Pulsed Electromagnetic Acceleration of Plasmas

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Abstract

A major shift in paradigm in driving pulsed plasma thruster is necessary if the original goal of accelerating a plasma sheet efficiently to high velocities as a plasma “slug” is to be realized. Firstly, the plasma interior needs to be highly collisional so that it can be dammed by the plasma edge layer (upstream) adjacent to the driving ‘vacuum’ magnetic field. Secondly, the plasma edge layer needs to be strongly magnetized so that its Hall parameter is of the order of unity in this region to ensure excellent coupling of the Lorentz force to the plasma. Thirdly, to prevent and/or suppress the occurrence of secondary arcs or restrike behind the plasma, the region behind the plasma needs to be collisionless and extremely magnetized with sufficiently large Hall parameter. This places a vacuum requirement on the bore conditions prior to the shot. These requirements are quantified in the paper and lead to the introduction of three new design parameters corresponding to these three plasma requirements. The first parameter, labeled in the paper as $\gamma_1$, pertains to the permissible ratio of the diffusive excursion of the plasma during the course of the acceleration to the plasma longitudinal dimension. The second parameter is the required Hall parameter of the edge plasma region, and the third parameter is the required Hall parameter of the region behind the plasma. Experimental research is required to quantify the values of these design parameters. Based upon fundamental theory of the transport processes in plasma, some theoretical guidance on the choice of these parameters are provided to help designing the necessary experiments to acquire these data.

1 Introduction

A coaxial pulsed plasma thruster (PPT) is a plasma accelerator consisting of a pair of coaxial cylindrical electrodes. Current from a pulsed power supply, typically a capacitor bank, enters at one of the electrodes, crosses the gap between the electrodes through a plasma, and returns to the capacitor bank via the other electrode. The current following in the electrodes generates an azimuthal magnetic field in the region between the electrodes. This magnetic field acts on the plasma current to produce the electromagnetic $j \times B$ (Lorentz) force on the plasma, accelerating the plasma down the tube. The ‘Holy Grail’ of PPT research has been to find a way to use the intrinsically highly efficient $j \times B$ (Lorentz) force to accelerate a plasma “slug” cleanly with a precise current pulse, leaving nothing inside the thruster after the current pulse with the plasma slug exiting the barrel with an uniform velocity. This has simply not been achieved to date. In a real PPT, the plasma is anything but a “slug” and displays a wide range of complex behavior. In fact, one may say that there is a tendency for the PPT to behave more like a quasi-steady-state MPD thruster. In that mode one may say that it has the worst of both worlds. Considerable amount of effort has been expended in understanding the complexities of the plasma behavior in PPT. In this paper, we take the opposite approach. We ask the question, “what plasma conditions must we provide mother nature in the PPT so that the plasma will behave in an ideal manner?” Preliminary considerations on the engineering implementation of the proposed “plasma agenda” are discussed in this paper as well as in the companion paper [1].

2 Performance Losses in PPT

The performance of past PPT’s in flight or in laboratories have been reviewed recently by several authors [2-4]. The principal performance losses and engineering issues may be summarized as follows:

1. Low propellant utilization efficiency. This is perhaps the most important performance loss and relates to the fact that even though some plasma pieces may be launched out of the thruster at a velocity $v$, the steady-state specific impulse achieved is considerably lower than $v/g$. Several mechanisms contribute to this. Some are of an engineering nature relating to the feeding of propellant and in principle they may be overcome with suitable engineering ingenuity[1]. The inherently more challenging issues are related to undesirable plasma behavior. For example, large amount of ablation of the breech was observed during the late phase of the discharge[4]. Due to poor coupling of the Lorentz force to this cold

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material, it acquires very little velocity. Though this ablation could be caused by direct radiation from the main plasma, more likely than not, the main source of this late-time ablation is due to the thermal transport from secondary arcs that may be formed by the following processes:

- Arc restrike at the breech, or at any parts of the accelerator between the breech and the main plasma.
- Instabilities of the main plasma leading to its fragmentation.

Any electrode erosion associated with these processes would further add to the parasitic propellant mass.

2. Low pulsed-power driver efficiency. This is caused by the usually large mismatch in the impedance of the pulsed-power driver (the capacitor, the switches, the transmission lines) and the PPT, especially in experimental laboratory PPT’s. PPT’s are low-inductance devices, whereas laboratory pulsed power supplies typically have large parasitic inductances. Significant amount of electrical energy remains in the external circuit after the main plasma has left the accelerator in typical experimental devices. This residual energy is often wasted in producing undesirable discharges and entropy inside the accelerator.

3. Electrode erosion.

4. Reliability. Fatigue in the various components is an important life-time issue for the repetitive operation of a pulsed plasma thruster system for any practical operation. External switches are particularly vulnerable to this issue.

3. A High-Energy Pulsed Plasma Thruster

To make our discussion concrete, we will introduce our discussion in the context of designing a pulsed electromagnetic plasma accelerator as a driver for an experiment in magnetized target fusion (MTF) [5]. For a near-term physics exploratory experiment to study the formation of a plasma shell for MTF, a pulsed plasma accelerator is required to launch pulsed plasma jets with a mass approximately 1 mg to more than 200 km/s [6]. Twelve of these plasma jets are used simultaneously to implode a target placed at about 0.5 m away. For this application, extremely low temporal and spatial jitters of the launch (less than 100 ns and 1 cm respectively). Good focusing and collimation properties of the plasma jet are additional requirements that are well beyond the usual requirements for propulsion applications.

The baseline concept for this purpose consists of a shaped coaxial plasma accelerator as shown in Figure 1. The plasma design in support of the concept is the main objective of the present paper and will be developed in the next two sections below. In this section, we will describe the engineering concepts and schemata that serve to implement the plasma design.

The accelerator consists of a pair of coaxial electrodes. The outer electrode has a diameter of 20 cm and is about 0.75 m long. The inner electrode has a diameter of 10 cm, and about 0.5 m long. Both electrodes have a gradual conical taper and an electromagnetic focusing section (much like a plasma focus device) to induce a
Figure 2. The three regions of the plasma: (1) The plasma edge region, (2) the plasma interior, (3) the plasma front. The curves denoted as B and j are the profiles of the magnetic field and current density.

4 The Plasma Physics Prescription

The principal ‘malfunction’ of the plasma dynamics in typical PPT’s is the failure of the magnetic field to confine the current distribution to a finite region in the accelerator. As a result secondary arcs are formed, either in the form of a restrike or a fragmentation of the main plasma due to plasma instabilities. As the velocity of the current “sheet” increases, the motional back emf (= I L/v) also increases. This voltage is applied at the breech to the accelerator. The bore of the accelerator is an extremely active environment electrically. It is permeated by a relative abundance of charged particles and neutrals left behind by the moving plasma, or from the products of ablation, or by the photoionization of the neutrals resulting from any gas in the bore before the pulse. If there is any significant amount of neutral particles present behind the plasma, Townsend avalanche could occur producing restrike. The presence of neutrals behind the plasma is a principal cause for the formation of secondary arcs.

The ratio (β) of the plasma (thermal) pressure (nkT) to the magnetic pressure (B²/2μo) is an important parameter in determining the quality of confinement of a plasma by a magnetic field. The intrinsic difficulty in accelerating a plasma by the self field of the driving current lies in the fact that the plasma β varies from 0 to ∞ over the thickness of the plasma current sheet. Thus, while the upstream (rear) part of the plasma may be strongly magnetized so that it can be ‘dammed’ by the magnetic field, the downstream (frontal) part of the plasma is at most weakly magnetized so that the only way to dam this part of the plasma is by collisions between the plasma particles.
This is done by driving the plasma with current density. The difficulty is compounded if the plasma is not fully ionized. We will at the outset demand that our plasma be nearly fully ionized by driving the plasma to temperatures much higher than is usual in past PPTs. This is done by driving the plasma with current density levels which are orders of magnitude higher than is usual, and thus obtaining plasma temperatures of several eV's (say, greater than 4 eV).

Our plasma strategy consists of firstly demanding the plasma interior to be highly dense with a high degree of collisionality to ensure that the plasma behaves like a fluid. Secondly, the plasma edge region (upstream) in contact with the 'vacuum' magnetic field to be strongly magnetized to ensure magnetic confinement. Thirdly we require that the region behind the plasma to be strongly magnetized and collisionless to the extent that the magnetic field provides electrical insulation like the way an ordinary insulator is used in preventing arcing. This last requirement puts an upper bound on the gas density in the accelerator before the pulse, i.e. the vacuum requirement for the accelerating chamber. Fourthly, we will seek to avoid any possible complications from any significant space charge effects by demanding quasi-neutrality. We will now quantify the underlined terms.

For collisionality in the plasma interior, we require that random walk of the particles due to collisions will not travel more than a prescribed percentage (denoted as \( \gamma_i \)) of the plasma dimension \( (L_p) \). The distance traveled by a random walk in \( N_c \) collisions is \( \ell_p = \sqrt{N_c \lambda} \) where \( \lambda \) is the mean free path between collisions. For a fully ionized plasma the electron-ion mean free path is the relevant collision length. Let the desired acceleration time be \( t_p \). Our first collisionality requirement leads to

\[
\lambda_e \sqrt{N_c} = \left( \frac{v_{tr}}{v_{ei}} \right) \sqrt{v_{tr} t_p} \leq \gamma_i L_p \tag{4.1}
\]

where \( v_{tr} \) is the thermal speed of the electrons \( \left( = \sqrt{\frac{kT_e}{m_e}} \right) \), \( v_{ei} \) is the electron-ion collision frequency given in SI units as \( [7] \),

\[
v_{ei} = b Z^2 n_e \ln \Lambda \left( \frac{T_e^{3/2}}{m_e} \right) = \frac{2 \pi^2 \epsilon_0^2 k_e^{3/2}}{12 \pi^2 \epsilon_0 k_e^{3/2} m_e^{1/2}} = 3.6332 \times 10^6
\]

The above expressions yield for the ion density required to be,

\[
n_i \geq \frac{1}{b} \left( \frac{\lambda_e}{v_{ei}} \right) \left( \frac{T_e^{3/2}}{Z^2 \ln \Lambda} \right) \left( \frac{t_p}{\gamma_i L_p} \right) \tag{4.3}
\]

where \( k \) is the Boltzmann's constant, \( m_e \) the electron mass, \( T_e \) the electron temperature. SI units are used throughout this paper, including the temperature being given in degrees Kelvin. In \( \Lambda \) is the usual Coulomb logarithm. Using the design of PEPA-1 as an example, with \( t_p = 5 \mu s \), and the reasonable choices of 0.1 for \( \gamma_i \) and the plasma dimension \( L_p \) half the interelectrode gap = \( 0.5(r_2 - r_1) = 2.54 \) cm, an electron temperature of 4 eV, and assuming \( Z = 1 \), the above expression gives \( n_\text{i} = 2.8 \times 10^{23} \) per m\(^3\), about two orders of magnitude higher than what is typical in past PPT experiments. With the above prescription of the main plasma, the plasma has a mass of about 0.98 mg, as required for our experimental accelerator PEPA-1.

For the plasma edge region to be strongly magnetized, the electron Larmor radius should be smaller than the electron-ion collision mean free path. This is the same as requiring the Hall parameter \((\alpha \times_t \tau_e)\) to be greater than 1. To indicate the trend and the order of magnitudes of the magnetic field required, a Hall parameter of at least 2 leads to a magnetic field of at least \( 3.3 \) T. Taking this to be the magnetic field at a point midway between the inner and outer electrodes, the driving current required to produce this magnetic field is \( 1.26 \) MA. Again, past PPT experiment uses magnetic field typically of the order of \( 0.1 - 0.5 \) T. We note that, current conduction in a strongly magnetized plasma can only occur across the magnetic field in the presence of a Hall electric field to provide the necessary E x B drift. Thus, the Hall current is an essential part for the proper operation of a pulsed plasma thruster. We also note that the choice of a Hall parameter of the order of unity will lead to some canting of the current sheet. But the canting of the current sheet by itself does not necessarily degrade the performance of the accelerator as the forward component of the net Lorentz force on the plasma is independent of the current distribution within the plasma so long as the current distribution is steady with respect to the current. Canting of the current sheet, however, could lead to higher, unbalanced plasma pressure and density against the electrodes resulting in higher skin friction and other plasma boundary effects.

Immediately behind the plasma, even stronger magnetization is required, so that any electrically charged particles that might be present in this region will be confined to move in circles about the magnetic field lines with radii considerably smaller than the collision mean free path and the bore dimensions. Collisions will allow the particles to diffuse across the field lines with the potential for producing the very dangerous
Townsend avalanche. Because of the proximity to the plasma, any charged particles present in this region will be assumed to have a kinetic temperature similar to that of the plasma.

In the region behind the plasma, the strong magnetization requirement applies to the ions as well as the electrons whose Larmor radius are much smaller than the ions for the same kinetic temperature. This requires that the ion Larmor radius to be substantially smaller than the smallest collision mean free path, i.e., the electron-ion mean free path. This is equivalent to requiring that the ion Hall parameter \((\omega \tau_n)\) for this 'vacuum' region to be extremely large. The difficulty to proceed with the design here is the lack of appropriate experimental data that could help us with making suitable design choices for this important parameter. This an obvious area for future experimental research.

Some theoretical guidance may be provided by a review of the transport processes in a plasma. According to Braginskii [8], in a single-fluid magnetohydrodynamic description of a plasma, the 'effective' electric field \(E'\) in the presence of a magnetic field \(B\) at a position in a plasma is related to the current density \(J\) flowing at that position by,

\[
E' = \frac{j_{\parallel}}{\sigma_{\parallel}} + \frac{j_{\perp}}{\sigma_{\perp}} \frac{j \times B}{n_e e} \tag{4.4}
\]

where,

\[
E' = E + v \times B + \frac{1}{ne} (\nabla p_e - R_r) \tag{4.5}
\]

In the above expressions, the subscripts \(\parallel\) and \(\perp\) indicate the components of the vector or the physical property parallel and perpendicular to the magnetic field, \(\sigma\) is the plasma electrical conductivity, \(n\) is the electron density, \(e\) is the electronic charge, \(p_e\) is the electron pressure, and \(R_r\) is the thermal force on the electrons due to electron temperature gradients.

In Expression (4.4), the second term involving the perpendicular resistivity \(1/\sigma_{\perp}\) is a result of momentum transport across the field lines due to electron-ion collisions. The perpendicular resistivity is at most twice the parallel resistivity \(1/\sigma_{\parallel}\). Looking at this term alone, it might seem that it is not possible to use magnetic field to suppress the flow of current behind the plasma. This is erroneous because the flow of such a current behind the plasma across the magnetic field lines will necessarily produce the Hall electric field given by the third term on the right hand side of the expression. This Hall electric field produces a \(E \times B\) drift in the opposite direction to the original current, and reduces the net current flow. To see this, the above expression can be inverted as was done by Braginskii to give the current density in terms of the electric field as,

\[
j = \frac{\sigma_{\parallel}}{(1 + \omega_{\parallel} \tau_n)} \left[ E'_{\parallel} + \omega_{\perp} e_{\perp} \times E' \right] \tag{4.6}
\]

where \(e_{\parallel}\) is a unit vector in the direction of the magnetic field, \(\omega_{\parallel}\) is the electron cyclotron frequency, and \(\tau_n\) is the electron-ion collision time. \(\omega_{\parallel} \tau_n\) is known as the Hall parameter. It is seen from the above expression that with a sufficiently large value of the Hall parameter, the current density across the field lines can be significantly suppressed. The above expression may also be used as a basis to explain the canting of the current sheet in traditional PPT's.

From the above expression, it follows that for large value of the Hall parameter, the current density is reduced by a factor in inverse proportion to the Hall parameter. Thus by choosing the Hall parameter suitably large, current flowing in the region behind the plasma can be suitably suppressed. To bootstrap the experimental development of PEPAT, we will use a value of at least \(10^5\) for the design of the experiment. This design choice is far from being an overkill because of the large electrode surface area behind the plasma compared to the conduction area for the plasma. To ensure that any current flowing in this region will be small compared to the main plasma current, a large value of this order would indeed be required for the Hall parameter. Using this value, the particle density in the region behind the plasma needs to be maintained at below \(4 \times 10^{19}\) per \(m^3\). This is a vacuum prescription for the accelerating chamber prior to the shot. Helium has the property of being difficult to break down. Using helium to flush out the accelerator before firing, a vacuum of below 1 milli-torr is required.

To check for the plausibility of our assumption for the plasma temperature, an estimate for the plasma temperature may be obtained by balancing the ohmic heating in the plasma against the radiation losses. Expression (4.6) may be used conveniently to provide an estimate for the ohmic heating in terms of the effective electric field (in the frame of the moving plasma) as follows:

\[
\dot{Q}_b = \iiint_V jE'dr, \quad V \rightarrow \text{plasma volume}
\]

\[
= \pi (r_2^2 - r_1^2) \omega_j E'
\tag{4.7}
\]
where, 

\[ J \cdot E' = \sigma E'^2 + \frac{\sigma}{1 + (\alpha_\lambda \tau_{\alpha})^2} E'^2 \]

\[ \sigma = e^2 n_{e} r_{n}, \quad \tau_{\alpha} = \frac{1}{\nu_{\alpha}} \tag{4.8} \]

and \( v_{n} \) is given by (4.2). The effective electric field is evaluated as the mean voltage gradient between the electrodes. Combining with the accelerator current determined earlier, the ohmic heating rate provides an estimate for the plasma resistance, which is then used iteratively to determine a self consistent value of the voltage gradient between the electrodes. The iterative procedure yielded a plasma voltage of approximately 750 V and an ohmic heating rate for the plasma of 9 x 10^8 W. The radiative loss calculated as a blackbody radiation assuming a reasonable value of 0.5 for the plasma emissivity is 6 x 10^8 W, indicating that the assumed plasma temperature of 4 eV can be sustained by the ohmic heating. The whole procedure may be iterated to provide a more consistent estimate of the plasma temperature.

Finally we check for quasi-neutrality. This requires that the Debye length is small compared with the plasma dimension, \( \Lambda_{D} \ll L \). With the above prescription for the plasma condition, the Debye length may be checked to be approximately 20 nm, which is sufficiently smaller than the plasma dimension.

We are now ready to run a lumped element (0-D) simulation code to model the performance of the accelerator, assuming that the plasma will stay together and be accelerated as a “slug”. This is done in the next section.

5 Lumped Element Modeling Results

The circuit and the dynamics of the plasma slug is simulated using a 0-D plasma accelerator code in which the plasma is modeled as a lumped mass element accelerated by the Lorentz force,

\[ F_L = \frac{1}{2} L' I^2 \]

where \( I \) is the current through the plasma and \( L' \) is the inductance gradient of the plasma gun. For the circuit, the capacitor bank is modeled as a pulse forming line as shown in Figure 3. Kirchoff’s Law for the circuit, the equation of motion for the plasma slug and the equation for the Coulomb electrode erosion can be written as:

\[ (L + L_{C}) \frac{d\bar{d}}{dt} = \bar{V}_{t} - \left( R_{h} + R_{g} + R_{p} + L' \nu \right) I \]

\[ L' = \left( \frac{\mu}{2\pi} \right) \ln \left( \frac{b}{a} \right), \quad L_{C} = L' z, \quad R_{p} = R' z \left( \frac{1}{A_{in}} + \frac{1}{A_{out}} \right) \]

\[ L_{i+1} \frac{d\nu_{i}}{dt} = \nu_{i+1} - \nu_{i}, \quad C_{i} \frac{dV_{i}}{dt} = -(I_{i} - I_{i+1}), \quad i \geq 1, \]

\[ \frac{d\nu_{p}}{dt} = \frac{1}{2} L_{p} \nu_{p} - \frac{1}{2} C_{p} \nu_{p}^{2} A_{p} \left( C_{i} \right), \quad \frac{dL_{p}}{dt} = m_{p} I \]

where \( L_{1}, L_{k} \) are the transmission inductance (bus-bar and the internal inductance of the capacitor) and the time-dependent inductance of the plasma gun, \( C_{i} \) is the capacitance of the capacitor connected to the plasma gun, \( R_{h}, R_{g}, R_{p} \) are the resistances of the transmission (bus-bars), the conductors of the gun, and the plasma sheet respectively, \( R' \) is the resistivity of the gun conductors, \( A_{in}, A_{out} \) are time-dependent cross-sectional areas of current conduction of the inner and outer conductors (electrodes) of the coaxial plasma gun, taking into account the skin depth due to the pulsed nature of the current, \( b \) and \( a \) are the outer and inner radii of the electrodes respectively. The PFN sections are numbered as section \( i = 1, 2, 3, ..., \) counting from the section nearest to the gun, \( C_{i} \) and \( L_{i} \) are the capacitance and the inductance of the \( i \)-th PFN section. \( I_{i} \) is the current through \( L_{i} \) and \( V_{i} \) the voltage on \( C_{i} \). For the
equation of motion, we have included the effects of the 
entrainment of the products of electrode erosion in the 
plasma as well as the effects of skin friction between the 
plasma and the electrodes. \( c_p \) is the skin-friction 
coefficient. The last equation gives the rate of mass 
entrainment in the plasma due to electrode erosion 
associated with charge transfer. The coefficient \( m'E \) 
gives the electrode erosion rate in kg/C of charge 
transfer.

The code (PFNX_plasma) uses a \( 4 \, (1/2) \) - order Runge-
Kutta-Fehlberg differential equation solver. Although 
the effect of tapering the electrodes with the attendant 
variation in \( L' \) as a function of the plasma current in the 
gun can be simulated, in order to keep the parametric 
exploration tractable, a nominal ratio of the radii of the 
outer and inner electrodes is assumed. Figures 4 to 7 
show the results of a run with the following circuit 
parameters: 4 capacitors each of 17.5 \( \mu \)F are connected 
to the accelerator in parallel and charged to 40 kV. The 
parallel inductance of the capacitors and the connection 
to the accelerator is held to 15 nH. Figure 4 shows the 
current pulse shape versus time, Figure 5 shows the 
plasma velocity versus time, Figure 6 shows the plasma 
velocity versus the length of acceleration. Figure 7 
shows the capacitor voltage versus time. The figures 
show the results for four values of the skin-friction 
coefficient, \( C_D = 0.001, 0.01, 0.1, \) and 1. The results 
show that the velocity goals of \( 200 \) km/s can be obtained 
if the skin friction coefficient is less than 0.01. If the 
skin friction coefficient is 0.1 or 1, the skin friction has a 
dramatic effect on the performance of the thruster.

6 Summary

In this paper, we give a discussion of the fundamental 
plasma physics issues governing the pulsed acceleration
of a plasma sheet by the self field of the driving current. A major shift in paradigm in driving pulsed plasma thruster is necessary if the original goal of accelerating a plasma sheet efficiently to high velocities as a plasma "slug" is to be realized. Firstly, the plasma interior needs to be highly collisional so that it can be dammed by the plasma edge layer (upstream) adjacent to the driving 'vacuum' magnetic field. Secondly, the plasma edge layer needs to be strongly magnetized so that its Hall parameter is of the order of unity in this region to ensure excellent coupling of the Lorentz force to the plasma for good magnetic confinement. Thirdly, to prevent and/or suppress the occurrence of secondary arcs or restrike behind the plasma, the region behind the plasma needs to be collisionless and extremely strongly magnetized with sufficiently large Hall parameter. These requirements are quantified in the paper.

Three new design parameters are introduced corresponding to these three plasma requirements. The first parameter, labeled in the paper as $\gamma_1$, pertains to the permissible ratio of the diffusive excursion of the plasma during the course of the acceleration to the plasma longitudinal dimension. The second parameter is the required Hall parameter of the edge plasma region, and the third parameter is the required Hall parameter of the region behind the plasma. Experimental research is required to quantify the values of these design parameters. Based upon fundamental theory of the transport processes in plasma, we provide some theoretical guidance on the choice of these parameters to help designing the necessary experiments to acquire these data.

Finally we remark that the plasma requirements prescribed in the paper need to be extended to include the considerations of plasma instabilities of the main plasma, especially the Rayleigh-Taylor instability. This will be treated in future work.

References


