The mismatch between fossil isotopic data and climate models known as the cool-tropic paradox implies that either the data are flawed or we understand very little about the climate models of greenhouse warming. Here we question the validity of the climate models on the scientific background of orbital noise in the Earth system. Our study shows that the insolation pulsation induced by orbital noise is the common cause of climate change and atmospheric concentrations of carbon dioxide and methane. In addition, we find that the intensity of the insolation pulses is dependent on the latitude of the Earth. Thus, orbital noise is the key to understanding the troubling paradox in climate models.

**Keywords:** Orbital signals; frequency noise; climate fluctuation; celestial mechanics; earth system science.

1. **Introduction**

One of the great scientific challenges in Earth Science is to predict and understand climate change. Wrestling to meet this challenge is the discipline of Earth System Science (ESS). Polar ice and ocean-sediment cores have provided a picture of the last million years of Earth's climate history and marked the emergence of ESS as the discipline that deals with the Earth as a complex, interacting system. ESS takes atmosphere, ocean, land and biosphere as the main components of the Earth and seeks to understand major patterns and processes in their dynamics. To do this, we need to study not only the processes that go on within each component but also interactions between these components. Because we still do not understand all these geophysical processes and interactions, we can not, yet, build a simple but realistic model that reproduces cycles of
the Earth's temperature and concentrations of the atmospheric greenhouse gases as
revealed by the ice core records [1]. But this is the central goal of ESS.

ESS has a fundamental problem. How can we link models of geophysical processes
to the greenhouse-gas concentration? The time scales involved in this question range
from hundreds to millions of years and involve processes that are highly nonlinear [2].
Therefore, we are in an uncharted territory, performing an uncontrolled experiment with
Earth dynamics that is terrifying in its scale and complexity.

Much of the current debate on the Earth's climate fluctuations is driven by the
observation of atmospheric concentrations of greenhouse gases. ESS climate models with
high levels of atmospheric carbon dioxide predict that global greenhouse warming would
cause heating in the tropics, but fossil isotopic data have indicated cool tropical
temperature during greenhouse episodes [3]. This mismatch between observation data
and ESS climate models known as the cool-tropic paradox implies that either the data are
flawed or we understand very little about the ESS climate models. On the scientific
background of orbital noise in the Earth system [1,2], here we question the validity of the
ESS climate models. We present results from calculations of insolation pulsation that is
induced by orbital noise of the Earth's changing obliquity. We show that global
greenhouse warming phenomena during the past half million years were the effect and
consequence of rapid temperature increases caused by insolation pulsation. In addition,
we find that the intensity of the insolation pulsation is dependent on the latitude of the
Earth. At high latitude, it provides extreme climate warming conditions for greenhouse-
gas concentration. However, it decreases markedly in low-latitude regions, predicting
cool tropical temperature during supposed greenhouse episodes.

2. Insolation Signal

Incoming solar radiation, generally called "insolation," is a key factor in climate studies.
Deviation of insolation depends on orbital parameters, which characterize the orbit of the
Earth around the Sun, and the Earth's axis of rotation. Equations of insolation deviations
at any point on the Earth are governed by [4]

\[ F_s = F_s^0 H \cos \psi_s \left( \frac{r_0}{r_5} \right)^2 \]  

where \( F_s^0 = 1368 \text{Wm}^{-2} \) is the average amount of sunlight impinging on the Earth, \( r_0 \) and
\( r_5 \) are the average and instantaneous distance from the center of the Earth to the center of
the Sun, respectively, \( \psi_s \) is the solar zenith angle. \( H = 1 \) when \( \cos \psi_s \geq 0 \) and \( H = 0 \)
when \( \cos \psi_s < 0 \). When \( \cos \psi_s < 0 \) the Sun is below the horizon and the solar flux is zero.
Eq. (1) has been developed in terms of 9 orbital parameters [4]. Averaging the diurnal
and annual cycles, the zonal insolation signal to zero order in the orbital eccentricity \( e \)
becomes

\[ S(t) = F_s^z(t) = F_s^0 \left( \frac{r_0}{a} \right)^2 \left[ \frac{1}{4} + \frac{5}{16} P_2(\sin \phi) \left\{ \frac{3}{4} \sin^2 \varepsilon(t) - \frac{1}{2} \right\} \right] \]  

where \( a \) is the semimajor axis of the orbit, \( \phi \) is the latitude, \( P_2(\sin \phi) \) is the ordinary
Legendre polynomial of order 2, and terms in \( e^2 \) and \( e^4 \) are dropped due to their small
values compared to 1. It is noted here that there is no precession \( \varepsilon(\sin \varpi) \) terms in Eq. (2) (\( \varpi \) is the argument of perihelion), such terms depend on season, and the average is zero. But the obliquity \( \varepsilon(t) \) does appear. The changing obliquity can be expressed by

\[
\varepsilon(t) = \varepsilon_0 + \sum_i A_i \cos(\gamma_i t + \zeta_i)
\]

where amplitude \( A_i \), frequency \( \gamma_i \) and phase \( \zeta_i \) for \( i = 1, 2, 3 \ldots \) are constants and \( \varepsilon_0 = 23.320556 \) degree at \( t = 0 \) refers to 1950 AD [5].

3. Frequency Noise of Insolation

The insolation signal \( S(t) \) has been embedded in the frequency noise of the Earth's obliquity variations [2,6]. Based on a convolution theory of the non-stationary time series of orbital motion, the frequency noise of the insolation signal can be expressed by [7–9]

\[
N(t) = \frac{d}{dt} \left[ \arctan \left( \frac{\langle \Delta F^* (t) \rangle}{\Delta F^* (t)} \right) \right]
\]

where \( \Delta F^* (t) = F^* (t) - 222.125 \text{ W m}^{-2} \) and

\[
\langle \Delta F^* (t) \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\Delta F^* (t)}{t-\tau} \, d\tau
\]

4. Insolation Pulsation

Insolation signal \( S(t) \) in Eq. (2) and the frequency noise of the insolation \( N(t) \) in Eq. (4) are shown in Fig. 1. The threshold model includes: (a) A threshold with two horizontal dashed lines, (b) A sub threshold signal of insolation at 65° N and (c) A frequency noise of the insolation \( N(t) \). The addition of the frequency noise of the insolation enables the insolation signal \( S(t) \) to cross the threshold. When the insolation signal \( S(t) \) plus its corresponding frequency noise \( N(t) \) increases across the threshold, an insolation pulse can be written to the time series. The quantized output is an insolation train. Insolation pulsation can be measured by calculating the integral of insolation deviations \( \Delta F^* (t) \) over time for having only half obliquity cycle [9]. It last about 10 kyr (one quarter of the obliquity period), and its total insolation surplus on the top of the atmosphere during the interglacial interval is about 10% more than that of the regular deglacial period of the obliquity variation. The insolation pulses during the past 420,000 years are also illustrated in Fig. 1.

The intensity of insolation pulsation (IIP) in terms of insolation surplus in Fig. 1 is dependent on latitude because the insolation signal \( S(t) \) in Eq. (2) and insolation noise \( N(t) \) in Eq. (4) are functions of the Legendre polynomial of \( \sin \phi \) where \( \phi \) is the latitude of the Earth. At high latitudes, the high IIP induces extreme climate warming conditions for greenhouse-gas concentration. But it decreases in low-latitude regions, predicting cool tropical temperature during supposed greenhouse episodes.
5. Atmospheric Greenhouse-Gas Concentration

The Earth's climate during the past one million years is punctuated by a series of large glacial-interglacial climate changes which are documented by complementary climate records, largely derived from deep sea sediments and ice cores. It has been shown that the variability of the glacial-interglacial climate change occurs with periodicity corresponding to that of the frequency noise of the Earth's changing obliquity [10,11].
Climate records give access to palaeoclimate series such as the Earth’s temperature and atmospheric greenhouse-gas concentrations [3]. There is a close and strong correlation between global temperature and atmospheric CO$_2$ and CH$_4$. However, this correlation does not mean that greenhouse-gas concentration is the cause of glacial-interglacial climate change. In fact, atmospheric greenhouse-gas concentration may be the effect and consequence of the rapid temperature increase due to the orbitally induced insolation.

Fig 2. Orbital insolation pulses cause atmospheric concentrations of carbon dioxide and methane and glacial-interglacial temperature changes [3].
Insolation pulses induced by frequency noise of the Earth's obliquity are shown in Fig. 2, in comparison with the time series of temperature and the CO₂ and CH₄ concentrations as obtained by Petit et al. [3]. Each insolation pulse lasting about 10 kyr will cause atmospheric concentration of CO₂ and CH₄ and rapid temperature increase for glacial-interglacial climate change as shown in (a), (b) and (c) in Fig. 2. During glacial inception, Petit et al. [3] have found that the CO₂ decrease lags behind the temperature decrease. This may imply that atmospheric composition fluctuation is the effect of temperature variation.

Climate records also give access to palaeoclimate series that include precipitation rate and moisture source conditions. Particularly, the interglacial periods are characterized by much warmer temperature, increased precipitation and large-scale atmospheric moisture concentrations. It seems reasonable to suggest that atmospheric moisture concentrations are also the direct effect of insolation pulsation on climate change.

6. Discussion

Global greenhouse warming is controversial because the ESS climate models cannot reproduce cycles of ice ages and concentrations of the atmospheric greenhouse gases. Furthermore, models of CO₂ greenhouse warming have confronted data from fossil planktonic foraminifera with the so called "cool-tropics paradox." The ESS climate models with high levels of atmospheric carbon dioxide predict that global greenhouse warming would cause heating in the tropics. But, in stark defiance of the global climate models [3,12,13], the planktonic data seemed to suggest that ancient climates have indicated cool tropical temperature during supposed greenhouse episodes [14–19]. This has led researchers to question the primary role of CO₂ in determining global temperature [20], while others have argued that the data may be biased [21–24]. In order to clarify the controversy and resolve this troubling paradox in climate models, we have studied how the global climate system is changing in response to insolation forcing [1,2, 6–11]. It has been found that the intensity of insolation pulsation, which is a function of latitude, is capable of reconciling the indisputably moderate polar and mid-latitudes conditions of the interglacial intervals with the planktonic data evidence that the tropical ocean was then barely warmer than the ocean at mid-latitudes. So small a latitude gradient would require an extraordinarily efficient heat-transport mechanism that only the insolation pulses can simulate.

The Milankovitch link [25] between orbital parameters and climate is rather old and weak. For the climate system subject to weak insolation signals, noise induced cooperative phenomena can often be set up, leading to stochastic resonance between the weak deterministic external insolation signal and the stochastic internal noise [26–31]. This resonance theory of climate is based upon internal mechanisms with the bio-cryo-ocean-atmosphere-lithosphere system believed to have response time on the order of the observed mean periodicity [32]. However, internal mechanisms or feed backs have difficulties in explaining the apparently episodic and irregular timing appearance of the glacial-interglacial temperature changes [33] and the cool-tropic paradox.
7. Conclusion

We conclude that concentration of greenhouse gases in the atmosphere is the effect and consequences of insolation pulsation due to frequency noise of the Earth's changing obliquity. The insolation pulse provides extra solar heating above the high latitudes of the Earth and increases the temperature rapidly to terminate the ice age glaciation. The fact that the timing of the insolation pulses coincides with the occurrence of the atmospheric CO₂ and CH₄ concentrations supports the idea that insolation pulsation is the cause of the glacial-interglacial climate changes. These findings, together with our clarification of the cool-tropics paradox in climate models of global greenhouse warming, are of relevance with respect to the continuing debate on the future of Earth's climate.

References


[33] L. Beaufort, *Climate importance of the modulation of the 100 kyr cycle inferred from 16 m. y. long miocene records*, Paleoceanography 9 (1994) 821-834.