CONTAINERLESS RIPPLE TURBULENCE

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ABSTRACT

One of the longest standing unsolved problems in physics relates to the behavior of fluids that are driven far from equilibrium such as occurs when they become turbulent due to fast flow through a grid or tidal motions. In turbulent flows the distribution of vortex energy as a function of the inverse length scale [or wavenumber ‘k’] of motion is proportional to $1/k^{5/3}$ which is the celebrated law of Kolmogorov. Although this law gives a good description of the average motion, fluctuations around the average are huge. This stands in contrast with thermally activated motion where large fluctuations around thermal equilibrium are highly unfavorable. The problem of turbulence is the problem of understanding why large fluctuations are so prevalent which is also called the problem of “intermittency”.

Turbulence is a remarkable problem in that its solution sits simultaneously at the forefront of physics, mathematics, engineering and computer science. A recent conference [March 2002] on “Statistical Hydrodynamics” organized by the Los Alamos Laboratory Center for Nonlinear Studies brought together researchers in all of these fields. Although turbulence is generally thought to be described by the Navier-Stokes Equations of fluid mechanics the solution as well as its existence has eluded researchers for over 100 years. In fact proof of the existence of such a solution qualifies for a $1M$ millennium prize.

As part of our NASA funded research we have proposed building a bridge between vortex turbulence and wave turbulence. The latter occurs when high amplitude waves of various wavelengths are allowed to mutually interact in a fluid. In particular we have proposed measuring the interaction of tipples [capillary waves] that run around on the surface of a fluid sphere suspended in a microgravity environment.

The problem of ripple turbulence poses similar mathematical challenges to the problem of vortex turbulence. The waves can have a high amplitude and a strong nonlinear interaction. Furthermore, the steady state distribution of energy again follows a Kolmogorov scaling law; in this case the ripple energy is distributed according to $1/k^{7/4}$. Again, in parallel with vortex turbulence ripple turbulence exhibits intermittency [Wright et al Science 278, 1609 (97)].

The problem of ripple turbulence presents an experimental opportunity to generate data in a controlled, benchmarked system. In particular the surface of a sphere is an ideal environment to study ripple turbulence. Waves run around the sphere and interact with each other, and the effect of walls is eliminated. In microgravity this state can be realized for over 2 decades of frequency.

Wave turbulence is a physically relevant problem in its own right. It has been studied on the surface of liquid hydrogen [Brazhnikov et al] and its application to Alfven waves in space is a source of debate.[P. Goldreich]. Of course, application of wave
turbulence perspectives to ocean waves has been a major success of V. E. Zakharov. [Kolmogorov Spectra of Turbulence, Springer, Berlin 1992].

The experiment which we plan to run in microgravity is conceptually straightforward. Ripples are excited on the surface of a spherical drop of fluid and then their amplitude is recorded with appropriate photography. A key challenge is posed by the need to stably position a 10cm diameter sphere of water in microgravity. Two methods are being developed. Orbitec is using controlled puffs of air from at least 6 independent directions to provided the positioning force. This approach has actually succeeded to position and stabilize a 4cm sphere during a KC 135 segment. Guigne International is using the radiation pressure of high frequency sound. These transducers have been organized into a device in the shape of a dodecahedron. This apparatus ‘SPACE DRUMS’ has already been approved for use for combustion synthesis experiments on the International Space Station.

A key opportunity presented by the ripple turbulence data is its use in driving the development of codes to simulate its properties. A head start on this aspect of the project is being developed at NASA Glenn Research Center.
SCIENCE REQUIREMENT

What are the differences in the way in which energy is distributed in turbulent flow and in thermal equilibrium?

**Thermal Equilibrium**

Experimentally controlled parameter

\[ T = \text{temperature} \]

**Turbulence**

\[ q = \text{energy throughput} = \text{ergs/cm}^2\text{sec} \]

Probability distribution for wave motion

\[ P(\{\varepsilon\}, T) \sim e^{-\Sigma (\varepsilon_i/k_B T)} \]

\[ P(\{\varepsilon\}, q) = ?? \]

\[ \varepsilon_i = \text{energy in mode } i \]

\[ \langle \varepsilon_i \rangle = k_B T \]

\[ \langle \varepsilon_k \rangle \sim \frac{q^{1/2} (\rho \sigma)^{1/4}}{k^{11/4}} \]

"Equipartition"

"Kolmogorov cascade"

Gaussian

Intermittency

boring

interesting

Competition between randomization and structure
Proposed Flight Experiment

Acoustic Positioning and Excitation

Pulsed Light Source

Fiber Optic

Optics

CCD Camera

12 cm

\( \omega + \omega \rightarrow 2\omega \)

\( 2\omega + 2\omega \rightarrow 4\omega \)

\( \cdots \)

Cascade
Big whorls have little whorls  
Which feed on their velocity;  
And little whorls have lesser whorls,  
And so on to viscosity  
(in the molecular sense)  
...  
Richardson 1926

The wind comes in gusts  
... attributed to Landau  
1942-Kazan

When I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics and the other is turbulent motion of fluids. And about the former I am really optimistic.

Sir Horace Lamb 1932
Excitation and Measurement of Ripples [Ground-based]
Power Spectrum of Ripple Turbulence

\[ \xi^2(k) \text{ (cm}^2/\text{cm}^{-1}) \]

\[ \text{wavenumber (cm}^{-1}) \]

Frequency (Hz)

Surface Acceleration

- 0.52 g
- 1.00 g
- 2.16 g

\[ k^{-15/4} \]

Threshold for turbulence

\[ k^{-4.2} \]
POWER SPECTRUM FOR RIPPLE TURBULENCE

\[ \omega^2 = (\sigma / \rho)k^3 \]

\( \sigma = \) surface tension

cascade from \( k \rightarrow 2k = \)

\[
\frac{\partial E_k}{\partial t} = \omega_k G^2 E_k \frac{E_k}{\sigma} - 4\mu k^2 E_k
\]

\( E_k \approx \sigma k^2 \xi^2 = \) ripple energy between \( k \) and \( 2k \)

\( \mu = \) kinematic viscosity \( \sim \) .01 cm\(^2\)/s for water,
\( G^2 = 8\pi^4/13 = \) nonlinear coefficient

\[
\frac{\partial E_k}{\partial t} = q = \frac{\text{ergs}}{\text{cm}^2\text{s}} = \text{energy throughput}
\]

in the steady state, \( E_k \sim \left[q\sigma / g^2 \omega_k \right]^{1/2} \).

therefore spectral density

\( e(\omega) \sim 1/\omega^{3/2} \)

\( \xi^2(\omega) = \frac{\sigma^{1/6} q^{1/2} / \rho^{2/3}}{\omega^{17/6}} ; \xi^2(k) = \frac{q^{1/2} \rho^{1/4} / \sigma^{3/4}}{k^{15/4}} \)

\[ \langle \xi^2(r) \rangle = \langle \xi^2(t) \rangle = \int \xi^2(k)dk = \int \xi^2(\omega)d\omega \]
OPERATIONAL DEFINITION OF TURBULENCE

\[ \left( \frac{\partial E_k}{\partial t} \right)_+ = G^2 \omega_k E_k \frac{E_k}{\sigma} - 4\mu k^2 E_k \]

1) Interaction rate due to nonlinear scattering:

\[ \frac{1}{\tau_+} \equiv \frac{1}{\tau_+} \equiv G^2 \omega_k \frac{E_k}{\sigma} \gg 4\mu k^2 \]

or \( M^2 > 1 / G^2 Q \),

where \( M \) is the Mach number and \( Q \) is the quality factor.

2) There must be many modes in the bandwidth determined by \( \tau_+ \):

\[ n(\omega) / \tau_+ \gg 1 \]

where \( n \) is the density of modes

\[ n(\omega) \approx \frac{S \omega^{1/3}}{3\pi (\sigma / \rho)^{2/3}} \]

and \( S \) is the surface area.

Turbulence, not Chaos!

\[
\begin{align*}
S &= 10 \times 10 \text{ cm}^2, \quad \omega = 100 \text{ Hz} \\
\frac{n}{5 \text{ seconds}} &= \frac{10^5}{10^4} \\
M^2 &\sim 1, \quad G \sim 5, \quad n / \tau_+ \sim 10^4
\end{align*}
\]
STOSSZAHL-ANSATZ

\[ <I_i> = \int I_i P d\Omega d\theta \]

Kinetic Equation for Waves:

\[ \frac{d}{dt} <I_i> - \epsilon^2 c^2 \sum \delta(\omega_{ijk}) <I_j I_k> \]

To close the equation requires a stosszahl ansatz - or equivalent:

\[ <I_i I_j> = <I_i> <I_j> \]

RENORMALIZED WAVE VELOCITY:

\[ \dot{\nu}_i = \omega_i + \int \phi(I) \nu d\nu d\theta = \omega_i + \int \phi(I) \nu d\nu d\theta \]

where \( \phi = \theta - \omega t \)

\[ P_i = i \pi \epsilon \sum_{ijk} \delta^{-}(\omega_{ijk}) c I^{3/2} \epsilon^i \Theta \nabla \bar{P} \]

\[ \frac{d}{dt} [<I_p I_q> - <I_p> <I_q>] \sim \epsilon^2 N C_{jk,q} \]
Figure 4: Power spectrum versus dimensionless frequency $\bar{\omega}/2\pi$ for the data shown in Figure 2. The vertical lines along the frequency axis are the natural frequencies $\bar{\omega}_l/(2\pi)$ for each of the symmetric spherical harmonics $l = 2, \ldots, 86$. 
\[ M_f^2 = M_0^2 \frac{f_0}{f} \]

\[ M_f^2 = (5) \frac{1}{Q_f G^2} = 4 \times 10^{-4} \left( \frac{f}{Hz} \right)^{\frac{\lambda}{3}} \]

Threshold for Turbulence

Breaking

Turbulent Spectrum

\[ \lambda = 0.6 \text{ mm} \]
Fluctuations Around Kolmogorov Spectrum are Large

\((\lambda_0 \nabla^2 \zeta)^2 > 5 \text{ rms for 16 pixels are indicated by blackening those regions}\)
Plot of local dissipation

Intermittency in Turbulence
Space-DRUMS®
Processing Chamber
and Material
Handling System

ISS Configuration
for inside Processing
Module

Launch ULF1