A NEW APPROXIMATE
CHIMERA DONOR CELL SEARCH ALGORITHM

4TH SYMPOSIUM ON OVERSET COMPOSITE GRID
& SOLUTION TECHNOLOGY
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PRESENTATION OUTLINE

OBJECTIVE/MOTIVATION

APPROACH
  > NUMERICAL APPROACH
  > DONOR CELL SEARCH ALGORITHM

TWO-ZONE ISOLATED WING RESULTS

THREE-ZONE WING/BODY RESULTS

CONCLUDING REMARKS
OBJECTIVE/MOTIVATION

OBJECTIVE:
> TO DEVELOP CHIMERA-BASED FULL POTENTIAL METHODOLOGY WHICH IS COMPATIBLE WITH OVERFLOW (EULER/NAVIER-STOKES CHIMERA FLOW SOLVER)
> TO DEVELOP A FAST DONOR CELL SEARCH ALGORITHM THAT IS COMPATIBLE WITH THE CHIMERA FULL POTENTIAL APPROACH

MOTIVATION:
> DESIRE TO SIGNIFICANTLY SPEED TURNAROUND TIME FOR AERODYNAMIC ANALYSIS AND DESIGN
> FULL-POTENTIAL+BL MUCH FASTER THAN NAVIER-STOKES (UP TO X100 FASTER)
> CHIMERA FULL POTENTIAL COULD USE EXISTING OVERFLOW INFRASTRUCTURE (GEOMETRY SETUP AND POST PROCESSING SOFTWARE)
> USER WILL HAVE FLOW SOLVER OPTION:
  - FULL POTENTIAL
  - EULER
  - NAVIER-STOKES
  (UTILIZE NAVIER-STOKES ONLY WHEN REQUIRED AND MORE APPROXIMATE FP APPROACH WHEN APPROPRIATE)
NUMERICAL APPROACH

> GOVERNING EQUATIONS: CONSERVATIVE FULL POTENTIAL EQUATION MAPPED FROM PHYSICAL DOMAIN $(X, Y, Z)$ TO COMPUTATIONAL DOMAIN $(\xi, \eta, \zeta)$

\[
\begin{align*}
\xi &= \xi(X, Y, Z) \\
\eta &= \eta(X, Y, Z) \\
\zeta &= \zeta(X, Y, Z)
\end{align*}
\]

> SPATIAL DIFFERENCING SCHEME:
- SUBSONIC FLOW: CENTRAL DIFFERENCING (SECOND-ORDER ACCURATE)
- SUPersonic FLOW (TWO OPTIONS):
  – FIRST-ORDER UPWIND
  – SECOND-ORDER UPWIND USING A SOLUTION LIMITER

> ITERATION SCHEME: ZONE-BY-ZONE FULLY IMPLICIT AF2 SCHEME

> ZONAL CHARACTERISTICS:
- IBLANK ARRAY MODIFICATIONS MADE IN SPATIAL AND ITERATION SCHEMES AS APPROPRIATE
- STANDARD CHIMERA UPDATES AT ALL ZONAL INTERFACES (TRI-LINEAR INTERPOLATION ON THE VELOCITY POTENTIAL)
- DENSITY EXTRAPOLATED AT ZONAL INTERFACES
GOVERNING EQUATIONS

CARTESIAN COORDINATES

\[(\rho \phi_x)_x + (\rho \phi_y)_y + (\rho \phi_z)_z = 0\]

\[\rho = \left[1 - \frac{\gamma - 1}{\gamma + 1}(\phi_x^2 + \phi_y^2 + \phi_z^2)\right]^{\frac{1}{\gamma - 1}}\]

TRANSFORMED COORDINATES \([\xi = \xi(X,Y,Z), \eta = \eta(X,Y,Z), \zeta = \zeta(X,Y,Z)]\)

\[\left(\frac{\rho U}{J}\right)_\xi + \left(\frac{\rho V}{J}\right)_\eta + \left(\frac{\rho W}{J}\right)_\zeta = 0\]

\[\rho = \left[1 - \frac{\gamma - 1}{\gamma + 1}(U\phi_\xi + V\phi_\eta + W\phi_\zeta)\right]^{\frac{1}{\gamma - 1}}\]

CONTRAVARIANT VELOCITY COMPONENTS

\[U = A_1 \phi_\xi + A_4 \phi_\eta + A_5 \phi_\zeta, \quad V = A_4 \phi_\xi + A_2 \phi_\eta + A_6 \phi_\zeta, \quad W = A_5 \phi_\xi + A_6 \phi_\eta + A_3 \phi_\zeta\]

METRIC QUANTITIES

\[A_1 = \nabla \xi \cdot \nabla \xi, \quad A_2 = \nabla \eta \cdot \nabla \eta, \quad A_3 = \nabla \zeta \cdot \nabla \zeta\]

\[A_4 = \nabla \xi \cdot \nabla \eta, \quad A_5 = \nabla \xi \cdot \nabla \zeta, \quad A_6 = \nabla \eta \cdot \nabla \zeta\]

\[J = \xi_x \eta_y \zeta_z + \xi_y \eta_z \zeta_x + \xi_z \eta_x \zeta_y - \xi_x \eta_y \zeta_z - \xi_y \eta_z \zeta_x - \xi_z \eta_x \zeta_y\]
SPATIAL DISCRETIZATION SCHEME

\[ L_{\phi_{i,j,k}}^n = \delta \xi \left( \frac{\tilde{p} U}{J} \right)_{i+1/2,j,k}^n + \delta \eta \left( \frac{\rho V}{J} \right)_{i,j+1/2,k}^n + \delta \zeta \left( \frac{\rho W}{J} \right)_{i,j,k+1/2}^n = 0 \]

\[ \tilde{p}_{i+1/2,j,k} = \rho_{i+1/2,j,k} - \nu_{i+1/2,j,k} \left[ \rho_{i+1/2,j,k} - \rho_{i-1/2,j,k} - \Psi_{i+1/2,j,k} (\rho_{i-1/2,j,k} - \rho_{i-3/2,j,k}) \right] \]

\[ \nu_{i+1/2,j,k} = \begin{cases} 2.46625(2 \rho^* - \rho_{i+1/2,j,k} - \rho_{i-1/2,j,k})C & \text{if } \rho_{i,j,k} \leq \rho^* \\ 0 & \text{if } \rho_{i,j,k} > \rho^* \end{cases} \]

\[ \Psi_{i+1/2,j,k} = \begin{cases} 1 - C_2 \Delta & \text{if } r_{i+1/2,j,k} \geq 0 \\ 0 & \text{if } r_{i+1/2,j,k} < 0 \end{cases} \]

\[ r_{i+1/2,j,k} = \frac{\rho_{i+1/2,j,k} - \rho_{i-1/2,j,k}}{\rho_{i-1/2,j,k} - \rho_{i-3/2,j,k}} \]

\[ \Psi_{i+1/2,j,k} = \Psi_{i+1/2,j,1} (C_3)^{-(k-1)} \]
AF2 CHIMERA ITERATION SCHEME
(C-GRID TOPOLOGIES)

UPPER SURFACE (FLOW ALIGNED WITH \( \xi \) DIRECTION):

SWEEP 1:
\[
(\alpha - IB_{i,j,k} \delta \xi) f_{i,j,k}^n = IB_{i,j,k} \alpha \omega L \phi_{i,j,k}^n
\]

SWEEP 2:
\[
(\alpha - IB_{i,j,k} \delta \eta \eta) g_{j,k}^n = f_{i,j,k}^n + IB_{i,j,k} \alpha^2 C_{i-1,j,k}^n
\]

SWEEP 3:
\[
(\alpha - IB_{i,j,k} \delta \zeta \zeta) C_{i,j,k}^n = g_{j,k}^n
\]

LOWER SURFACE (FLOW ALIGNED WITH - \( \xi \) DIRECTION):

SWEEP 1
\[
(\alpha + IB_{i,j,k} \delta \xi) f_{i,j,k}^n = IB_{i,j,k} \alpha \omega L \phi_{i,j,k}^n
\]

SWEEP 2:
\[
(\alpha - IB_{i,j,k} \delta \eta \eta) g_{j,k}^n = f_{i,j,k}^n + IB_{i,j,k} \alpha^2 C_{i+1,j,k}^n
\]

SWEEP 3:
\[
(\alpha - IB_{i,j,k} \delta \zeta \zeta) C_{i,j,k}^n = g_{j,k}^n
\]
CHIMERA
INTERPOLATION
SCHEME

SCHEME CHARACTERISTICS:

> TRI-LINEAR INTERPOLATION OF
  VELOCITY POTENTIAL USED TO
  OBTAIN FRINGE POINT VALUES

> IBLANK ARRAY USE IDENTICAL TO
  TRADITIONAL CHIMERA APPROACH

> SPECIAL DENSITY COMPUTATION
  REQUIRED AT FRINGE POINTS

> SPECIAL LOGIC REQUIRED WHEN
  PERFORMING DONOR CELL
  SEARCHES FOR FRINGE POINTS ON
  VORTEX SHEET
DONOR CELL SEARCH ALGORITHM

PROBLEM STATEMENT

GIVEN LOCATION OF ARBITRARY IGBP (INTERGRID BOUNDARY POINT)

\[ x^r, y^r, z^r \]

FIND THE THREE INDICES \( i^d, j^d, k^d \) THAT DEFINE THE CELL SURROUNDING 
\( x^r, y^r, z^r \)

THIS "DONOR CELL" IS DEFINED BY THE \( x, y, z \) VALUES AT THE FOLLOWING 8 POINTS:

\[ i^d, j^d, k^d \]
\[ i^d, j^d + 1, k^d \]
\[ i^d + 1, j^d, k^d \]
\[ i^d + 1, j^d + 1, k^d + 1 \]

\[ i^d, j^d + 1, k^d \]
\[ i^d + 1, j^d, k^d + 1 \]
\[ i^d, j^d, k^d + 1 \]
\[ i^d + 1, j^d + 1, k^d \]
DONOR CELL SEARCH ALGORITHM
BASIC IDEA

RELATIONAL INFORMATION BETWEEN COMPUTATIONAL DOMAIN \((\xi, \eta, \zeta)\) AND
PHYSICAL DOMAIN \((x, y, z)\)

\[
\begin{align*}
  d\xi &= \xi_x dx + \xi_y dy + \xi_z dz \\
  d\eta &= \eta_x dx + \eta_y dy + \eta_z dz \\
  d\zeta &= \zeta_x dx + \zeta_y dy + \zeta_z dz
\end{align*}
\]

USE NUMERICAL APPROXIMATION TO COMPUTE DONOR CELL LOCATION

\[
\begin{align*}
  i^d - i^n &= \xi_x^n (x^r - x^n) + \xi_y^n (y^r - y^n) + \xi_z^n (z^r - z^n) \\
  j^d - j^n &= \eta_x^n (x^r - x^n) + \eta_y^n (y^r - y^n) + \eta_z^n (z^r - z^n) \\
  k^d - k^n &= \zeta_x^n (x^r - x^n) + \zeta_y^n (y^r - y^n) + \zeta_z^n (z^r - z^n)
\end{align*}
\]

WHERE \(\xi_x^n, \xi_y^n, \text{etc.}\) ARE STANDARD NUMERICALLY EVALUATED METRICS AND
\(x^n, y^n, z^n\) IS AN ARBITRARY STARTING POINT IN THE DONOR GRID. TO IMPROVE
ACCURACY THESE QUANTITIES ARE EVALUATED AT CELL CENTERS, i.e., AT
\(i^n + 1/2, j^n + 1/2, k^n + 1/2\)
DONOR CELL SEARCH ALGORITHM

ALTERNATE NOTATION

\[ M^d = M^n + H^n (R^r - R^n) \]

WHERE

\[ M = \begin{pmatrix} i \\ j \\ k \end{pmatrix}, \quad H = \begin{pmatrix} \xi_x & \xi_y & \xi_z \\ \eta_x & \eta_y & \eta_z \\ \zeta_x & \zeta_y & \zeta_z \end{pmatrix}, \quad R = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \]

MODIFICATION FOR INTEGER CHOPPING

\[ M^d = M^n + H^n (R^r - R^n) + l^n_s \]

WHERE \( l^n_s \) IS A COLUMN MATRIX DEFINED BY \( l^n_s = 0.5 \text{sgn}[H^n (R^r - R^n)] \)

WHEN STARTING CELL \( x^n, y^n, z^n \) IS FAR REMOVED FROM DESIRED DONOR CELL, MUST ITERATE

\[ M^{n+1} = M^n + H^n (R^r - R^n) + l^n_s, \quad n = 1, 2, \ldots, \text{MAXIT} \]

WHERE \( \text{MAXIT} \sim 10 \)
DONOR CELL SEARCH ALGORITHM

GRID INDEX LIMITS:

if \( M^{n+1} < 1 \) then \( M^{n+1} = 1 \)
if \( M^{n+1} > M_{\text{max}} - 1 \) then \( M^{n+1} = M_{\text{max}} - 1 \)

DONOR CELL CONVERGENCE CRITERIA:

\[ |H^n (R^r - R^n) + I_s^n| \leq 1.0 + \text{TOL} \quad \text{WHERE TOL} \sim 0.05 \]

NEAREST NEIGHBOR CONVERGENCE CRITERIA:

IF DONOR CELL CONVERGENCE CRITERIA IS NOT SATISFIED AND

\( n = \text{MAXIT} \)

OR

\( i^{n+1} = i^n, \quad j^{n+1} = j^n, \quad k^{n+1} = k^n \)

THEN AN ALTERNATE CRITERIA IS USED

\[ |H^n (R^r - R^n) + I_s^n| \leq 2.0 \]
# DONOR CELL SEARCH STATISTICS

**ONERA M6 WING, \(M_\infty = 0.84, \alpha = 3.06^\circ\)**  
**FIRST LINE IN EACH TABLE FROM EXACT SEARCH ALGORITHM**

**FINE GRID: INNER GRID = 201X41X17, OUTER GRID = 109X49X74, IGBP=15299**

<table>
<thead>
<tr>
<th>TOL</th>
<th>NMISS</th>
<th>NNN</th>
<th>NAVG</th>
<th>CPU (sec)</th>
<th>MFLOPS</th>
<th>CL</th>
<th>IT</th>
<th>CD</th>
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</thead>
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**COARSE GRID: INNER GRID = 101X23X9, OUTER GRID = 55X25X38, IGBP=4259**

<table>
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<tr>
<th>TOL</th>
<th>NMISS</th>
<th>NNN</th>
<th>NAVG</th>
<th>CPU (sec)</th>
<th>MFLOPS</th>
<th>CL</th>
<th>IT</th>
<th>CD</th>
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</table>

**NMISS** - NUMBER OF IGBPs OUTSIDE THEIR DONOR CELLS AS DETERMINED BY EXACT SEARCH ALGORITHM  
**NNN** - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONOR CELLS AS DETERMINED BY NEAREST NEIGHBOR TEST  
**NAVG** - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP
MACH NUMBER CONTOUR COMPARISONS
WING ROOT SYMMETRY PLANE
ONERA M6 WING, $M_\infty = 0.84, \alpha = 3.06^\circ$

TOL = 0.0

TOL = 1.0
DONOR CELL SEARCH STATISTICS

GRID REFINEMENT STUDY
THREE GRID-ZONE CASE
RAE WING + B2 BODY
TOL=0.1

<table>
<thead>
<tr>
<th>GRID</th>
<th>GRIDS</th>
<th>TOTAL POINTS</th>
<th>IGBP</th>
<th>NNN</th>
<th>NAVG</th>
<th>CPU</th>
<th>MFLOPS</th>
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<td>201X41X17</td>
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<td>101X41X62</td>
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<td>26423</td>
<td>6</td>
<td>4.99</td>
<td>0.714</td>
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</table>

IGBP - INTERGRID BOUNDARY POINTS
NNN - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONORS
NAVG - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP
CPU - CPU TIME (SEC) ON A SINGLE-PROCESSOR CRAY C-90
MFLOPS - PROCESSING RATE (MILLION FLOATING POINT OPERATIONS PER SEC)
DONOR CELL SEARCH STATISTICS
RAE WING A WITH B2 FUSELAGE

THREE-ZONE GRID
WING GRID = 201X41X17, FUSELAGE GRID = 145X25X58, OUTER GRID = 101X41X62
IGBP = 26423

<table>
<thead>
<tr>
<th>TOL</th>
<th>NNN</th>
<th>NAVG</th>
<th>CPU (Sec)</th>
<th>MFLOPS</th>
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NNN - NUMBER OF NEAREST NEIGHBOR CELLS USED AS DONORS
NAVG - NUMBER OF DONOR CELL SEARCH ITERATIONS PER IGBP
WING SURFACE PRESSURE COMPARISONS

RAE WING A + BODY B2
\[ M_\infty = 0.82, \alpha = 2^\circ \]

THREE-ZONE GRID
- WING GRID = 201X41X17
- FUSELAGE GRID = 145X25X58
- OUTER GRID = 101X41X62

\[ t_{R\text{AVG}} = 73 \text{ sec} \]
\[ t_{OH} = 17 \text{ sec} \]

2Y/B = 0.40

2Y/B = 0.17

2Y/B = 0.85
FUZELAGE SURFACE PRESSURE COMPARISONS
RAE WING A + BODY B2, $M_{\infty} = 0.82, \alpha = 2^\circ$

$\varphi = -15^\circ$

$\varphi = +0^\circ$

$\varphi = -0^\circ$

$\varphi = +15^\circ$
MACH NUMBER CONTOURS

RAE WING A WITH B2 FUSELAGE, $M_\infty = 0.9$, $\alpha = 0^\circ$
WING GRID = 201X41X17, FUSELAGE GRID = 145X25X58, OUTER GRID = 101X41X62
CONCLUDING REMARKS

> A NEW DONOR CELL SEARCH ALGORITHM SUITABLE FOR USE WITH A CHIMERA-BASED FULL POTENTIAL SOLVER HAS BEEN PRESENTED AND EVALUATED

> NEW SEARCH ALGORITHM IS EXTREMELY FAST AND SIMPLE PRODUCING DONOR CELLS AS FAST AS 60,000 PER SEC (CRAY C-90 SINGLE PROCESSOR)

> NEW SEARCH ALGORITHM IS APPROXIMATE, IE, DONOR CELLS MAY BE APPROXIMATED BY A NEAREST NEIGHBOR CELL

  - ON A FINE GRID THIS ONLY HAPPENS 3-4% OF THE TIME WITH NO IMPACT ON SOLUTION ACCURACY OR CONVERGENCE EFFICIENCY

  - EVEN WHEN THE NUMBER OF NEAREST NEIGHBOR CELLS USED TO APPROXIMATE ACTUAL DONOR CELLS IS FORCED UP TO 70%, THERE IS NO IMPACT ON ACCURACY OR CONVERGENCE EFFICIENCY
FUTURE DIRECTIONS

> MULTI-ZONE COMPLEX GEOMETRY APPLICATIONS
> GLOBALLY IMPLICIT SCHEMES/MULTI-GRID SCHEMES
> BOUNDARY LAYER APPLICATIONS
> WIND TUNNEL WALL AND MODEL SUPPORT COMPUTATIONS
> PARALLELIZATION STUDIES
> STRUCTURES COUPLING
> DESIGN/OPTIMIZATION APPLICATIONS