Hybrid Verification of an Interface for an Automatic Landing

Meeko Oishi, Ian Mitchell, Alexandre Bayen, Claire Tomlin
Hybrid Systems Lab, Stanford University, Stanford, CA
moishi,imitchell,bayen,tomlin@stanford.edu
Asaf Degani
NASA Ames Research Center, Moffett Field, CA
adegani@mail.arc.nasa.gov

Abstract

Modern commercial aircraft have extensive automation which helps the pilot by performing computations, obtaining data, and completing procedural tasks. The pilot display must contain enough information so that the pilot can correctly predict the aircraft’s behavior, while not overloading the pilot with unnecessary information. Human-automation interaction is currently evaluated through extensive simulation. In this paper, using both hybrid and discrete-event system techniques, we show how one could mathematically verify that an interface contains enough information for the pilot to safely and unambiguously complete a desired maneuver. We first develop a nonlinear, hybrid model for the longitudinal dynamics of a large civil jet aircraft in an autoland/go-around maneuver. We find the largest controlled subset of the aircraft’s flight envelope for which we can guarantee both safe landing and safe go-around. We abstract a discrete procedural model using this result, and verify a discrete formulation of the pilot display against it. An interface which fails this verification could result in nondeterministic or unpredictable behavior from the pilot’s point of view.

1 Introduction

One of the key enabling technologies for increased automation in human-machine systems is verification, which allows for heightened confidence that the system will perform as desired. To verify system safety, the safety specification is first represented as a desired subset of the state space in which the system should remain. The process of verifying safety then involves computing the subset of the state space which is backwards reachable from this “safe set” of states; if this backwards reachable set intersects any states outside the desired region, then the system is deemed unsafe. We can restrict system behavior by pruning away system trajectories which lead to unsafe states, to synthesize a controller which, if enforced, guarantees safety.

In the past several years, a method [1] and a numerical tool [2, 3] have been developed for verifying the safety of hybrid systems. Previous work, for example [4], has focused on applications of hybrid system theory to fully automated systems, assuming that the controller itself is an automation. Here we consider the problem of controlling semi-automated systems, in which the automation and a human controller share authority over the control of the system [5]. In particular, we consider the problem of verification of an interface between a semi-automated hybrid system and a human controller, and we pose the question: Is the information displayed to the human controller about the hybrid system evolution sufficient for the human controller to act in such a way that the system remains safe? We consider this problem within the framework of an example: the automatic landing system (autoland) of a large civil jet airliner.

The autoland system of modern aircraft is one of the most safety-critical components, and is subject to stringent certification criteria [6]. Modeling the aircraft’s behavior, which incorporates logic from the autopilot as well as inherently complicated aircraft dynamics, results in a high-dimensional hybrid system with many continuous and discrete states. Most of the information is abstracted away, so that only a subset of this information is displayed to the pilot. Here, we are interested in verifying that the cockpit interface provides the pilot with enough information so that the pilot can safely land or safely go-around.

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### 2.3 State and Input Bounds

Each mode in the procedural automaton is subject to state and input bounds, due to constraints arising from aircraft aerodynamics and desired aircraft behavior. These bounds, shown in Table 2, form the boundary of the flight envelope \( \mathcal{W}_0 \). Bounds on \( V \) and \( \alpha \) are determined by stall speeds and structural limitations for each flap setting [22]. Bounds on \( \gamma \) and \( T \) are determined by the desired maneuver [23]. Additionally, at touchdown, \( \theta \in [0^\circ, 12.9^\circ] \) to prevent a tail strike, and \( h \geq -1.829 \text{ m/s} \) to prevent damage to the landing gear.

### 3 Safety Analysis

The state bounds just described define flight envelopes for each of the discrete modes. These envelopes are not necessarily controlled invariant. Thus, we need to determine what subsets of these envelopes are actually controllable given the input authority available to the autopilot. Because the nonlinear dynamics of our model (1) make analytic determination of the controllable subsets impossible, we employ a previously developed computational algorithm for finding controlled invariant sets for this problem [3].

#### 3.1 Computing Reachable Sets

For each discrete mode of the autoland system, we define the target set as the region outside the flight envelope \( \mathcal{W}_0 \), denoted \( (\mathcal{W}_0)^c \) for the complement of \( \mathcal{W}_0 \). Given some dynamically evolving system and some target set, we define the backward reachable set \( \mathcal{V}^b(t) \) as the set of all system states which reach the target set in time \( t \). The autopilot inputs \( \alpha \) and \( T \) try to drive the state away from the target set, to keep the aircraft within \( \mathcal{W}_0 \).

Computing the reachable set in a discrete system with a finite number of states—and hence a finite number of possible transitions—is a straightforward but possibly time consuming task of enumerating all the states which have a path to the target set. Computing reachable sets for a continuous system is a much more difficult undertaking; for example, how should the uncountably many states in any nontrivial target set be represented?

An algorithm has been developed for computing the reachable sets of continuous nonlinear systems, based on a time dependent Hamilton-Jacobi (HJ) partial differential equation (PDE) [2, 3]. For \( \dot{x} = f(x,u) \), \( x \in \mathbb{R}^n \), input \( u \in U \) tries to keep the system from reaching the target set. Define a continuous function \( J_0 : \mathbb{R}^n \rightarrow \mathbb{R} \) such that \( (\mathcal{W}_0)^c = \{ x \in \mathbb{R} | J_0(x) \leq 0 \} \). As shown in [2],

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**Table 1:** Aerodynamic constants for autoland modes indexed by \( \dot{x} = f(x,u) \).

<table>
<thead>
<tr>
<th>( C_D )</th>
<th>( C_L )</th>
<th>( K )</th>
<th>Flaps Setting</th>
<th>Landing Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4225</td>
<td>0.024847</td>
<td>0.04831</td>
<td>Flaps-20</td>
<td>Down</td>
</tr>
<tr>
<td>0.7043</td>
<td>0.025151</td>
<td>0.04831</td>
<td>Flaps-25</td>
<td>Down</td>
</tr>
<tr>
<td>0.8212</td>
<td>0.025455</td>
<td>0.04831</td>
<td>Flaps-30</td>
<td>Down</td>
</tr>
<tr>
<td>0.4225</td>
<td>0.019704</td>
<td>0.04589</td>
<td>Flaps-20 Up</td>
<td>Up</td>
</tr>
<tr>
<td>0.7043</td>
<td>0.020009</td>
<td>0.04589</td>
<td>Flaps-25 Up</td>
<td>Up</td>
</tr>
<tr>
<td>0.8212</td>
<td>0.020313</td>
<td>0.04589</td>
<td>Flaps-30 Up</td>
<td>Up</td>
</tr>
</tbody>
</table>

**Table 2:** State bounds for autoland modes of \( H_{\text{procedure}} \).

<table>
<thead>
<tr>
<th>Mode</th>
<th>( V ) [m/s]</th>
<th>( \gamma ) [degrees]</th>
<th>( \alpha ) [degrees]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flare</td>
<td>[55.57, 87.46]</td>
<td>[-6.0°, 0.0°]</td>
<td>[-9°, 15°]</td>
</tr>
<tr>
<td>Toga-Max</td>
<td>[63.79, 97.74]</td>
<td>[-6.0°, 0.0°]</td>
<td>[-8°, 12°]</td>
</tr>
<tr>
<td>Toga-Up</td>
<td>[63.79, 97.74]</td>
<td>[0.0°, 13.3°]</td>
<td>[-8°, 12°]</td>
</tr>
</tbody>
</table>
| Altitude   | [63.79, 97.74]| [-0.7°, 0.7°]          | [-8°, 12°]
so that trajectories which begin inside or on the boundary of the safe flight envelope in one mode will remain within or on the boundary of the safe flight envelope in all other modes in that hybrid subsystem. Only across user-controlled switches can the system become unsafe, because we can make no guarantees about the user's actions. $G_{\text{procedure}}$ has modes $Q_{\text{procedure}}$, events $\Sigma_{\text{procedure}}$, and transition function $\delta_{\text{procedure}}$.

We verify the correspondence between $G_{\text{interface}}$ and $G_{\text{procedure}}$ according to the verification methodology in [7]. We associate each mode in $Q_{\text{interface}}$ and $Q_{\text{procedure}}$ to a certain specification class [7]. Specification classes are a way of indicating a type of behavior or quality of the system – for example, modes which the system should avoid belong to a specification class Unsafe.

The interface and the abstracted procedural model are related through their events: events in $\Sigma_{\text{procedure}}$ map to events in $\Sigma_{\text{interface}}$. We define the map through $\Sigma_{\text{procedure}} \rightarrow \Sigma_{\text{interface}}$ by examining the events in each set and creating a correspondence between them by hand [7]. Events in $\Sigma_{\text{procedure}}$ which do not have a corresponding transition in $\Sigma_{\text{interface}}$ map to the empty event $\varepsilon$ [7].

The two systems are verified through the creation of a composition, defined by the map $\pi$. The composition $G_{\text{composition}}$ allows us to keep track of the modes and events in both systems ($G_{\text{interface}}$ and $G_{\text{procedure}}$) at the same time. The process of creating the composition uncovers possible problems: error states, blocking states, and augmented states [7].

The composition begins with each initial state in each system for a given specification class, and is repeated for each pair of initial states. If each event $\alpha$ in $G_{\text{procedure}}$ such that $p \xrightarrow{\alpha} p'$ has a corresponding event $\pi(\alpha)$ in $G_{\text{interface}}$ such that $q \xrightarrow{\pi(\alpha)} q'$, then the composite state $(p, q) \xrightarrow{\pi(\alpha)} (p', q')$ exists. If $p$ and $q$ have the same specification class, and $p'$ and $q'$ have the same specification class, then the composition continues throughout the model. An error state exists when $p'$ and $q'$ have different specification classes [7].

Other problems occur when the composition fails. If for a transition $\alpha \in \Sigma_{\text{procedure}}$ from $p \xrightarrow{\alpha} p'$ there is no corresponding transition $q \xrightarrow{\pi(\alpha)} q'$, then the composition has reached a blocking state [7]. (The interface blocks a transition which occurs in the abstracted procedural model.) Alternatively, if there is a transition $\pi(\alpha') \in \Sigma_{\text{interface}}$ from $q \xrightarrow{\pi(\alpha')} \pi(\alpha) = q'$ but no corresponding transition $\alpha \in \Sigma_{\text{procedure}}$ from $p \xrightarrow{\alpha} p'$, then the composition has reached an augmented state [7]. (The interface indicates a transition which is not possible in the abstracted procedural model.)

highly automated aircraft, including the option not to enforce a recommended switch.

The pilot activates various knobs, buttons, and toggles to change the system’s mode. Interaction between the pilot’s actions and the system’s modes are encapsulated by a finite-state machine representation of the interface $G_{\text{interface}} = (Q_{\text{interface}}, \Sigma_{\text{interface}}, \delta_{\text{interface}})$. Modes $Q_{\text{interface}}$ are determined by the indications on the display; events $\Sigma_{\text{interface}}$ are determined by internal transitions in the system, or by the pilot’s actions. The transition function is $\delta_{\text{interface}}$. The interface for an autoland/go-around maneuver is shown in Figure 6.

To compare the interface against the procedural model, we implement the controller for safety $u^*(x)$ in $H_{\text{procedure}}$ and create a discrete abstraction $G_{\text{procedure}}^*$ based on the resultant closed-loop hybrid system. We partition the state-space in each mode into the interior, boundary, and complement of the safe flight envelope in that particular mode. Across the user-controlled switch $\sigma_{\text{TOGA}}$, we partition the state space according to the intersection of $W_F$ in Flare and $W_T$ in Toga-Up, resulting in nine regions in each mode. Across all other switches in $H_F$ and $H_T$, we enforce safety by implementing $u^*(x)$ so that trajectories which begin inside or on the boundary...


