Final Technical Report for NAG5-7574

HIGH-RESOLUTION GRAVITY AND TIME-VARYING
GRAVITY FIELD RECOVERY USING GRACE AND CHAMP

Investigation Time Period: August 1, 1998 - July 31, 2002

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SUMMARY

This progress report summarizes the research work conducted under NASA's Solid Earth and Natural Hazards Program 1998 (SENH98), Contract No. NAG5-7574, entitled High-Resolution Gravity and Time-Varying Gravity Field Recovery Using GRACE and CHAMP, for the time period from August 1, 1998 through July 31, 2002, which included a no-cost extension time period.

The investigation has conducted pilot studies to use the simulated GRACE and CHAMP data and other in situ and space geodetic observable, satellite altimeter data, and ocean mass variation data to study the dynamic processes of the Earth which affect climate change. Results from this investigation include (1) a new method to use the energy approach for expressing gravity mission data as in situ measurements with the possibility to enhance the spatial resolution of the gravity signal; (2) the method was tested using CHAMP and validated with the development of a mean gravity field model using CHAMP data, (3) elaborate simulation to quantify errors of tides and atmosphere and to recover hydrological and oceanic signals using GRACE, results show that there are significant aliasing effect and errors being amplified in the GRACE resonant geopotential and it is not trivial to remove these errors, and (4) quantification of oceanic and ice sheet mass changes in a geophysical constraint study to assess their contributions to global sea level change, while the results improved significant over the use of previous studies using only the SLR-determined zonal gravity change data, the constraint could be further improved with additional information on mantle rheology, PGR and ice loading history.

A list of relevant presentations and publications is attached, along with a summary of the SENH investigation generated in 2000.

HIGH-FREQUENCY ENHANCEMENT OF GRACE/CHAMP SIGNAL

The primary purpose is to address the potential advantage to preserve the high-frequency content of gravity mapping mission measurements such as CHAMP and GRACE at a localized region and time; as opposed to the uniform resolution of the gravity field determination, e.g., for one month in the case of GRACE. A method is developed for the difference of gravitational potential between two close Earth-orbiting satellites in terms of measured range-rates, velocities and velocity differences and specific forces, using the conservation of energy approach, assuming that the non-conservative forces are measured by onboard accelerometers [Jekeli, 2000]. The method has been generalized to apply to high-low GPS tracking satellite equipped with 3-axis accelerometer, such as CHAMP.
Based on energy considerations, the model specifically accounts for the time variability of the potential in inertial space, principally due to Earth’s rotation. Analysis shows the latter to be a significant (±1 m²/s²) effect that overshadows by many orders of magnitude other time dependencies caused by solar and lunar tidal potentials. Also, variations in Earth rotation with respect to terrestrial and celestial coordinate frames are inconsequential. Results of simulations contrast the new model to the simplified linear model (relating potential difference to range-rate) and delineate accuracy requirements in velocity vector measurements needed to supplement the range-rate measurements. The conclusion is that accuracy in the differenced velocity vector of 2\times 10^{-5} \text{ m/s} would be commensurate within the model to the anticipated accuracy of 10^{-6} \text{ m/s} in GRACE range-rate precision [Jekeli, 2000].

A concise discussion of this approach using GRACE is as follows. The potential difference between two satellites expected from the satellite-to-satellite tracking mission in low-low mode like GRACE can be computed by measuring the range-rates, velocity vectors, and position vectors in the inertial frame:

\[ V_{12} = V_2 - V_1 = \lvert \dot{x}_1 \rvert \hat{n}_{12} \]

This model relates the in situ inter-satellite range rate measurements to the gravitational potential difference between two satellites, \( V_{12} \). This model, however, is not appropriate to take full advantage of the current instrument’s capability. Especially, it does not include the time-variable effect of gravitational potential due to the Earth rotation, which is significant on the order of ±1 m²/s². Considering some of these significant effects, the new and rigorous model is developed using the energy conservation principle [Jekeli, 1999]. By correcting the mistakes in (27) of Jekeli [1999], reformulating it in terms of the disturbing potential (Earth’s gravitational potential minus normal gravitational potential) difference, \( T_{12} \), instead of the residual potential difference, and assuming the energy dissipation term is corrected by accurately measuring non-gravitational accelerations, the correct model is given by:

\[ T_{12} = \lvert \dot{x}_1 \rvert \hat{n}_{12} + v_1 + v_2 + v_3 + v_4 + \dot{\delta} V R_{12} - \dot{\delta} E_0, \]

where \( v_1 = \left( \dot{x}_2 - \dot{x}_1 \right) \hat{a}_{12}, \quad v_2 = \left( \dot{x}_1 - \right. \left. \dot{x}_1 \right) \hat{a}_{12}, \quad v_3 = \hat{a} \times \hat{a}_{12} \cdot \hat{a}_{12}, \quad \text{and} \quad v_4 = \frac{1}{2} \| \ddot{a}_{12} \|^2. \)

The superscript, 0, denotes the quantity based on the reference field as GRS80 that is known, and the symbol, \( \delta \), indicates the residual quantity between the true field and the reference field. The sixth term of the right hand side is the potential of rotation difference between two satellites, which can be computed with a linear combination of positions and velocities of two satellites in both fields. The last term is the residual energy constant of the system. The constant term is not our interest because it corresponds to zero degree and zero order spherical harmonics and will not be estimated. This model indicates that the accurate potential difference between two satellites can be obtained by measuring the inter-satellite range rate as well as the position vectors and
velocity vectors, which are available from GPS. Again, the on-board accelerometers are assumed to measuring all non-conservative forces acting on the satellites and that the dissipating energy is removed accordingly.

The expected range rate accuracy from K-band ranging of GRACE mission is about 0.1 \( \mu \text{m/s} \) [Kim et al., 2001; Kim in UT/CSR, personal communications, 2002] and this corresponds to the potential difference accuracy in the level of \( 10^{-3} \text{ m}^2/\text{s}^2 \). In order to take a full advantage of this high-precision range rate measurements, the commensurate accuracy of a single satellite’s position and velocity should be less than 7 cm and 5 \( \mu \text{m/s} \), respectively, and that of inter-satellite baseline position and velocity should be less than 0.1 mm and 2 \( \mu \text{m/s} \), respectively, [Jekeli, 1999]. These high precision orbital parameters might be obtainable with the aid of high precision range and range-rate measurements together with the Blackjack class GPS receiver. The registration of the observable causes error as well, because of the imperfect orbit. It, however, is not very sensitive to GRACE potential “difference” observable, because the orbit error of two satellites would be highly correlated [Jekeli, 2000]. The orbit, therefore, would be fixed when the relationship between the observable and the unknown geopotential coefficients is established.

**CHAMP MEAN GRAVITY FIELD SOLUTION**

We demonstrate the technique and using an efficient method to determine mean gravity field model using data from accelerometer- and GPS-equipped satellites, such as CHAMP. On the basis of the conservation of energy principle, in situ (on-orbit) and along track disturbing potential observable were computed using 16-days of CHAMP data. The global disturbing potential observable was then used to determine a 50×50 test gravity field solution (OSU02A) by employing a computationally efficient inversion technique based on conjugate gradient [Han et al., 2002a]. An evaluation of the model using independent GPS/leveling heights and Arctic gravity data, and comparisons with existing gravity models, EGM96 and GRIM5C1, and new models, EIGEN1S and TEG4 which include CHAMP data, indicate that OSU02A [Han et al., 2002b] is commensurate in geoid accuracy and, like other new models, it yields some improvement (10% better fit) in the polar region at wavelengths longer than 800 km.

In the spatial domain, we compared the models (all truncated at degree 50) in terms of the RMS geoid differences over all longitudes per latitude. Fig.1 shows these RMS differences for the cases of EGM96-OSU02A, TEG4-OSU02A, GRIM5C1-OSU02A, and EIGEN1S-OSU02A. OSU02A is closest to GRIM5C1 over the middle latitude, while the difference between them increases toward the poles. EGM96 and GRIM5C1 do not include CHAMP data and this reflects their relatively larger differences with respect to OSU02A over the Polar Regions. On the other hand, EIGEN1S, TEG4 and OSU02A models include CHAMP data, and the difference between OSU02A and EIGEN1S as well as TEG4 is not as pronounced over the poles, specifically the Antarctic region. Other assessment using GPS leveling and surface gravity data over Arctic region
(Data courtesy, S. Keynon, NIMA), indicate that the OSU02 model is comparable to other models [Han et al. 2002b].

The demonstration of the technique using CHAMP data would allow one to process GRACE data in form of disturbing potential, with a possibility to enhance the high-frequency signals regionally using data from these gravity mapping missions.

![Graph showing comparisons between different gravity field models](image)

**Fig. 1.** Global geoid difference (RMS over longitude bands) between different gravity field models

**GRACE SIMULATIONS USING IN-SITU GEOPOTENTIAL DATA**

We use the developed technique. New models of Earth's static and time-variable gravity fields will be available with a resolution of every month, which is an adequate period considering the accuracy requirement and the data coverage. For a successful recovery of these gravity fields, some temporal effects such as ocean tidal and atmospheric mass redistribution should be corrected from the GRACE measurements based on the available recent model and data. The remaining residual effects due to imperfect model and data may affect the GRACE monthly estimates significantly. Even if they are perfect, the time-variable aliasing effects may also considerably contaminate the GRACE monthly products, because the temporal sampling interval of the GRACE products is relatively low comparing periods of temporal mass redistribution signals. In addition, monthly surface water mass recovery may suffer due to short period variations of surface water mass, even if we have perfect GRACE measurements. We quantify the effects due to the inherent modeling error and temporal aliasing caused by ocean tides, atmosphere, and ground surface water mass on monthly mean GRACE gravity estimates. We show the simulation results on GRACE range-rate perturbations due to modeling error along the
orbit, and analyze their effects and temporal aliasing on the estimated gravitational coefficients by fully inverting monthly GRACE data. For the ocean tide model, some constituents like $S_2$ may cause errors 3 times larger than the measurements noises at degrees less than 15 in monthly mean estimates, while other constituents like $K_1$, $O_1$, and $M_2$ are reduced below the measurement noise level by one month averaging. For the atmosphere, ECMWF–NCEP produces errors in GRACE range-rate measurements as strong as measurement noises. They corrupt all recovered coefficients and provide 30% more errors in the global monthly geoid estimates up to maximum degree 120. However, the analysis based on daily CDAS-1 data for continental surface water mass redistribution indicates that the daily soil moisture and snow depth variations affect less the monthly mean GRACE recovery than the measurement noise. The temporal aliasing effect of daily variation is less than 30% of the effect of measurement noise in terms of both range-rates and geoid heights. The following sections are more detailed descriptions of quantification of the ocean tide and atmosphere effects on GRACE and recovery of hydrological signals from GRACE.

Fig. 2 shows four global maps of four (month-long) time-series of each tidal constituent error. The error tends to be small over the ocean areas, which may not represent a realistic error level in the model because the two models, NAO99 and CSR4.0, used the same TOPEX/POSEIDON (T/P) data. For constituents such as $K_1$, $O_1$, and $M_2$, the short-wavelength sectorial variation of the error is dominant, while the long wavelength features are dominant in the $S_2$ error map. The errors for $K_1$, $O_1$, and $M_2$ show consecutive positive and negative values along the longitude with a resolution of about 5–6 degrees, corresponding to spherical harmonic orders 30–36. We can expect that these sectorial variations affect the harmonic coefficients of all degrees and corresponding specific orders in the monthly mean gravity field estimates.

In the presence of these four systematic tidal model errors (but no measurement noise), four monthly mean gravity fields were recovered up to degree and order 120. For the purpose of comparison, the simulation was also performed in the presence of measurement noise only (but no tidal errors). The true coefficients were subtracted from each of the five sets of estimated geopotential coefficients to determine the distinct effects due to the tidal constituent errors and measurement noise. Figure 6 shows the degree variances of the errors existing in the recovered spherical harmonic coefficients. Except for the $S_2$ error, the tidal errors affect the low degree coefficients (below degree 30) less than the measurement noise does. The degree variances jump at degree 31 and remain constant at higher degrees. However, it should be mentioned that not all coefficients beyond degree 30 were corrupted by tidal error, because its effect is limited to a certain order and all degrees (Fig. 3). The curve for the $S_2$ error indicates that it remains and significantly corrupts the low degree harmonic coefficients in the monthly averaged field. It does not average out, having nearly the same power as its mean error shown in Figure 1, because its aliasing period is much longer than one month.

In summary, the large errors at resonant orders of GRACE orbit are not easily removable and alternative technique would have to be developed. In principal, some of the tides
except the deep aliased tides (e.g., $S_2$) can be recovered simultaneously with the geopotential coefficients.

**Fig 2.** The time-varying tidal model errors computed along GRACE orbit for 30 days in terms of the range-rate; (a) $K_1$, (b) $O_1$, (c) $M_2$, and (d) $S_2$.

**Fig 3.** Order variances of errors existing in the recovered spherical harmonic coefficients.
In this Section, we present the atmospheric simulation results conducted to assess the atmospheric modeling error and the effects of its temporal variations on the recovered gravity field coefficients. A monthly mean gravity field was recovered up to degree and order 120 in the presence of the measurement noise only, and in the presence of noise combined with the residual atmospheric perturbations (6 hour ECMWF – 6 hour NCEP). Fig. 4 shows that the geoid error (limited to degree 30) due to noise only (left) and noise combined with residual atmospheric perturbation (right). Both maps show the sectorial anomaly and the second one shows significantly larger errors, especially around Antarctica. The overall magnitude of error increases from sub-mm to mm level in the geoid height up to degree 30 due to mismodeling of the atmosphere. This implies that the changes of monthly mean geoid estimates (coefficients less than degree 30) would be significantly corrupted by the atmospheric modeling error. In addition, the RMS of global geoid error (degree up to 120) also increases from 1.90 cm to 2.51 cm, which verifies that temporal aliasing is not limited to low degree and order coefficients. Fig. 5 (top) shows partial degree variances of errors in the recovered resonant coefficients (just m=30, 31, 32, and 33) in three ways; (1) monthly solution in the presence of measurement noise only, (2) monthly solution in the presence of measurement noise and atmospheric modeling error, (3) monthly mean of daily solutions in the presence of measurement noise and atmospheric modeling error. By comparing the cases (2) and (3), we see some improvement in the recovered resonant coefficients below degree 90. However, the mean of daily solutions is not better than monthly solution at degrees higher than 90. It might be due to the fact that one day is too short a time span to recover such relatively high degrees (n ≥ 90). Fig. 5 (bottom) shows the order variances of errors for the above three cases. They were computed from degree 30 to degree 90. Each order (degrees lumped) shows slight improvement (sub-mm) in the geoid height.

In summary, atmospheric error including aliasing is a significant problem to overcome, the problem is compounded by the problem that all unmodeled errors are amplified at the GRACE resonant period.

We show results of recovery of hydrological signal using GRACE (Fig. 6). Monthly data computed as the mean Water Storage Anomaly (WSA) from CDAS-1 (hydrological model data) in the form of equivalent water height for January, 2001. No model data were used over Antarctica and Greenland areas. The overall magnitude is in a decimeter level. The monthly mean WSA is the anomalous quantity with respect to the 5 years mean field. For a month, WSA is not static of course, hence its short period temporal variability aliases the monthly mean WSA estimate. In order to quantify how much the temporal variability of WSA affects the GRACE range-rate measurements, one month of daily WSA data and monthly mean WSA data were used to compute the corresponding range-rate perturbations. To quantify the aliasing effect on the geoid, the geoid changes using the ‘truth’ MWSA coefficients and the recovered coefficients from the second test were computed the degree and order up to 30, because the error and signal spectra cross at degree 30. Fig. 6 (upper left) shows the ‘truth’ geoid change due to the monthly mean
WSA, and (upper right) shows the recovered geoid change in the presence of the measurement noise and daily WSA. Fig. 6 (lower left) shows the effect of noise and daily WSA, i.e., difference between

Fig. 4. Errors in the geoid height \( N_{\text{max}} = 30 \) due to noise only (left panel) and noise with residual atmospheric perturbation (right panel).

Fig. 5. Degree variances \((m = 30, 31, 32, \text{ and } 33)\) of errors in the geoid height (top panel); Order variances \((30 \leq n \leq 90)\) of errors in the geoid height (bottom panel).
Fig. 6. The ‘truth’ geoid change (upper left); The recovered geoid change (upper right); The effect of noise and aliasing (lower left); The aliasing effect only (lower right).

(upper panels), and (lower right) shows the effect of daily WSA (aliasing effect), i.e., difference between the recovered coefficients from the first and second tests. The global RMS values of the geoid change signal, the effect of noise, and the effect of aliasing are 2.49, 0.16, and 0.05 mm, respectively. The ratios between noise to signal, aliasing to signal, and aliasing to noise are 7, 2, and 30 %, respectively. From Figure 17 (d), we see that the effect of temporal variability of continental surface water is not limited to the continental region, instead it corrupts the gravity field globally (including the oceanic area). The overall effect of the hydrological aliasing is at the sub-mm level with respect to the global monthly mean geoid, which is three times smaller than the effect of measurement noise.

ICE SHEET AND OCEANIC MASS BALANCE AND GEOPHYSICAL CONSTRAINTS

We use satellite solutions to the low degree zonal coefficients of the Earth's gravitational potential, through $J_2$, as well as ocean and ice sheet mass variation observations from radar and laser altimetry to invert for large scale features of rates of thickness change of the polar ice sheets, and the lower mantle viscosity [Shum et al., 2000a; 2000b; Trupin and Shum, 2000a, 2000b; Trupin et al., 2000a, 2000b]. Fig. 7 shows the geographical variation of the interior Antarctica elevation change observed by ERS-1 and ERS-2 radar altimeter data, 1992-1996 [Wingham et al., 1998] (left) and the Greenland elevation
Fig. 7. Interior Antarctica elevation change observed by ERS-1 and ERS-2 radar altimeter data, 1992-1996 [Wingham et al., 1998] (left) shown geographically. Right panel shows a map of Greenland elevation change observed by airborne laser altimeter, 1993-1999 [Krabill et al., 2000].

Fig. 8. Ice sheet mass balance constraint results using the ERS-1/2 ice elevation change data, the Greenland airborne elevation change data, and the oceanic mass variation data. The best-fit lower mantle viscosity is $4.5 \times 10^{21}$ Pa-s. Sea level contributions from Greenland = 0.14, and Antarctica = 0.16 mm/yr, respectively. Total contribution is 0.3 mm/yr.
Fig. 9. Ice sheet mass balance constraint results using the $J_7$ solution from Cheng et al. [2000], and the oceanic mass variation data. The best-fit lower mantle viscosity is $10 \times 10^{21}$ Pa·s. Sea level contributions from Greenland = 0.52, and Antarctica = 0.11 mm/yr, respectively. Total contribution is 0.63 mm/yr.

Fig. 10. Thickness change for Greenland and Antarctica that is inverted from observed gravity using an iterative approach that makes no assumptions about the shape of the ice covered area.

change observed by airborne laser altimeter, 1993-1999 [Krabill et al., 2000]. Fig. 8 shows the ice sheet mass balance constraint results using the ERS-1/-2 ice elevation change data, the Greenland airborne elevation change data, and the oceanic mass
variation data. The best-fit lower mantle viscosity is \(4.5 \times 10^{21}\) Pa-s. Sea level contributions from Greenland = 0.14, and Antarctica = 0.16 mm/yr, respectively. This result (total contribution of 0.3 mm/yr) is in poor agreement with the satellite gravity results (Fig. 9, total contribution of 0.63 mm/yr) which we have added the oceanic mass variation effects from satellite altimetry. The later result (Fig. 9) is in more agreement with results e.g., quoted in the IPCC [2001] studies as the mass balance contributions of ice sheets to sea level. It would appear that the ice sheet observations are inconsistent with the gravity solutions even with the variations in ice loading history and lithosphere parameters (viscosity, thickness, etc). Further study is needed.

We also have explored one other technique in the inversion method. Thickness change for Greenland and Antarctica that is inverted from observed gravity using an iterative approach that makes no assumptions about the shape of the ice covered area. This method is capable of resolving mass balance with latitude only from the zonal coefficients \(l=2\) through \(l=8\). Fig. 10 shows that the best-fit lower mantle viscosity is \(1 \times 10^{23}\) Pa-s, based on the new method and data. In this run, Antarctica contributes 1.2 mm/yr to the global oceans and Greenland contributes 0.4 mm/yr to the sea level rise. The approach, though less realistic than other approaches that use a template can be more readily adapted to gravity observations of high angular order. It is anticipated using GRACE data of ice sheet mass balance, the new method is more suitable to exploit the more accurate data for improved geophysical constraints.

**RELEVANT PUBLICATIONS AND PRESENTATIONS**

Relevant publications and presentations are listed. The list is as follows.


Garcia, R., Local geoid determination from GRACE mission, PhD dissertation, Ohio State University, March 2001.


Han, S., C. Jekeli, and C. Shum, Efficient method for static and temporal gravity field recovery using GRACE and CHAMP, 27th General Assembly of the EGS in Nice, France, April 21-26, 2002a.


Shum, C., A. Trupin, and C. Zhao, Ice sheet mass balance from temporal gravity field observations, invited paper, Session G3-03, Temporal Gravity Field, EGS2000, Nice, France, April, 2000a.


Shum, C., Proposed GRACE calibration in Antarctica, National Astronomical Observatory, Mizusawa, Iwate, Japan, September 2, 2000.


Shum, C., Global gravity field modeling: geophysical applications, Invited lecture at the Crustal Deformation Laboratory, Geography and Crustal Dynamics Research Center, Geographical Survey Institute, Tsukuba, Japan, February 28, 2001.

Shum, C., Global gravity field modeling: an interdisciplinary research problem, Invited lecture at the Department of Land Surveying and Geo-informatics, Hong Kong Polytechnic University, Hong Kong, March 5, 2001.


Shum, C., S. Han, and S. Ge, Contributions of COSMIC to temporal gravity field solutions, COSCIC Workshop, Boulder, Colorado, August 21-23, 2002.
Shum, C., Contribution of GRACE and COSMIC to temporal gravity field studies, Invited lecture, Purple Mountain Astronomical Observatory, Nanjing, China, September 10, 2002.


VonFrese, R., L. Potts, H. Kim, C. Shum, P. Taylor, J. Kim, and S. Han, CHAMP gravity anomalies over Antarctica, CHAMP meeting in GFZ, Potsdam, Germany, January 2002.

Project Schedule: August 1, 1998 - July 31, 2002

Measurements show an increase in the mass of the Earth's core, which is linked to changes in the Earth's interior. This study will provide an improved understanding of the processes that cause these changes.

Anticipated Benefits:

- Improved estimates of the Earth's interior structure
- Better understanding of the Earth's interior processes
- Enhanced ability to model the Earth's interior

Strategic Importance:

- Better understanding of the Earth's interior
- Improved ability to model the Earth's interior
- Enhanced ability to model the Earth's interior processes

Funding: $300,000

Conservation and estimation changes are expected to be small. The study will provide an improved understanding of the Earth's interior processes and better modeling of the Earth's interior.

Figure Caption: Predicted ice sheet thickness change and mass balance over a 100-year period. The model shows that the ice sheet is predicted to thin by 10% over the next 100 years.

High-resolution gravity field measurement and time-varying gravity field recovery using GRACE and CHAMP

May 1, 2000

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