin{cases}
\frac{1}{2} (M \pm |M|), & \text{if } |M| \geq 1, \\
M_{(2)}^{\pm} (1 \mp 2M_{(2)}^{\mp}), & \text{otherwise},
\end{cases}
\]  

(7)

where

\[
M_{(2)}^{\pm} (M) = \pm \frac{1}{4} (M \pm 1)^2.
\]  

(8)

4. Define the interface pressure \( p_I \) in terms of the above-defined Mach numbers,

\[
p_I = P_{(5)}^{+} (M_L) p_L + P_{(5)}^{-} (M_R) p_R
\]  

(9)

where similarly we use the following polynomials,

\[
P_{(5)}^{\pm} (M) = \begin{cases}
\frac{1}{2M} (M \pm |M|), & \text{if } |M| \geq 1, \\
M_{(2)}^{\pm} [(1 \pm 2 - M) \mp 3M_{(2)}^{\mp}], & \text{otherwise}.
\end{cases}
\]  

(10)

Then, we get

\[
P_{nl} = p_I \begin{pmatrix} n_{lx} \\ n_{ly} \\ n_{lz} \\ 0 \end{pmatrix}
\]  

(11)

5. Assemble the interface numerical flux by inserting \((M_t, P_{nl})\) in Eq. (3).

2.3 Flow Solvers

As mentioned before, the flow code DRAGONFLOW is made up of two well-validated NASA codes, OVERFLOW [1] and USM3D [5], which are respectively structured- and unstructured-grid codes. We have spent a significant effort to add an interface and modify both codes, more so for the OVERFLOW code. The interested readers should consult with
the cited references for details of either code. The details on implementing the DRAGONFLOW code are to be addressed in Section 4. Nonetheless, there are differences between these codes that warrant concerns and cares when combining them into the present framework, such as time integration schemes, order of spatial discretizations, and choice of turbulence models.

A spatial-factorized implicit algorithm is employed in the OVERFLOW code, while a point implicit one in the USM3D code. Since we are only interested in seeking steady state solutions, the differences of convergence rate due to the incompatibility between the structured and unstructured grid regions can be tolerated. However, if unsteady solutions are of interest, this difficulty can be overcome by a dual time integration approach in which the solution within each physical time step will be iterated to achieve convergence. This subject is left for a future research.

As for the spatial discretization, there is inconsistency in the order of accuracy in that a third-order and a second-order accurate schemes are used respectively in the structured grid-based and unstructured grid-based codes. But in both cases a similar limiter function is applied. We use the aforementioned flux scheme AUSM+ to express the inviscid flux at the cell faces in both the OVERFLOW and USM3D codes. Also, the viscous fluxes are approximated, as usual, by a centered scheme.

3 Data Communication through Grid Interface

3.1 Generic Concept

In the Chimera method, communication through grid interfaces is made through the hole boundary or the outer boundary. Since the interface treatment methods are not necessarily satisfying any form of conservative constraints, the solutions on overlaid grids are often mismatched with each other. More seriously, this may ultimately lead to spurious or incorrect solutions, especially when a shock wave or high-gradient region passes through boundaries of overlaid grids, as will be seen later.

For the DRAGON grid, conservation laws are solved on the same basis on both the structured and unstructured grids. Figure 1 shows an interface connecting both structured and unstructured grids, assuming the cell center scheme is used. As described earlier, the numerical fluxes, evaluated at the cell interface, are based on the conditions of neighboring cells (denoted as L and R cells, respectively). For the unstructured grid, the interface flux $F_{nb}$ will be evaluated using the structured-cell value as the right (R) state and the unstructured-cell value as the left (L) state. Consequently, the interface fluxes, which have been evaluated for the unstructured grid, can now be directly applied in computing the cell volume residuals for the structured grid.

Thus, the data communication through grid interfaces in the DRAGON grid guarantees satisfying of the conservation laws. It is considered seamless in the sense that no manipulation of data, which introduces uncertainties, is required, and the solution is obtained on the same basis for both structured and unstructured grid regions. Consequently this strictly enforces the conservation property locally and globally.
3.2 Fictitious Structured Grids

In Figure 1, the cell center scheme is assumed for both structured and unstructured grids. However, the OVERFLOW code used in the structured grid regions is developed based on a nodal-based scheme, where the flow variables are solved and stored at the grid points, rather than being offset to the center of the cells defined by the grid points. The surface of the unstructured grids within interfacial areas between the two types of grids could not match a surface described by nodes in the structured grids, in order to establish reasonable data communication through grid interfaces. For the DRAGONFLOW code, special treatment has been proposed to tackle the difficulty associated with the difference between cell-based and node-based schemes. This can be neatly resolved by a novel use of fictitious structured grids based on the original structured grids.

From a structured grid with \( l \times m \times n \) nodes, an artificial grid with \( (l + 1) \times (m + 1) \times (n + 1) \) nodes can be constructed by means of introducing middle nodes between original nodes. It is clear that the new grid is shifted from the original one by a half cell size in the interior of the original grid. This new grid is referred to as a fictitious grid, and the associated nodes are named as fictitious nodes. Figure 2 gives an illustration of the fictitious grid. Cell center \((j,k)\), which is node \((j,k)_o\) of the original grid, is surrounded by nodes \((j,k)_f, (j+1,k)_f, (j+1,k+1)_f\) and \((j,k+1)_f\) of the fictitious grid. Subscripts \(o\) and \(f\) denote “original” and “fictitious” grids, respectively.

3.3 Establishing DRAGON Grid Interfaces

It is obvious that the faces described by the fictitious nodes are the faces of finite volumes centered at the original nodes. For the fictitious grids, if we mimic the gridding method of DRAGON grids based on the original structured grids, as proposed in Reference [22], then we can generate a DRAGON grid based on the fictitious grids. Moreover, the DRAGON interfacial boundary nodes of the unstructured grid coincide with the corresponding nodes.
Figure 2: Diagram of a fictitious grid associated with its original structured grid.

on the fictitious grid. Because the numerical fluxes are still conservative in the DRAGON grid interface, the conservation property will not be affected by means of using a fictitious grid.

During the creation of the DRAGON grid based on the fictitious grids, IBLANK values for the fictitious grids can be automatically specified, taking into account the IBLANK values of the original grids. A fictitious node is specified 1 as its IBLANK value, if and only if there is an original node nearby with IBLANK value 1. According to this convention, a special case depicted in Figure 3 should be taken care of.

Figure 3: Special case to fix IBLANK values on the original structured grid.
The faces of DRAGON grid interface are constructed from certain faces of the finite volumes centered at the original nodes. For an original node of interest, there are six possible faces, which could become faces of the DRAGON grid interface. Figure 4 illustrates the numbering systems of the original grid, fictitious grid and flux definition, given Faces A and B as possible faces of the DRAGON grid interface, within a frame of a two-dimension problem. In this case, Faces A and B are defined as \((j, k, -1)\) and \((j, k, 1)\), where \pm 1 denotes the directions of the face. For Face A (idir = -1), \((j, k; -1)\) represents the index of cell center \(j_D\), flux \((j - 1)j_f\) at the \(j - 1\) face and the fictitious grid index \(j_f\). While for Face B (idir = 1), \((j, k; 1)\) represents the index of cell center \(j_D\), flux \(j_fj\) at the \(j\) face and the fictitious grid index \((j + 1)j\). Generally, for three-dimensional cases, the notation of face directions is given in Figure 5.

![Diagram of 2D DRAGON grid interfaces](image)

Figure 4: Diagram of 2D DRAGON grid interfaces.

![Diagram of 3D DRAGON grid interfaces](image)

Figure 5: Notation of face directions of 3D DRAGON grid interfaces.
3.4 Nodal Variables near DRAGON Grid Interfaces

In the DRAGONFLOW procedure, values of nodal variables of structured grids near the interfaces are passed to the counterpart in the side of unstructured grids. These values are used to compute flux values on the interfaces. Therefore, the information from the side of structured grids contributes to the evolution of the solution in the unstructured grid regions. On the other hand, the fluxes computed are passed to the OVERFLOW to influence the solution in the structured grid regions.

Figure 6 is a diagram of passing $Q$ values from structured to unstructured grids. The corresponding values are stored in Arrays $Q_b$ and $Q_{gh}$ in the USM3D part. It should be mentioned that $Q$ values in the OVERFLOW and USM3D parts are in terms of conservative variables and primitive ones, respectively. A conversion has been conducted when $Q$ values are passed from structured to unstructured grids. $Q_1$, $Q_2$, $Q_b$ and $Q_{gh}$ in Figure 6 are all primitive. As every face of the interface from the structured grid side is split into at least two faces in the unstructured grid side, $Q_b$ values of several unstructured grid elements are identical. And same situation occurs for $Q_{gh}$ values. When the structured grids are distorted very much near the DRAGON grid interfaces, the accuracy of assigning $Q_b$ and $Q_{gh}$ values in this way is lowered, referring to Figure 7. This is a main area that accuracy could be affected.

![Figure 6: Diagram of Q values from structured to unstructured grids.](image)

![Figure 7: Diagram of a neighborhood of the DRAGON grid interface, when the structured grid is distorted.](image)
During the flux calculation in the USM3D part, high-order spatial differencing is used for high-order cases. Referring to Figures 8 and 9, $Q$ value on Face $f_{123}$, that is, $Q_{f_{123}}$, can be deduced by Equations 12 and 13, where $Q_c$ denotes $Q$ value at the center of the element.

$$\nabla Q_c \cdot \Delta r = \frac{\partial Q}{\partial r} \Delta r = \left[ \frac{1}{3} (Q_{nd1} + Q_{nd2} + Q_{nd3}) - Q_{nd4} \right] \Delta r$$  \hspace{1cm} (12)$$

$$Q_{f_{123}} = Q_c + \text{limiter}\left\{ \frac{1}{12} (Q_{nd1} + Q_{nd2} + Q_{nd3}) - Q_{nd4} \right\}$$  \hspace{1cm} (13)$$
Figure 10: Contributions to interior nodal values.

Table 1: Various averaging schemes for nodal values.

<table>
<thead>
<tr>
<th>lapavg</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>Pseudo-Laplacian averaging on interior nodes; Inverse-distance averaging on boundary nodes. Boundary nodal values have no contributions from values of interior cell centers.</td>
</tr>
<tr>
<td>0</td>
<td>Inverse-distance averaging to all nodes. Boundary nodal values have no contributions from values of interior cell centers.</td>
</tr>
<tr>
<td>1</td>
<td>Pseudo-Laplacian averaging to all nodes. Boundary nodal values are computed from ghost cells and all interior cells sharing the boundary nodes.</td>
</tr>
<tr>
<td>2</td>
<td>Revised inverse-distance averaging, same as that of lapavg=3, at the first step. From the second step, set lapavg to 1.</td>
</tr>
<tr>
<td>3</td>
<td>Inverse-distance averaging to all nodes. Boundary nodal values are computed from ghost cells and all interior cells sharing the boundary nodes.</td>
</tr>
</tbody>
</table>
We treat the DRAGON grid interface as a boundary, however routine UBC in the USM3D code does nothing with this special boundary. Routine QNODE in the USM3D part computes distance-averaged $Q$ values of node points, and it remains almost the same even after the addition of the special boundary type. Table 1 lists five lapavg values for the QNODE routine. Figures 10 and 11 show the schemes of calculating contributions to interior and boundary nodal values.

![Diagram showing contributions to boundary nodal values.]

**Figure 11: Contributions to boundary nodal values.**

Nodal value $Q_{\text{gh}}$ may have slight influence on dtr(*) in subroutine UCTIME, when $dt \leq 0$. It will results in changes in fluxes and resid(*) in subroutine AUSM, when iorder $\geq 2$. It will results in changes in viscous fluxes in subroutine GFLUX, when ivisc $> 0$.

3.5 Flux Values near DRAGON Grid Interfaces

As mentioned before, the fluxes calculated based on the information on tetrahedral elements are used to update corresponding fluxes associated with the structured grids, due to the requirement of flux conservation on the DRAGON grid interfaces. Therefore, topology relationship between the two types of grids near the DRAGON grid interfaces is established before hand, and Figure 12 illustrates this corresponding relation.

In the OVERFLOW part, nodal $Q$ values and fluxes are stored in the way as shown in Figure 13. When periodic or annular periodic boundary conditions are encountered, attention should be paid on the $Q$ values near boundaries.

For the three-dimensional case, all the fluxes are arranged in $J-K$ planes for $J$-direction fluxes (in Files reuj.F and reuj2.F), $J-K$ planes for $K$-direction fluxes (in Files reuk.F and reuk2.F), and $J-L$ planes for $L$-direction fluxes (in Files reul.F and reul2.F). Originally, there are $(n_j - 1) \times (n_k) \times (n_l - 2)$ $J$-direction fluxes, $(n_j) \times (n_k - 1) \times (n_l - 2)$ $K$-direction fluxes, and $(n_j) \times (n_k - 2) \times (n_l - 1)$ $L$-direction fluxes. Referring to Figure 14,
some fluxes shown as shaded will not be updated, according to the design of the OVERFLOW code. Therefore, there are only \((n_j - 1) \times (n_k - 2) \times (n_l - 2)\) J-direction fluxes, \((n_j - 2) \times (n_k - 1) \times (n_l - 2)\) K-direction fluxes, and \((n_j - 2) \times (n_k - 2) \times (n_l - 1)\) L-direction fluxes, which are to be updated.

For the OVERFLOW code, the \(Q_j\) value on node \(j\) is denoted as \(Q_j\), and the corresponding fluxes near node \(j\) are denoted as \(F_{j-1}\) and \(F_j\) (Refer to Figure 15). For the first-order flux splitting/upwind option, there is

\[
F_j = f(Q_j, Q_{j+1}), \quad F_{j-1} = f(Q_{j-1}, Q_j).
\]

(14)

For the higher order cases (i.e. second- and third-order), there exists interpolated/limited states

\[
Q_j^+ = \text{Int}(Q_{j-1}, Q_j, Q_{j+1}),
\]

(15)
so that

\[ F_j = f(Q^*_j, Q^*_{j+1}), \quad F_{j-1} = f(Q^*_{j-1}, Q^*_j). \]  \hspace{1cm} (16)

Various forms for higher order interpolations and limiter functions are abundant in literature; the one used in the OVERFLOW code is the third-order MUSCL scheme with the Koren's limiter [9]. Figure 15 depicted this generic scheme of flux calculation, in terms of the nodal values used. As \( Q \) values at five nodes are taken into account, for the flux computation for the third-order option, it is necessary to construct and correct \( Q \) values at certain nodes with IBLANK value 0. By circling the targeted nodes, Figure 16 shows the correction of nodal values associated.

![Figure 15: Calculation of fluxes on structured grids.](image)

![Figure 16: Correction of some nodal values on the structured grids.](image)
4 Implementation of the Solver

4.1 Programming Philosophy

The adoption of OVERFLOW and USM3D codes for the DRAGON grid is based on the concept of domain splitting. A DRAGON grid is dealt with as a combination of a Chimera grid and a collection of unstructured grids. In the DRAGONFLOW suite, both OVERFLOW and USM3D are presented in form of module libraries, and a master module controls the invoking of these individual modules. The alternative invoking of these solvers in each time step, and the immediate data exchange on the DRAGON grid interfaces, leads to a seamless coupling of these two solvers. Figure 17 depicts the flow chart of the DRAGONFLOW suite. A significant effort has been made to add an interface and modify both codes, more so on the OVERFLOW code.

![Diagram](image)

Figure 17: Flowchart of the DRAGONFLOW procedure.

The DRAGONFLOW code keeps the structure of the OVERFLOW code. Directory chimera has been renamed as dragon. Files from USM3D code resides in a directory named usm3d. The drivers of the DRAGONFLOW is located in files

```
dragon/dragon.F
dragon/dragfl.F
```

Files, which are modified to deal with flux exchange between the OVERFLOW and USM3D parts, are listed as follows:

```
usm3d/usm.F
ns/euler/rhse.F
ns/euler/reuj.F
ns/euler/reuj2.F
ns/euler/reuk.F
```
Q values, $Q_b$ and $Q_{gh}$, associated with the tetrahedral elements near the DRAGON grid interfaces need to be updated from the structured grid side. This operation has been implemented in
dragon/interface.F

Special treatment, of correcting $Q$ values at certain nodes with IBLANK value 0, has been implemented in the following two files, for higher order cases.
ns/step/fillblk.F
ns/step/step.F

Files, which are modified to deal with viscous flux exchange between the OVERFLOW and USM3D parts, are listed as follows:
usm3d/vis513.F
ns/viscous/rhsv.F
ns/viscous/rvj.F
ns/viscous/rvj2.F
ns/viscous/rvk.F
ns/viscous/rvk2.F
ns/viscous/rl1.F
ns/viscous/rl2.F
ns/viscous/rjk.F
ns/viscous/rjk2.F
ns/viscous/rjl.F
ns/viscous/rjl2.F
ns/viscous/rkl.F
ns/viscous/rkl2.F

4.2 Extension to the Family of Boundary Conditions in the OVERFLOW

An enhancement has been made to the family of boundary conditions in the OVERFLOW part, devoted to the applications such as turbomachinery. This new type of boundary condition is referred to as annular periodic condition. In this case, a convention that $X$ axis is the reference axis to the annular periodic condition, is applied (See Figure 18). Also, it is assumed that $J - I$ planes at the ends of $K$ direction are the corresponding planes. For an annular periodic pair $(P, P')$ at $x = \text{const}$ and $r = \text{const}$, the periodic boundary condition requires $u_r(k = 1) = u_r(k = -1)$, $u_r(k = 1) = u_r(k = -1)$, and $u_\theta(k = 1) = u_\theta(k = -1)$. Files affected for this enhancement are listed as follows.
include/grdsav.h include/grdtop.h
grid/metjfv.F grid/metric.F
grid/perchk.F grid/topchk.F
Figure 18: Diagram for the annular periodic boundary condition.
5 Benchmark Tests

5.1 Shock Tube Problem

First, a 2D shock tube problem was considered. To focus only on the issue of grids, the flow was solved by the basic first-order accurate discretization in both time and space. This case serves to show the effect of interpolation in the Chimera grid and the validity of the DRAGON grid method for a transient problem as a plane shock moves across the embedded-grid region. The shock wave is moving into a quiescent region in a constant-area channel with a designed shock speed \( M_s = 4 \). Solutions were obtained using three grid systems, namely (1) single grid, (2) Chimera grid, and (3) DRAGON grid, as displayed in Figure 19. The single grid solution is used for benchmark comparison.

The pressure distributions along the centerline of the channel, as plotted in Figure 20, shows that the Chimera scheme predicts a faster moving shock in the tube, while the present DRAGON grid and the single grid results coincide, indicating that the shock is accurately captured and conservation property well preserved when going through the region of the embedded DRAGON grid.

![Figure 19: Grids for moving shock problem (\( M_s = 4 \)): (a) single grid, (b) Chimera grid, and (c) DRAGON grid.](image)
5.2 2D Supersonic Flow in a Symmetric Convergent Channel

Now we consider a supersonic flow of Mach 1.8 through a symmetric convergent channel. The configuration and pressure pattern are sketched in Figure 21, where both the top and bottom walls are bent, thus creating two wedge shocks of equal strength. The wedge angle is sufficiently large so that there is a Mach stem after the interaction of the two wedge shocks, resulting in a subsonic flow there. However, the exit flow is still supersonic, due to the acceleration of the fans emitting from the shoulders of the steps.

This test case is designed to provide a comparison in terms of conservation property between Chimera and DRAGON grids. The problem has been solved by using a single grid, two Chimera grids and a DRAGON grid, respectively. This configuration is treated as a three-dimensional geometry. Grids are generated to fill the whole domain, although the geometry is symmetric. Figure 22(a) depicts the pressure contours of the flow computed based on the single grid. Figures 22(b) and 22(c) show those computed based on two Chimera grids, I and II. The Chimera Grids I and II have the same grid point distributions as the single grid, apart from the overlapping areas. The nodal points in the overlapping areas are modestly displaced from those identical to the single grid. The difference between the Chimera Grids I and II is due to the locations of the overlapping areas. Chimera Grid II shown in Figure 22(c) is constructed in such a way that a segment of shock almost lies on one of the grid boundary. This could be the worst case that causes a large error in the simulation. However, Chimera Grid I shown in Figure 22(b) could be considered as a typical one for general applications of Chimera grids. Figure 22(d) depicts the pressure contours of the flow computed based on a DRAGON grid. This DRAGON grid is constructed based on Chimera Grid II, and a segment of shock almost lies on the DRAGON grid interface. In Figure 22(d), the contours are plotted based on the original structured grids and the corresponding unstructured grid forming the DRAGON grid. Therefore, there are discontinuities in the contour plot near the DRAGON grid interface, as there are gaps of a half cell size between the original structured grids and the unstructured grid.

Figure 23 presents the computed pressure distributions on Lines AB and CD, referring to Figure 21. It confirms that variable interpolation between grids of a Chimera grid causes the loss of conservative property, and could introduce globally significant errors in some cases. However, the DRAGON grid scheme overcomes this shortcoming.
Figure 21: Geometry and pressure pattern of a symmetric convergent channel, through which a supersonic flow is passing.

Figure 22: Pressure contours of the flow: (a) Single grid; (b) and (c) Chimera Grids I and II. (d) DRAGON grid.
5.3 3D Supersonic Flow in a Convergent Duct with Two Bent Side Walls

Next we consider a supersonic flow of Mach 3 through a convergent channel. The geometry is sketched in Figure 24, where both the top and front side walls are bent by a wedge angle of 10 degrees, thus creating two wedge shocks of equal strength and subsequent interactions between them. A strip of unstructured-gridded region, denoted by the shaded region, is placed in the mid-section of the channel. This test case is designed to provide a check on the three-dimensional code against conservation property of the DRAGON grid through a shock. The flow consists of two planar wedge shocks, two embedded shocks, a corner shock and two shear lines (slip surfaces) as shown in Figure 25, where the location of Section IJKL is depicted in Figure 24. The two wedge shocks intersect and generate, near the corner edge AD (see Figure 24), a corner shock region manifested by the slip surfaces emanating from the triple point. Figure 26 displays the density contours on the top and front side faces, and the exit plane. As seen in Figures 27 and 28, the corner shock region progressively becomes larger as the wedge shocks sweep towards the opposite walls. Eventually these two wedge shocks reflect and interact with the flow previously generated by the corner shocks, making the flow field even more complicated.

In Figures 29, 30 and 31, we give an inside view of variables on two planes: density contours on the symmetric plane ABCD (as denoted in Figure 24), and Mach number and pressure contours on the midplane EFGH. This reveals quite a rich feature of shock-shock interactions inside the duct, while the shock configuration on the duct walls (Figure 26) is relatively simple. The slip surfaces issuing from the shock triple points are evident in Figure 30 and they subsequently interact with the downstream shock.
It is evident that the shock profiles pass through the DRAGON grid interfaces seamlessly without creating spurious waves, guaranteeing the conservation property. In each of the above figures, we also include the results based on a single structured grid for validation. We see that both sets of results are essentially the same, except some minute variations in the core region, which are a remanent result of the unstructured grid. This is clearly indicated in Figure 29 where the shock becomes thinner in the unstructured grid region because the grid size is reduced roughly by $\frac{1}{6}$. A quantitative measure of the difference between the single grid and DRAGON grid solutions at the exit plane is given in Figure 32, indicating a close agreement of the two solutions.

Figure 24: Sketch of a convergent duct where the unstructured grid is in the shaded region for the DRAGON grid.

Figure 25: Flow structure in Section IJKL of the supersonic flow passing a convergent duct.
Figure 26: Density distributions of the flow on the surface of the convergent duct: (a) Single grid; (b) DRAGON grid.

Figure 27: Mach number distributions of the flow at various stations: (a) Single grid; (b) DRAGON grid.

Figure 28: Mach number distributions of the flow at various stations (split view): (a) Single grid; (b) DRAGON grid.
Figure 29: Density distributions of the flow at the symmetric plane ABCD: (a) Single grid; (b) DRAGON grid.

Figure 30: Mach number distributions of the flow at a typical cutting plane EFGH: (a) Single grid; (b) DRAGON grid.

Figure 31: Pressure distributions of the flow at a typical cutting plane EFGH: (a) Single grid; (b) DRAGON grid.

Figure 32: (a) Density, (b) Mach number and (c) pressure distributions of the flow on Line HG. Boxes and circles denote results based on the single grid and the DRAGON grid, respectively.
5.4 Subsonic Flow through a Wavy-Wall Duct

We consider a subsonic flow in a wavy-wall duct, of which the problem setting is illustrated in Figure 33. This test case is designed to provide a check on the three-dimensional code against conservation property of the DRAGON grid for a subsonic flow. We have performed computational simulation based on a single grid and a DRAGON grid, respectively. The DRAGON grid has unstructured grid regions of a shape as Label "3D", and the geometry is sketched in Figure 34. Figure 35 provides additional views of the DRAGON grid. As the result, Figures 36, 37, and 38 depict the density, Mach number and pressure distributions for both cases based on the single grid and the DRAGON grid. It is evident that the results of both cases are essentially the same. The flow passes through the DRAGON grid interfaces seamlessly without creating spurious diffusion, and this guarantees the conservation property.

And further, Figure 39 shows a quantitative measure of the difference between the single grid and DRAGON grid solutions on lines AB and CD, which are defined in Figure 34. This figure indicates a close agreement of the two solutions in terms of density, Mach number and pressure distributions.

![Figure 33: Problem setting of flow in the wavy-wall duct.](image)

![Figure 34: Sketch of a wavy-wall duct where the unstructured grid is in the shaded region for the DRAGON grid.](image)
Figure 35: (a) A view of the DRAGON grid; (b) A layer of the structured grid in the DRAGON grid.

Figure 36: Density distributions of the flow in the wavy-wall duct: (a) Single grid; (b) DRAGON grid.

Figure 37: Mach number distributions of the flow in the wavy-wall duct: (a) Single grid; (b) DRAGON grid.

Figure 38: Pressure distributions of the flow on the surface of the wavy-wall duct: (a) Single grid; (b) DRAGON grid.
Figure 39: (a) Density, (b) Mach number and (c) pressure distributions of the flow on Lines AB (boxes) and CD (circles). Doted and solid lines denote results based on the single grid and the DRAGON grid, respectively.

6 Numerical Simulations

6.1 Inviscid Flow around a Linear Cascade

We have conducted a 3D calculation of an inviscid flow around a cascade in a turning duct. This solution exhibits the capability of DRAGON grid scheme in dealing with problems involving complex geometries. Also calculation based on a Chimera grid, which is the base grid of the DRAGON grid, has been conducted in order to make a comparison. Figure 40 illustrates the 3D DRAGON and Chimera grids for the computational domain of concern, and Figure 41 shows the unstructured grid of the corresponding DRAGON grid. Figures 42, 43 and 44 depict the pressure, density and Mach number contours of the solutions. Finally Figure 45 shows the pressure distributions on the surfaces of the six vanes, where (a), (b), (c), (d), (e) and (f) are for the first to last vanes from the center outwards, respectively. While no measurements are available for comparison, both Chimera and DRAGON grid solutions, at least serving as a code-to-code validation, are in agreement with each other, especially on the pressure side of the vanes. But there are noticeable differences on the suction side of the vanes. From the contour plots, Figures 42-44, The DRAGON grid solutions appear to be smoother and free of noises emanating from the interpolation surfaces.
Figure 40: 3D DRAGON and Chimera grids of the cascade.

Figure 41: Unstructured grid of the DRAGON grid of the cascade.

Figure 42: Pressure distributions of solutions based on the DRAGON and Chimera grids.
Figure 43: Density distributions of solutions based on the DRAGON and Chimera grids.

Figure 44: Mach number distributions of solutions based on the DRAGON and Chimera grids.
Figure 45: Pressure distributions around the six vanes: (a), (b), (c), (d), (e) and (f) are for the first to last vanes from the center outwards, respectively.
6.2 Viscous Flow through an Annular Cascade

Lastly, we show the results of a flow simulation for an annular cascade of turbine stator vanes developed and tested at NASA Lewis [6]. The annular ring has 36 untwisted vanes, of constant profile from hub to tip, with a hub-tip radius ratio of 0.85 and a tip diameter of 508 mm. The vanes themselves are 38.10 mm high, and have an axial chord of 38.23 mm and a blade chord of 55.52 mm (See Figure 46). Design flow conditions are for a fully axial inflow with an exit-hub-static to inlet-total pressure ratio of 0.6705. The designed inflow Mach number is 0.2. The Reynolds number based on the axial chord is $1.73 \times 10^6$[3], or $4.522 \times 10^6$ in a unit of meter used as an input value for the solver. These conditions correspond to average inlet and exit Mach numbers of 0.211 and 0.697, respectively. However, as in [3] we assume the free stream Mach number of 0.2 in the calculation.

A DRAGON grid as shown in Figures 47 and 48 is used for the flow simulation. It consists of a background H-type grid placed to cover the channel between the vanes, an O-type viscous grid used to resolve the region around the vane, and an unstructured grid (See Figure 49) located in the overlapping region between the H- and O-type grids. The H-type grid has $60 \times 31 \times 33$ points, and O-type grid $85 \times 20 \times 33$ points. All the three component grids have viscous layers near the hub and the tip. The initial spacing at the wall is 0.0002 of a blade chord, that is, 0.0111 mm.

Previous calculations have been obtained using a single structured grid [2, 3]. The computed surface static pressure distributions of the present DRAGON grid solution are plotted in Figure 50, and they are seen to be in very good agreement with the measured data [6] at three spanwise locations, 13.3%, 50%, and 86.6%, respectively. Note that the plot shown in Figure 50 has been scaled with respect to the inlet-total pressure $P_0$. Figure 51 depicts the pressure contours on the vane surfaces and on the hub. The pressure side displays a basically two-dimensional behavior while a significant spanwise variation is seen on the suction side.
Figure 46: Core turbine stator vane geometry at the mean section.

Figure 47: 3D DRAGON grid for a turbine vane.
Figure 48: DRAGON grid of the annular turbine cascade.

Figure 49: Unstructured grid of the corresponding DRAGON grid.
Figure 50: Computed and measured static pressure distributions at 13.3%, 50% and 86.6% of span from hub.

Figure 51: Pressure distribution for the annular turbine cascade showing two vanes.
7 Concluding Remarks

In the present work, we have concentrated on the extension of the DRAGON grid method into three-dimensional space. This method attempts to preserve the advantageous features of both the structured and unstructured grids, while eliminating or minimizing their respective shortcomings. The flow solutions confirm the satisfaction of conservation property through the interfaces of structured-unstructured regions, and the results are in very good agreement with the measured data, thus demonstrating the reliability of the method. Future plan includes further validations and applications to engineering problems with complex geometry.
Appendix

File Descriptions

For the execution of the DRAGONFLOW code, information on topology relation between the two types of grids near the DRAGON grid interface is required. This information is provided through a set of four files, that is, *.fls, *.fdef, *.fcm and *.pcm.

File *.fls provides information on face list numbers. Its file format is as follows:

```
NGRID_STR
DO IG=1,NGRID_STR
   JDIM(IG),KDIM(IG),LDIM(IG)
   DO II=1,LDIM(IG)
      NFMLS=NFMLS+1
      II,LFMLS(NFMLS)
   ENDDO
   DO II=1,LDIM(IG)
      NFMLS=NFMLS+1
      II,LFMLS(NFMLS)
   ENDDO
   DO II=1,KDIM(IG)
      NFMLS=NFMLS+1
      II,LFMLS(NFMLS)
   ENDDO
ENDDO
```

Note:

```
LFMLS(*)  --- numbers of quadrilateral faces in i, j, and k-directions, respectively. The faces are grouped according to loops DO II=1,LDIM(IG), DO II=1,LDIM(IG) and DO II=1,KDIM(IG).
```
Face definition file *.fdef provides definition information for DRAGON interfaces. Every interface can be defined as \((i, j, k; \text{idir})\) as shown in Figure 5. The file format is as follows:

```plaintext
NSURF
DO IS=1,NSURF
   ISURQ(IS),LFDEF(IS,IM),IM=1,5
ENDDO

Notes:
ISURQ       --- overall numbering of quadrilateral faces;
LFDEF(IS,1) --- i;
LFDEF(IS,2) --- j;
LFDEF(IS,3) --- k;
LFDEF(IS,4) --- idir (1, -1, 2, -2, 3, -3); and
LFDEF(IS,5) --- ig_str;
```

File *.fcm provides information on face couple mapping. Its file format is as follows:

```plaintext
NSURF_I,NSURT_I
DO J=1,NSURF_I
   (LFPCM(J,IM),IM=1,4)
ENDDO
DO J=1,NSURT_I
   LFVCT(J)
ENDDO

Notes:
LFPCM(J,1) --- index of the quadrilateral face. NSURQ faces are ordered according to the *.fdef file.
LFPCM(J,2) --- ID of an unstructured volume grid.
LFPCM(J,3) --- starting index of slave (triangular) faces.
LFPCM(J,4) --- ending index of slave (triangular) faces.
LFVCT(J) --- a pointer to overall face list.
   That is ID of a slave face.
```

Point couple mapping file *.pcm provides information on point couple mapping on DRAGON interfaces, with respect to fictitious grids. The file format is as follows:

```plaintext
NSURQ
DO IP=1,NSURQ
   IP,IPRTET,IG_UNS,IG_STR,J,K,L
ENDDO

Notes:
NSURQ     --- Number of points on quadrilateral DRAGON interface.
IP         --- index of the point on quadrilateral DRAGON interface.
IPRTET     --- index of surface point of the unstructured grid.
IG_UNS     --- unstructured grid ID.
IG_STR     --- structured grid ID.
J,K,L      --- definition of the point in the fictitious grid.
```

Generation of Relation Files

Four auxiliary files are used for creating information contained in Files *.fis, *.fdef, *.fcm and *.pcm. These files are named as *.fat, *.pat, *.pcm0 and *.vis.

Face attribution file *.fat provides information on attributions of DRAGON interfaces. Every triangular face can be defined as \((v_1, v_2, v_3)\), and the file format is as follows:
NSURF
DO IS=1,NSURF
   IS,V1,V2,V3,BCID,PATCHID,ISDRAGON
ENDDO
Notes:
V1,V2,V3    --- definition of a triangular face. The nodal numbering
            is based on the surface triangulation.
BCID        --- boundary type.
PATCHID      --- patch ID, which is the index of a quadrilateral face.
ISDRAGON    --- flag indicating whether the face is on
            the DRAGON interface.

Point attribute file *.pat provides information about attributes of points on DRAGON
grid interfaces, with respect to fictitious grids. The file format is as follows:

NPSURQ
DO IP=1,NPSURQ
   IPSURQ,IG_STR,J,K,L
ENDDO
Notes:
NPSURQ    --- Number of points on quadrilateral DRAGON interface.
IG_STR    --- index of the point on quadrilateral DRAGON interface.
J,K,L      --- definition of the point in the fictitious grid.
Note that faces are grouped according to their parents, that is, patchids.

Initial point couple mapping file *.pcm0 provides information on point couple mapping on
DRAGON interfaces. The file format is as follows:

NPSURQ
DO IP=1,NPSURQ
   IPTET,IPSURQ
ENDDO
Notes:
NPSURQ    --- Number of special points on the surface of the unstructured
            grid, which match to points on quadrilateral DRAGON interface
            (NPSURQ = NPSUR_TET).
IPTET      --- index of surface point of the unstructured grid.
IPSURQ     --- index of the point on quadrilateral DRAGON interface.

For cases with viscous flows involved, viscous layers can be generated based on an intermediate
unstructured grid. In these cases, additional information is required, which is written in a viscous
layer information file *.vis. The file format is as follows:

NPSURQ_I,NSURQ_I
DO IS=1,NSURQ_I
   ISMAP
ENDDO
DO IP=1,NPSURQ_I
   COORD(1),COORD(2),COORD(3)
ENDDO
DO IP=1,NPSURQ_I
   (IPATY(J),J=1,4)
ENDDQ

Notes:

NPSURQ_I    --- Number of points on quadrilateral DRAGON interface.
NSURQ_I     --- Number of patches on quadrilateral DRAGON interface.
ISMAP       --- Numbering index of surface patches. The numbering splits patches into groups, which correspond to areas of unstructured grids and viscous layers. The former is with positive indices while the later negative indices.
COORDQ(*)   --- Coordinates of points on quadrilateral DRAGON interface.
IPATT       --- Attribution of the points on quadrilateral DRAGON interface (IG_STR,J,K,L)
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A Flow Solver for Three-Dimensional DRAGON Grids

Meng-Sing Liou and Yao Zheng

National Aeronautics and Space Administration
John H. Glenn Research Center at Lewis Field
Cleveland, Ohio 44135–3191

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DRAGONFLOW code has been developed to solve three-dimensional Navier-Stokes equations over a complex geometry whose flow domain is discretized with the DRAGON grid—a combination of Chimera grid and a collection of unstructured grids. In the DRAGONFLOW suite, both OVERFLOW and USM3D are presented in form of module libraries, and a master module controls the invoking of these individual modules. This report includes essential aspects, programming structures, benchmark tests and numerical simulations.

Computational fluid dynamics; Grid generation; Turbomachinery; Turbine vanes; Cascade flow; Supersonic flow; Subsonic flow; Viscous flow