PITCHED AND YAWED CIRCULAR JETS IN CROSS-FLOW

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ABSTRACT

Results from an experimental investigation of flow field generated by pitched and yawed jets discharging from a flat plate into a cross-flow are presented. The circular jet was pitch at $\alpha = 20^\circ$ and $45^\circ$ and yawed between $\beta = 0^\circ$ and $90^\circ$ in increments of $15^\circ$. The measurements were performed with two X-wires providing all three components of velocity and turbulence intensity. These data were obtained at downstream locations of $x = 3, 5, 10$ and $20$, where the distance $x$, normalized by the jet diameter, is measured from the center of the orifice. Data for all configurations were acquired at a momentum-flux ratio $J = 8$. Additionally, for selected angles and locations, surveys were conducted for $J = 1.5, 4$, and $20$. As expected, the jet penetration is found to be higher at larger $\alpha$. With increasing $\beta$ the jet spreads more. The rate of reduction of peak streamwise vorticity, $\omega_{max}$, with the downstream distance is significantly lessened at higher $\beta$ but is found to be practically independent of $\alpha$. Thus, at the farthest measurement station $x = 20, \omega_{max}$ is about five times larger for $\beta = 75^\circ$ compared to the levels at $\beta = 0^\circ$. Streamwise velocity within the jet-vortex structure is found to depend on the parameter $J$. At $J = 1.5$ and $4$, ‘wake-like’ velocity profiles are observed. In comparison, a ‘jet-like’ overshoot is present at higher $J$.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$D$</td>
<td>Nozzle diameter</td>
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<tr>
<td>$J$</td>
<td>Momentum-flux ratio, $J = (\rho_j U_j^2)/(\rho U_\infty^2)$</td>
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<tr>
<td>$U$</td>
<td>Mean jet or free-stream velocity normalized by $U_\infty$</td>
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<tr>
<td>$VR$</td>
<td>Velocity ratio, $VR = U_j/U_\infty$</td>
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<tr>
<td>$u, v, w$</td>
<td>Mean streamwise, normal and spanwise velocity components normalized by $U_\infty$</td>
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<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates normalized by $D$</td>
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Greek Symbols

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<tr>
<th>Symbol</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>Jet pitch angle relative to tunnel floor, degrees</td>
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<tr>
<td>$\beta$</td>
<td>Jet yaw angle relative to direction of cross-flow, degrees</td>
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<tr>
<td>$\rho$</td>
<td>Density</td>
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<tr>
<td>$\omega$</td>
<td>Vorticity normalized by $D/U_\infty$</td>
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Subscripts

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<tr>
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<tr>
<td>$j$</td>
<td>Jet</td>
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<tr>
<td>$max$</td>
<td>Maximum</td>
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<td>$\infty$</td>
<td>Free stream</td>
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INTRODUCTION

Jets in cross-flow (JICF) have applications in a variety of technologically important systems and processes. In one form or another JICF is involved in active flow control, aircraft performance and stability, mixing augmentation, film and effusion cooling, etc. Before discussing the objectives of this study, we will review the flow features of JICF and some pertinent works from the literature. The presence of the high momentum transverse jet in a cross flow has the similar effect as that of a solid body. The retarded flow at the jet’s ‘leading’ edge creates an increased pressure, while the ‘trailing’ edge is characterized by low pressure. The cross-flow deflects the jet into its characteristic trajectory and deforms the jet cross-section. At the same time, the cross flow shears the jet fluid around its perimeters, and the resulting vorticity distribution ultimately develops into a counter-rotating vortex pair. It has been shown that this streamwise vortex pair, which is the salient feature of a JICF, can persist for hundreds of diameters downstream.

Investigation of the flow field of an inclined JICF dates back to the 1950s, when Wallis [1] showed that a pitched and yawed jet produces a vortex system similar to one from a wing-type vortex-generator. Wu et al. [2] documented the flow
topology of normal jets with different cross-sectional shape and $1 \leq VR < 9$ using flow visualization techniques. Note that for incompressible flow, the velocity ratio $VR$ is the square root of the momentum-flux ratio $J$. In Ref. 2, comparison of flow fields was made in an attempt to identify conditions that enhance asymmetry. Johnston and Nishi [3] studied jets pitched at $45^\circ$ and yawed at $90^\circ$ and $180^\circ$ over a range of velocity ratios, $0.4 \leq VR \leq 1$. The emphasis was on the investigation of potential active flow control method as an alternative to the existing solid vortex generators. Lin et al. [4] examined $45^\circ$ pitched jets at $1.7 \leq VR \leq 6.8$, as part of a comparative study on passive and active methods for flow control. Compton and Johnston [5] investigated the development of the mean velocity field from a single jet at $45^\circ$ pitch, and at yaw angles up to $180^\circ$. Velocity ratios were varied from 0.7 to 1.3. The study indicated that an optimal yaw angle producing maximum vorticity might be between 45 and 90 degrees. Honami et al. [6] carried out a study of a jet at $\alpha = 30^\circ$, $\beta = 90^\circ$, and $0.5 \leq VR \leq 1.2$, for film cooling purposes. Increase in velocity ratio was shown to enhance asymmetry of the vortical system and reduce film-cooling effectiveness.

While the aforementioned investigations focused on the mean flow features, Zhang [7] carried out LDA measurements for pitch and yaw angle of $45^\circ$ at velocity ratios up to 1.5, and also provided data on the turbulent stress field. Johnston and Khan [8-10] performed flow visualization as well as LDV measurements at $\alpha = 30^\circ$ and $45^\circ$, $\beta = 45^\circ$, $90^\circ$, and $1 \leq VR \leq 1.5$. Quantitative information on the flow field included both mean and turbulent flow features. It was found that for a velocity ratio of 1, $30^\circ$ pitch and $60^\circ$ yaw produced the vortex with the peak mean vorticity. Jonhson [11] reviewed experimental and computational results on pitched and yawed JICF. The velocity ratio was found to be the principal design parameter if other factors were kept within reasonable limits. Bray [12] performed detailed five-hole probe surveys examining effects of pitch angle, yaw angle, diameter and Mach number ratio, as well as streamwise distance. In his low speed study, $VR$ ranges from 0.7 to 2, and pitch and yaw angles were set to 30, 45 and 60 degrees. Comparison between vane and air-jet vortices was also made. Bray and Garry [13] presented a correlation for maximum vorticity of a pitched and yawed JICF.

It is obvious that a lot of work has been done in previous studies on the subject area. Yet, it should also be clear that because of the vast parameter space, a coherent understanding has not been in place. Many aspects of the flow field such as the structure, trajectory and evolution of the streamwise vortices as a function of pitch and yaw angles and momentum-flux ratio, remained far from completely clear. Moreover, the literature lacks detailed measurements at low pitch, enhanced angular resolution in yaw, and high momentum-flux ratios. Such information is increasingly in demand by the designer of propulsion components. This provided the motivation for revisiting the subject and carry out the present study. The objective was to obtain detailed quantitative data on the flow field evolution for systematic variation of certain parameters.

The experimental conditions and parametric ranges are described in the next section.

**EXPERIMENTAL SETUP**

The investigation was conducted in a NASA GRC open circuit low-speed wind tunnel with 0.76 m x 0.51 m test section. The tunnel has a free stream turbulence level of 0.1%. The jet was produced with an inclined nozzle of diameter $D = 19.05$ mm. The nozzle was a straight hole cut through a clear plastic disc, of 25.4 mm thickness. The disc was mounted flush on the test section floor. Two discs were used to provide two pitch angles, ($\alpha = 20^\circ$ and $45^\circ$), measured between the nozzle centerline and the floor of the test section. Each disk could be rotated to vary the yaw angle $\beta$, measured between the nozzle centerline and the direction of the cross flow. The jet was yawed in $15^\circ$ increments between $0^\circ$ and $90^\circ$. A flow-conditioning screen was placed at the nozzle inlet, which was connected to compressed air supply through a flexible hose. An orifice meter fitted to the supply line was used to monitor the mass flow. The mass flow was used to calculate the mean jet velocity, $U_j$. All data were acquired for a constant free stream velocity of $U_\infty = 8$ m/s. The Reynolds number, based on free stream conditions and nozzle diameter was 9800.

The measurements were performed by hot-wire anemometry. Two X-wires of different orientations could be traversed under automated computer control. The probes were stepped through the same grid points allowing the measurement of all three components of mean velocity and turbulence intensity. The origin of the coordinate is located at the center of the jet orifice. The streamwise (i.e., the cross-flow) direction is denoted by $x$, the direction normal to the tunnel floor is denoted by $y$, and the spanwise direction along the tunnel floor by $z$. At the downstream locations of 3, 5, 10 and 20 jet diameters from the orifice, the turbulent boundary layer had a thickness of 0.60, 0.64, 0.684 and 0.822 jet diameters, respectively. The data for all configurations were acquired at a momentum-flux ratio of 8. Additionally, for selected arrangements ($\alpha = 20^\circ$ and $45^\circ$, $\beta = 75^\circ$, $x = 10$) the momentum-flux ratio was set at 1.5, 4, and 20. The test-matrix involved a total of 48 cross-sectional surveys.

**MEAN VELOCITIES**

Contours of streamwise velocity distribution, $u$, for both pitch angles and zero yaw, indicated symmetrical distribution around the $z = 0$ plane. Cross-flow vectors ($v, w$) showed strong lateral flow toward the symmetry plane. As the fluid passage is restricted by the wall and the symmetry condition, flow is forced upward deforming the jet into a kidney shape. For a given downstream location, the upward penetration of the jet and the curvature of the jet cross-section are more pronounced at the higher pitch angle. These data are not shown for space limitation. Only key results for the pitched and yawed cases are presented. However, the discussion will
Fig. 1 Downstream Evolution of Streamwise Velocity Distribution ($\alpha = 20^\circ, \beta = 75^\circ, J = 8$)

Fig. 2 Streamwise Velocity Distribution for Various Momentum-Flux Ratios ($\alpha = 45^\circ, \beta = 75^\circ, x = 10$)
draw on the entire dataset and comparison will be made with data from the literature wherever possible.

Detailed streamwise velocity contours for a representative case (α = 20°, β = 75° and J = 8) are shown in Fig. 1, capturing the evolution of the jet cross-section with increasing downstream distance. A comparison with corresponding data at zero yaw angle (not shown) indicates that the yawed jet has spread much more. The trends in Fig. 1 are further discussed in the following.

In Fig. 2, the influence of momentum-flux ratio, for a given pitch and yaw, is examined at a fixed downstream location (x = 10). It can be seen that the jet-vortex field at values of J greater than about 4 is characterized by velocities higher than \( U_w \). On the other hand, velocity deficits are observed at low momentum-flux ratios, as seen in the works of Compton and Johnston [5], Khan and Johnston [9-10], and Lee et al. [14] for \( VR = 1 \). Reference 9 identified two possible causes for the deficit; namely, the low momentum boundary layer fluid entrained into the core and vortex bursting very close to the orifice. The present results not only confirm the velocity deficit at low J, but also capture a systematic trend. Velocity overshoot becomes the prominent feature at higher momentum-flux ratios. Note that at intermediate values of J both deficit and overshoot take place within the jet-vortex structure. The present results, obtained at \( x = 10 \), moreover indicate that the velocity deficit trend can persist farther downstream than previously found.

The evolution of the streamwise velocity distribution with increasing downstream distance, for zero yaw, is discussed further. The shape of the velocity profile in the plane of symmetry for momentum-flux ratio of 8 changes as follows. Velocity profiles at 20° pitch exhibit one peak. With increasing \( x \), its magnitude decreases and the profiles become 'flatter'. The location of the peak shifts upwards. Velocity profiles at 45° pitch, on the other hand, indicate two peaks at measurement planes close to the orifice. However, further downstream the profiles are characterized by just one maximum value. Note that a similar dual-peak profile was observed by Sherif and Fletcher [15]. The present results also show that the 20° pitch case, compared to 45° case, is characterized by larger velocity magnitudes occurring closer to the wall.

As seen in Fig. 1, yaw angle introduces asymmetry in the velocity distributions. The location of the maximum velocity shifts away from the tunnel centerline with increasing downstream distance. A distorted kidney shape is initially discernible, with peak streamwise velocity located in a region closer to the centerline. In comparison to the zero yaw case, maximum velocities for \( \beta = 75° \) are closer to the wall and have considerably lower values. The kidney shape changes its orientation from ‘horizontal’ to ‘vertical’, and at the last measurement station only one local maximum is detected. Overall, the flow field for \( \alpha = 45° \) and \( \beta = 75° \) indicates a similar behavior of outward translation and simultaneous counterclockwise rotation with downstream distance. In the latter case, two distinct regions of velocity peaks are still present, (see the third figure from top in Fig. 2), even at the last measurement location. The magnitudes of the peak streamwise velocity for four representative cases are shown in Fig. 3, as a function of \( x \). It can be seen that a higher pitch angle results in considerably lower magnitudes at all measurement stations.

![Fig 3 Maximum Streamwise Velocities at Various Measurement Planes (J = 8)](image)

The jet was yawed in 15° increments between 0° and 90°, and data were collected for \( x = 3 \) and 10. Fig. 4 shows the maximum velocity as a function of the yaw angle. It can be seen that the peak velocity decreases with increasing yaw angle. For both pitch angles this trend is non-linear at \( x = 3 \) but becomes almost linear at \( x = 10 \). As expected, the detailed data showed that the location of \( U_{max} \) shifts upwards with the higher pitch, and outwards with higher yaw.

![Fig 4 Maximum Streamwise Velocities vs. Yaw Angle (J = 8)](image)

Recall that at low J, the jet-vortex is characterized by velocity deficit. This is examined further in Fig. 5 with velocity profiles through the point of minimum \( u \), for \( x = 10 \) and \( J = 1.5 \) for selected angles. At zero yaw, velocity profiles for both pitch values exhibit one local minimum and one local maximum. The magnitude of the minima for both pitch are
comparable and about \( u = 0.85 \), the one at higher pitch occurring farther away from the wall. The profiles for the 75° yaw case have one pronounced minimum, 0.69 for 20° pitch, and 0.79 for 45° pitch. These minima coincide with the core of the stronger vortex. At this high yaw angle, the weaker vortex has already been diffused. The deficit values compare well with the measurements of Khan and Johnston [9-10]. A configuration with \( \alpha = 30° \) and \( \beta = 60° \) in Ref. 10, exhibited velocity minima of 0.7 and 0.85, at \( x = 10 \), for \( VR = 1 \) and 1.5, respectively.

![Fig. 5 Velocity Profiles Through the Point of Minimum Velocity, \( (J = 1.5, x = 10) \)](image)

**MEAN STREAMWISE VORTICITY**

Contours of streamwise vorticity distribution, \( \omega_x \), for selected configurations at a fixed momentum-flux ratio are presented in Figures 6-7. In the case of zero yaw, counter-rotating vortex pair moves away from the wall and apart from each other with the downstream distance. For increased yaw, the vortex with the positive vorticity becomes the dominant structure. Its strength during the downstream evolution is as large as six times that of the weaker vortex. There is also a net transport of the vortex system laterally in the yawed direction. The dominant vortex facilitates a movement of low momentum flow close to the wall into the weak vortex. At the upstream locations additional concentrations of negative vorticity are observed near the wall, presumably due to reorientation of the boundary layer. Merger of structures with negative vorticity is achieved faster for the lower pitch angle, due to the strong vortex and its proximity to the wall. The region of concentrated negative vorticity is then quickly diffused and dissipated further downstream. At the last measurement station, essentially a single vortex with the positive vorticity remains.

![Fig. 6 Downstream Evolution of Streamwise Vorticity Distribution (\( \alpha = 20°, \beta = 75°, J = 8 \))]
It can be seen in Figs. 6-7 that the peak streamwise vorticity, $\omega_{\text{max}}$, decreases with the downstream distance. This trend is clearly shown in Fig. 8 for both pitch cases. At zero yaw, $\omega_{\text{max}}$ is found to become half of the initial value by $x = 5$, and by the last measurement station it is reduced to about one tenth. The rate of decrease for the $\beta = 75^\circ$ is more gradual. Thus, a vortex of considerable strength remains at $x = 20$. The values at different pitch angles but same yaw compare closely, however, as already stated, the rate of decrease is much lower at the higher yaw case.

![Fig. 7 Downstream Evolution of Streamwise Vorticity Distribution (\(\alpha = 45^\circ, \beta = 75^\circ, J = 8\))](image)

![Fig. 8 Maximum Streamwise Vorticity as a Function of Downstream Distance (\(J = 8\))] (image)

The result reveals that if an application requires longer persistence of streamwise vorticity, yawed jets are advantageous. The result also appears to be generic across the range of momentum ratios investigated in the present experiments, as clearly evident from the data shown in Fig. 9. The changes in $\omega_{\text{max}}$ are small for all cases when the momentum-flux ratio is increased beyond a value of about four.

![Fig. 9 Maximum Streamwise Vorticity as a Function of Momentum-Flux Ratio (\(x = 10\))] (image)
The streamwise vorticity is also examined as a function of yaw angle for J = 8 at two downstream locations, x = 3 and 10. At the first measurement station and 20° pitch, the vortex pair changes its structure with an increase in yaw. For β = 60° and higher, the flow field seems to result in two regions of loosely dispersed positive vorticity instead of a concentrated one (see Fig. 6). Such an occurrence is also observed in Ref. 9 for the case of VR = 1, α = 30°, and β = 60°. The origin of the two regions with positive vorticity is not clear but obviously the flow is in an early stage of the rolling up process at the upstream location (x = 3), and the vorticity has not had the chance to develop into a vortex. For both pitch angles, at x = 10, well-defined concentrated vorticity is ultimately developed as the generated vorticity has the necessary distance to evolve.

The variation of peak streamwise vorticity as a function of yaw angle for the positive vortex is summarized in Fig. 10 for J = 8, at x = 3 and 10. For all cases the trends are non-linear. At the measurement plane x = 3, maximum streamwise vorticity for 20 and 45 degree pitch is achieved respectively at 15° and 30° yaw. At x = 10, peak vorticity values for α = 20° and 45° are found at β = 60° and 45°, respectively.

**CONCLUSIONS**

An experimental investigation of pitched and yawed JICF was carried out. To complement the existing literature, this work provided data for the low pitch angle and high momentum-flux ratios, with enhanced angular resolution in yaw.

The results for both pitch angles reveal that yawed jets are advantageous for applications requiring longer persistence of peak streamwise vorticity. Maximum streamwise vorticity for 20 and 45 degree pitch at J = 8 and x = 3 is attained respectively at about 15 and 30 degree yaw. Peak vorticity values for the same α–J combinations farther downstream (x = 10), are found to shift to the higher yaw angles: 60° and 45°, respectively. In the range of momentum-flux ratios examined, both velocity deficit and overshoot in the jet-vortex structure are observed. The deficit occurs at lower J while overshoot is typical at higher J. An examination of the velocity gradients near the wall reveals that the proximity of the vortex portion of the jet-vortex system might be somewhat more effective in boundary layer separation control.

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**REFERENCES**


