Computation of Turbulent Heat Transfer on the Walls of a 180 Degree Turn Channel With a Low Reynolds Number Reynolds Stress Model

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ABSTRACT

The Low Reynolds number version of the Stress-ω model and the two equation k-ω model of Wilcox were used for the calculation of turbulent heat transfer in a 180 degree turn simulating an internal coolant passage. The Stress-ω model was chosen for its robustness. The turbulent thermal fluxes were calculated by modifying and using the Generalized Gradient Diffusion Hypothesis. The results showed that using this Reynolds Stress model allowed better prediction of heat transfer compared to the k-ω two equation model. This improvement however required a finer grid and commensurately more CPU time.

INTRODUCTION

Our computational turbine heat transfer group has long been interested in developing the necessary tools to compute the external (blade surface) and internal (cooling passage) heat transfer. We have adopted the k-ω model of Wilcox [1] for its robustness and the absence of distance to the wall in its formulation. It was therefore natural for us to choose the Reynolds Stress model (RSM) based on the ω equation of Wilcox [1] for our first venture into this type of modeling.

We have in the past presented the solution to the problem of flow and heat transfer in a 180 degree channel[2] as predicted using the k-ω model. The geometry and experimental measurements chosen come out of the work of Arts et al. [3]. In this work we will explore the use of the Stress-ω model and contrast the solutions using the two models.

The RSMs are good candidates for this effort due to the exactness of their production terms and their ability to better represent the flow history among other advantages. The exactness of the production terms among other advantages has the potential to better simulate the stagnation flow, reattachment and curvature effects. Advances in Reynolds Stress modeling are continuously being made. Much of the effort has been placed in the modeling of the pressure-strain correlation in these models which are of significant magnitude and are responsible for the redistribution between different components of the Reynolds Stress tensor. To make the correlations valid near walls many authors use the so called reflection terms which in most instances require the unit wall normals to the wall. These quantities are not always clearly definable away from walls and are thus not desirable. Some workers instead have tried to use various invariances of the anisotropy of Reynolds Stress matrix for this purpose [4]. It is not the intent of this paper to provide a comprehensive summary of RSMs and the interested reader may refer to [5] which is a recent review paper on this subject. More recently the method of elliptic relaxation which solves an additional set of six differential equations to modify the redistribution tensor in the vicinity of the walls has been gaining momentum[6]. The scheme has been applied to some two dimensional or axisymmetric cases [6-9], but has not yet been proven for three-dimensional complex problems. Due to the inclusion of six additional equations the computational cost is presumably much higher than the simpler alternative.
There have been attempts made to solve the heat transfer problem in channels using RSMs. For example, Iacovides and Raisee[10], have performed internal cooling passage calculations with a Reynolds Stress Model. Iacovides, Launder and Li[4] also applied their RSM model to flow and heat transfer in a U bend. Recently Chen et al.[11,12] applied their \( \varepsilon \) based RSM to an internal cooling channel heat transfer and achieved good results. The RSM model used in that work was reflection free and was applied all the way to the wall, although the distance to the wall was used in a damping function.

The present Stress-\( \omega \) model is also valid all the way to the wall[1]. It does not use reflection terms and there is no need for wall functions or the use of a two layer model which have limited validity. This feature would make the model useful in heat transfer calculations. The model was implemented in our code Glenn-HT (NASA Glenn Heat Transfer Code) and solved using the code's explicit scheme.

NOMENCLATURE

\( C_p \) constant pressure specific heat
\( D \) hydraulic Diameter
\( h \) heat transfer coefficient
\( k \) Kinetic energy of turbulence
\( Pr \) Prandtl number
\( Re \) Reynolds number
\( T \) static temperature/\( T_0 \)
\( y^* \) dimensionless distance from a wall, \( y^* \equiv \sqrt{\frac{\gamma w}{v}} \)
\( \varepsilon \) Turbulence dissipation rate
\( \gamma \) specific heat ratio
\( \tau \) Reynolds stress
\( \omega \) Specific dissipation rate of turbulence

Subscripts
\( t \) total conditions
\( w \) wall value

FORMULATION

Stress-Omega Model

As described in the introduction the Stress-\( \omega \) Reynolds Stress model of Wilcox was adopted for the present work. The equations for the Reynolds stresses \( \tau_{ij} = -\rho \bar{u}_i \bar{u}_j \) are given below:

\[
\frac{\partial \tau_{ij}}{\partial t} + U_i \frac{\partial \tau_{ij}}{\partial x_j} = -\tau_{ik} \frac{\partial U_j}{\partial x_k} - \rho e_{ij} - \Pi_{ij} + \frac{\partial}{\partial x_k} \left[ \mu \frac{\partial \tau_{ij}}{\partial x_k} + \rho C_{ijk} \right]
\]

where the terms underlined on the right hand side are modeled.

The Stress-\( \omega \) model uses the ‘standard’ modeling practice for the underlined terms except for the last term.

From the Kolmogorov hypothesis of local isotropy.

\[
\varepsilon_{ij} = \frac{2}{3} \varepsilon \delta_{ij}
\]

Using the definition of \( \varepsilon = \beta \omega \varepsilon \) allowing for ‘isotropic damping’ near walls,

\[
\varepsilon_{ij} = \frac{2}{3} \beta \omega \delta_{ij}
\]

The term \( \Pi_{ij} \) is the pressure-strain Correlation of Launder-Reece and Rodi(1975)[16] written as:

\[
\Pi_{ij} = \beta C_i \omega \left( \tau_{ij} + \frac{2}{3} \rho \delta_{ij} \right) - \alpha_m \left( P_{ij} - \frac{2}{3} P \delta_{ij} \right)
\]

\[
- \beta_m \left( D_{ij} - \frac{2}{3} P \delta_{ij} \right) - \gamma_\omega \rho \left( S_{ij} - \frac{2}{3} S_{kk} \delta_{ij} \right)
\]

where

\[
P_{ij} = \tau_{im} \frac{\partial U_m}{\partial x_j} + \tau_{jm} \frac{\partial U_m}{\partial x_i}
\]

\[
D_{ij} = \tau_{im} \frac{\partial U_m}{\partial x_j} + \tau_{jm} \frac{\partial U_m}{\partial x_i}
\]

\[
\Pi_{ij} = \frac{1}{2} P_{kk}
\]

also

\[
C_{ijk} = \left[ \sigma * \mu T \frac{\partial \tau_{ij}}{\partial x_k} \right]
\]

For calculation of \( \omega \) the standard equation is used [1]:

\[
\rho \frac{\partial \omega}{\partial t} + \rho U_j \frac{\partial \omega}{\partial x_j} = \alpha \rho \omega / k T \frac{\partial U_j}{\partial x_k} - \beta \rho \omega^2 + \frac{\partial}{\partial x_k} \left[ (\mu + \sigma \mu_T) \frac{\partial \omega}{\partial x_k} \right]
\]

above

\[
\nu_T = \alpha * \frac{k}{\omega} \quad \text{and} \quad Re_T = \frac{k}{\omega \nu}
\]

The closure coefficients are given as

\[
\alpha^* = \frac{\alpha_0^* + Re_T / R_k}{1 + Re_T / R_k}
\]

\[
\alpha = \frac{13 \alpha_0 + Re_T / R_\omega}{25} \cdot \frac{3 + Re_T / R_\omega}{3 \alpha_0^* + Re_T / R_{\omega_0}}
\]

\[
\beta^* = 0.09 \cdot \frac{4/15 + Re_T / R_\beta}{1 + (Re_T / R_\beta) \cdot f^*}
\]
\[ \alpha = \frac{1 + \hat{\alpha}_w Re_{T}/R_k}{1 + Re_{T}/R_k} \]  
(12)

\[ \hat{\beta} = \hat{\beta}_w \frac{Re_{T}/R_k}{1 + Re_{T}/R_k} \]  
(13)

\[ \hat{\gamma} = \hat{\gamma}_w + Re_{T}/R_k \]  
(14)

\[ C_1 = \frac{9}{5/3 + Re_{T}/R_k} \]  
(15)

where the constants with the \( \omega \) subscript are the constants of the original LRR model for the pressure-strain correlation model given as follows:

\[ \hat{\alpha}_w = \frac{8 + C_2}{11} \quad , \quad \hat{\beta}_w = \frac{(8C_2 - 2)}{11} \quad \text{and} \quad \hat{\gamma}_w = \frac{(60C_2 - 4)}{55} \]

Further definitions follow:

\[ \beta = \frac{9}{125} f_\beta \quad , \quad \sigma^* = \frac{1}{2} \quad , \quad C_2 = \frac{13}{25} \]

\[ \alpha^*_0 = \frac{1}{3} \hat{\beta}_0 \quad , \quad \alpha_0 = 0.105 \quad , \quad \hat{\gamma}_0 = 0.007 \]  
(16)

\[ R_k = R_\beta = 12 \quad \text{and} \quad R_\omega = 6.20 \]

Following the suggestions of Menter [17], the production terms are modified and written in terms of vorticity magnitude \( \Omega \). This reduces the overshoots of heat transfer rates in the vicinity of stagnation points.

### Turbulent Heat Flux

Both models integrate to the walls and no wall functions are used. A value for Prandtl number (Pr) equal to 0.72 is used. Viscosity is a function of temperature through a 0.7 power law [18] and \( C_p \) is taken to be a constant.

When using the 2-equation model for the calculation of turbulent thermal fluxes, eddy viscosity model and a constant value of 0.9 for turbulent Prandtl number, \( Pr_t \) was used. For the Reynolds stress model the turbulent heat fluxes were calculated using the Generalized Gradient Diffusion Hypothesis (GGDH) given in for example Iacovides et. al. [3] among other places.

For our purposes we will rewrite the above in the following form:

\[ \rho \frac{\partial k}{\partial t} + \rho U_j \frac{\partial k}{\partial x_j} = \mu_T \Omega^2 - \rho \beta^* \omega_k + \frac{\partial}{\partial x_j} \left( \mu + \sigma_T \frac{\partial k}{\partial x_j} \right) \]  
(18)

rewriting using \( k \) and \( \omega \), we found the following form
The ratio of \( \frac{k}{u_j u_j} \) can be regarded as an anisotropic turbulent Prandtl number relating an anisotropic thermal diffusivity to an isotropic momentum diffusivity.

Using the eddy viscosity hypothesis with a constant Prandtl number in Stress-\( \omega \) model as is done for the \( k-\omega \) model produced heat transfer results that were too low. This was found to be due to the fact that the Stress-\( \omega \) model produces lower levels of \( k \) than does the \( k-\omega \) model.

**CHANNEL GEOMETRY AND FLOW CONDITIONS**

The test problem was taken from the experiment of Arts et al.[6] It is an aggressive 180 degree turn channel with a rectangular cross section. The inlet and exit channels have the same cross section as shown in Fig. 1. The overall length of the channel is 8W. The divider has a thickness of W/5 and extends to within one width of the end wall. The divider has a semi-circular end. The experiments were performed for two channels with aspect ratios (H/W) of 1. and 0.5, the latter of which is considered here. The condition of symmetry has been used so that only half of the channel has been gridded. In the experimental work two Reynolds Numbers were considered, namely 17,000 and 35,000. For the present numerical work the channel Reynolds number of 17000 was simulated.

**COMPUTATIONAL METHOD**

**Computational Scheme**

The simulations in this study were performed using a multi-block computer code called Glenn-HT [2]. This code is a general purpose flow solver designed for simulations of flows in complicated geometries. The code solves the full compressible, Reynolds-averaged Navier-Stokes equations using a multi-stage Runge-Kutta-based multigrid method. It uses the finite volume method to discretize the equations. The code uses central differencing together with artificial dissipation to discretize the convective terms. The overall accuracy of the code is second order. To achieve good convergence the turbulence \( \omega \) equation and the Reynolds stresses were implicitly coupled in a pointwise fashion in the Runge-Kutta stage calculations.

**Boundary Conditions**

Boundary condition treatment is dealt with in [2]. Here we will not repeat the treatment of standard flow variables but extend the discussion to include specification of Reynolds stresses. The boundaries are treated as follows.

**Inlet:** At the inlet, the incoming profiles of \( k \) and \( \omega \) need to be specified. Typically, the details of the profiles are unknown so reasonable assumptions need to be made. Approximate values of \( k \) and \( \omega \) can be computed based on turbulence intensity and some measure of a length scale. In cases such as the present where the flow is sensitive to the exact values of \( k \) and \( \omega \) at the inlet, the profiles need to be specified more carefully.

In the present problem the channel is extended upstream of where the inlet boundary condition is specified and heating begins. For that reason an inlet profile corresponding to a fully developed channel flow is assumed. For the \( k-\omega \) model this specification is done algebraically [2]. For the case where the detailed boundary conditions on the Reynolds stresses are necessary the approach taken was to solve the fully developed channel flow and transfer the resulting profile to the inlet of the 180 degree channel. The problem was solved on a grid identical to the grid used at the inlet but extended in the axial direction. The channel was not very long so in order to achieve fully developed flow, compressible flow periodicity in axial direction was enforced.

**Exit:** The static pressure is specified at the exit and other variables are extrapolated.

**Symmetry:** Symmetry boundary conditions are trivial for all the variables except the Reynolds stresses some of which vanish and others have a vanishing normal derivative to the symmetry plane.

**Walls:** At solid walls the specific dissipation rate, \( \omega \), can be specified as follows:

\[
\omega = S_R \frac{\partial}{\partial n}(u)\bigg|_{wall} \tag{27}
\]

where

\[
S_R = \begin{cases} 
(50 \frac{K_R}{K})^2 & \text{if } K_R < 25 \\
100 \frac{K_R}{K} & \text{if } K_R > 25 
\end{cases} \tag{28}
\]

and \( K_R \) is equivalent sand-grain roughness height in turbulent...
Figure 2. (a) Grid Topology (b) Coarse Grid, (c) Medium Grid and (d) Fine Grid and (e) distribution of $y+$ for grid ‘(d)’

Figure 3. Velocity vectors (a) Downstream of the turn and (b) near the bottom wall and (c) Streamline tracing the major vortices in the flow.

Figure 1 shows a typical grid topology and the grid constructed for this problem. It covers half of the channel and a symmetric boundary condition is used. The multiblock grid generated using a commercial package called GridPro, consists approximately 3.4E6, 5.2E6 and 1.06E6 cells for the coarse, medium and the fine grid. Grid spacing adjacent to the walls produce averaged dimensionless spacing ($y+$) near unity (Fig. 2(e) corresponding to grid ‘(d)’) with a stretching ratio of grid

wall units. Typically, $K_R=5$ is chosen, corresponding to a hydraulically smooth surface.

The wall temperature is specified as a constant value for heat transfer surfaces. The Reynolds stresses and the turbulent thermal fluxes are zero at the walls.

GEOMETRY MODELING AND GRID SYSTEM

The wall temperature is specified as a constant value for heat transfer surfaces. The Reynolds stresses and the turbulent thermal fluxes are zero at the walls.
spacing away from the walls equal to 1.1 for the fine grid and 1.2 for the medium and coarse grid calculation.

RESULTS AND DISCUSSION

In an earlier paper [2] we reported our flow and heat transfer solution for the present problem using the k-ω model. The calculations for that model were repeated here for the sake of comparison of heat transfer results. The result was identical to that presented here (as expected) but the grid was 3.5 times coarser than the present Medium grid.

Figure 3 shows the details of the flow downstream of the turn and very near the bottom wall of the channel as predicted with the Reynolds Stress model on the fine grid. The simulations show that the flow around the corner is quite complex. Fig 3(a) shows the multiple vortices generated in the turn as Fig. 3(b) show the complex flow structure near the bottom wall. One can discern a large vortex at the inner wall of the turn with its axis generally in the normal to the bottom wall direction. There is another vortex that starts near the inner turn and generally follows the flow direction. This vortex is accompanied by an opposing one on the other side of the plane of symmetry. One can discern also a corner vortex on the inner wall of the return leg with its axis in the flow direction. Other vortical structures are also present as evidenced by Figure 3.

Heat Transfer

Figure 4(a) shows the experimental measurement of the rate of heat transfer on the bottom wall in terms of a Nusselt number ratio Nu/Nu0. The data is from the experimental measurements of Arts et al. [16] and corresponds to an aspect ratio of 0.5 and Reynolds number of 18000. The contours are Nusselt number ratio Nu/Nu0, where

\[ Nu = \frac{hD}{k} \quad (29) \]

In this equation D is the hydraulic diameter and k is the thermal conductivity evaluated at the reference temperature T_ref defined as the arithmetic average of the inlet and exit centerline temperatures. The heat transfer coefficient h is defined by the following expression:

\[ h = \frac{q_w}{T_w - T_{ref}} \quad (30) \]

Nu0 is the Nusselt number for a fully developed channel flow defined as:

\[ Nu_0 = 0.023Re_\nu^{0.8}Pr^{0.4} \quad (31) \]

Figure 4(b) shows the calculated heat transfer rate using the k-ω model. This was accomplished using the coarse grid.

Figure 4. Nu/Nu0 for the case of channel flow with AR=0.5 and Red=17000. (a) Experimental measurement of Arts et al.[16], (b) calculation using k-ω model and (c), (d) and (e) calculation using Stress-ω model with the 340,000, 523,000 and 1.06E6 cells.
The results are similar to those obtained with a yet coarser grid (≈100,000 cells) in [2] and thus it is ensured that further refinement would not change the character of the solution substantially. The results have two major differences with the data. First the rise in the rate of heat transfer near the endwall (the wall facing the inlet) is not matched by the experiment and secondly the rise in heat transfer near the inner wall of the return leg is not captured by the k-ω model.

Figure 4(c) shows the same case solved using the Stress-ω model and the coarse grid of 340,000 cells while Fig. 4(d) is for the medium grid of 523,000 cells. As can be seen there are substantial differences between the two solutions. This suggested that further refinement was still necessary. A finer grid of 1.06E6 cells was considered. The solutions using RSM have similar characteristics. Namely a rise in heat transfer near the endwall as with the k-ω model, not supported by the experiment and the two other regions of high heat transfer on the return leg. The high heat transfer region on the return leg adjacent to the partition is captured by the RSM. The rise in heat transfer on the return leg near the outer wall is present for the Stress-ω model as for the k-ω model in agreement with the experimental data.

It is observed therefore that the Reynolds Stress model utilized can help capture the flow physics and by extension the rate of heat transfer better than the two-equation model version. A much finer grid is required in order to obtain a satisfactory solution with the RSM. In the present case the coarsest grid to capture the physics of the flow with the RSM was 3 times finer than the grid used for the k-ω model in [2]. The expenditure however appears to be worthwhile if a greater resolution of the heat transfer distribution is desired. In the present case however the agreement with measurements in the U-turn area suggests that the present model is still not quite able to predict the turbulence levels where there exists flow acceleration and impingement. It should be noted that the present problem owing to the severe turn in the flow and presence of severe three-dimensionality and strong secondary flows is a difficult test for any turbulence model.

**SUMMARY AND CONCLUSIONS**

The Low Reynolds number versions of Stress-ω model and the two equation k-ω model of Wilcox were used for the calculation of turbulent heat transfer in a 180 degree turn simulating an internal coolant passage. The RSM was implemented within a Finite Volume code (Glenn-HT) and solved using the existing explicit scheme. For the present problem care was taken to specify reasonable inlet boundary conditions for all the variables, especially those for the Reynolds stresses by first solving a fully developed flow in a channel.

A rather systematic grid study was implemented. The results showed that using the Reynolds Stress model allowed better prediction of heat transfer compared to the k-ω two equation model. This improvement however required specification of a finer grid and a larger CPU time.

**REFERENCES**


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