Modeling of Unsteady Three-Dimensional Flows in Multistage Machines

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Abstract

This report describes progress made on NASA Glenn Research Center Grant NAG3-2627 for the period April 2001 through September 2002. This grant was funded by NASA as part of the GUIde Consortium III program. This report details work to date, as well as plans for the upcoming reporting period.

Introduction

Despite many years of development, the accurate and reliable prediction of unsteady aerodynamic forces acting on turbomachinery blades remains less than satisfactory, especially when viewed next to the great success investigators have had in predicting steady flows. Hall and Silkowski (1997) have proposed that one of the main reasons for the discrepancy between theory and experiment and/or industrial experience is that many of the current unsteady aerodynamic theories model a single blade row in an infinitely long duct, ignoring potentially important multistage effects. However, unsteady flows are made up of acoustic, vortical, and entropic waves. These waves provide a mechanism for the rotors and stators of multistage machines to communicate with one another. In other words, wave behavior makes unsteady flows fundamentally a multistage (and three-dimensional) phenomenon.

In this research program, we have has as goals (1) the development of computationally efficient computer models of the unsteady aerodynamic response of blade rows embedded in a multistage machine (these models will ultimately be capable of analyzing three-dimensional viscous transonic flows), and (2) the use of these computer codes to study a number of important multistage phenomena.

Recently, Hall and Smelova (1999) have developed a three-dimensional unsteady multistage Euler analysis under an AFOSR grant. For a given multistage fan or compressor, they first generate a computational mesh for each blade row. The steady and unsteady multistage flows are computed using the Euler and linearized Euler equations and conventional computational fluid dynamic (CFD) techniques, with so-called mixing planes (the inter-row computational boundaries of the computational grid) used to couple together the solutions computed in the individual blade rows. For the unsteady flow solution, several linearized unsteady flow calculations are performed simultaneously, one
corresponding to each spinning mode in the model. The only coupling among the various spinning modes is at the inter-row boundaries. The method is very efficient. A typical unsteady multistage flow calculation might take on the order of ten times the computational time required for a single row steady flow computation.

For flows in which nonlinear effects are thought to be important, most researchers have relied on time marching techniques. However, these methods, as previously discussed, are computationally expensive. Recently, Hall, Thomas and Clark (2000) have developed a novel technique for computing unsteady nonlinear flows in turbomachinery cascades. The unsteady flow is represented by a Fourier series in time with frequencies that are harmonics of the excitation frequency -- blade passing frequency in the case of wake/rotor interaction, or the blade vibratory frequency in the case of flutter. Borrowing from the structural dynamics community, they use a harmonic balance technique to write a set of coupled partial differential equations for the unknown flow solution at each harmonic. A pseudo-time term is introduced into the harmonic balance equations so that the equations may be solved by using conventional time marching computational fluid dynamic techniques.

Both the linearized multistage analysis and the single row harmonic balance analyses described above have a number of distinct advantages over more conventional time-domain solutions. First, because the solutions are computed in the frequency domain, the time-marching algorithm is only used to converge the solution to steady state. Thus, acceleration techniques, including local-time time marching with multi-grid acceleration, can be used. Second, complex periodicity conditions may be applied for each harmonic, so that the computational domain for each blade row may be reduced to a single blade passage. Finally, for many applications, just a few spinning modes in the multistage analysis, or higher harmonics in the harmonic balance analysis, need to be incorporated where engineering accuracy is sufficient. The result is that these methods are potentially two orders-of-magnitude faster than conventional time-marching methods, making them ideal for both routine design work and parametric studies to gain insight into complex flow physics.

Research Progress and Challenges During this Reporting Period

In this research program, we will further develop both the multistage and harmonic balance analyses, and to use them to study problems of importance to the gas turbine industry. At the GULde Consortium Meeting held in August 2001, we gave an oral presentation of our plans for this grant, including a timeline for completion of the major tasks (see Table 1). The steering committee recommended that the tasks be completed in a different order, with an emphasis on the early completion of Task 5, the development of a linearized multistage Navier-Stokes analysis of multistage flows. This Task is dependent on Task 3, the development of a block structured version of the multistage code. We have, therefore, revised our timeline for the completion of the major tasks (see Table 2).

During the present reporting period, our efforts have been focused on Tasks 3 and Task 5. With regard to Task 3, in a previous AFOSR grant, investigators at Duke University developed a three-dimensional linearized multistage analysis of inviscid (Euler) unsteady flow in turbomachinery. However, a number of modifications to the previous computer code are required if the code is to model viscous flows. First, the original computational code used an H-grid structure. H-grids, while relatively simple to use in a CFD code, do not have adequate resolution to resolve boundary layers, especially near the leading and trailing edges of an airfoil. Therefore, in the present research, we are implementing using an H-0-H grid structure, with an O-grid around the airfoils, with H-
grid extensions upstream and downstream of the O-grids. During this reporting period, we have made good progress on this task. We now have a single blade row Navier-Stokes code that uses and H-O-H grid structure. We have included the additional viscous terms in the Navier-Stokes, including Reynolds stress terms modeled using the Spalart-Allmaras turbulence model. We are currently in the process of debugging this analysis, and hope to have both steady and linearized unsteady three-dimensional viscous analysis capability shortly.

Simultaneously, we have continued development of an inviscid multistage analysis. (This analysis will be the combined with the single row block structured Navier-Stokes analysis to form a multistage Navier-Stokes analysis.) Some preliminary results have been obtained. Figure 1 shows a typical example. Shown is the computed unsteady pressure difference across the surface of a vibrating rotor airfoil that has a helicoidal shape. For this example, the rotor has 38 airfoils, and the downstream stator has 50 airfoils. In the non-rotating frame of reference, the flow is entirely axial. Both the rotor and the stator do no turning of the flow field, and the steady axial Mach number is 0.35. The hub to tip ratio is 0.5. The rotor airfoils vibrate in torsion. Also shown in Figure 1 is the semi-analytical solution due to Namba (private communication). Note the generally good agreement between the two theories. In this example, nine spinning modes are used to couple the unsteady flows in the rotor and stator blade rows.

Shown again in Figure 2 is the same solution shown in Figure 1, along with the solution computed for the rotor in isolation, i.e., without multistage effects. Note the substantial differences in the two solutions. Clearly multistage effects have a dramatic influence on the unsteady aerodynamic response of the rotor, and should not be neglected.

Shown in Figure 3 is the convergence history for the shown in Figure 1. Note that the flow is computed in the frequency domain; frequencies appear as parameters so that the flow may be solved as a steady state problem. Thus, the goal is to drive the residual of the solution to zero. The solution is seen to converge more than six orders of magnitude in only 2000 iterations. Furthermore, only a single blade passage is required for each blade row, making the calculation extremely efficient.

In August 2002, the principal investigator gave a brief presentation of the progress of this project at the GUIde Consortium annual meeting. These vu-graphs are attached for reference. The project has suffered a minor setback, which was reported at the annual GUIde Consortium meeting. The doctoral student working on the project, Dmytro Voyotovich, a native of the Ukraine, returned to the Ukraine for a short visit in June of 2002. To return to the United States, he was required to apply for a new student visa. However, because of heightened security measures in the wake of September 11th, the time required to obtain such visas has been greatly increased. He has been recently notified that he will receive his visa shortly. However, as of October 20, 2002, he but still has not received his visa, and cannot return to complete his studies. In his absence, a post-doctoral student, Kivanc Ekici, has largely taken over Dmytro's portion of the project, and is now making good progress.

The main technical issue that we have considered in recent months was to revisit the issue of the coupling of unsteady solutions in the individual blade rows via spinning modes at the interface boundary between the individual blade rows. The results for model problems, when compared to known solutions, were reasonably close, but did show some small but systematic errors, and more troubling, did not converge to the correct solution as the number of spinning modes was increased. Instead, small but persistent errors remained in the solution. We have systematically looked for this error, and concluded...
that the error occurs only when we use multiple spinning modes in our solution, and further, that the error is most likely being introduced at the interface boundaries that connect the individual blade rows. We have just recently found and corrected this problem.

Figure 4 shows a typical example. Shown is the computed unsteady two-dimensional aerodynamic force for "Configuration B," a model multistage test case (stator/rotor/stator) presented in an earlier paper by Silkowski and Hall (1997). In this example, a single mode is used to couple solutions in the three blade rows. Also shown for comparison are the computed results using previous two-dimensional theories using semi-analytical flow solvers. Figure 5 shows the same example, but now with nine coupling modes. Note the excellent agreement we now achieve, at least with previous two-dimensional theories. We are in the process of performing similar validation tests on a semi-analytical three-dimensional solution due to Namba for unsteady multistage flow.

Having eliminated a systematic error in our technique, we will now concentrate our efforts on extending the method to three-dimensional Navier-Stokes Analyses. We have already completed large portions of this analysis. We have developed an unsteady linearized Navier-Stokes analysis, although the method does not yet model tip flows, nor does it account for no-slip conditions on the hub and tip casings. These features will be added and combined with the current Euler analysis to produce the desired three-dimensional Navier-Stokes analysis of unsteady flows in multistage machines (Tasks 3 and 5).

References


Table 1. Original timeline for research.

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Table 2. Revised timeline for research.

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Figure 1. Computed unsteady pressure jump across airfoil surface. Shown are real and imaginary parts of unsteady pressure difference across airfoil surface. Also shown is semi-analytical solution due to Namba (personal communication).
Figure 2. Computed unsteady pressure jump across airfoil surface. Shown are real and imaginary parts of unsteady pressure difference across airfoil surface. Also shown is uncoupled solution, i.e., solution computed without multistage effects.
Figure 3. Convergence history of unsteady multistage calculation.
Figure 4. Real and imaginary part of unsteady aerodynamic lift for plunging rotor blade embedded in a multistage machine (stator/rotor/stator). Current three-dimensional Euler analysis is compared to an isolated blade row analysis (LINSUB [Whitehead, 1987]), and the coupled mode method of Hall & Silkowski (1997). Note almost exact agreement with previously derived multistage analysis. These solutions were computed using one coupling mode.
Figure 5. Real and imaginary part of unsteady aerodynamic lift for plunging rotor blade embedded in a multistage machine (stator/rotor/stator). Current three-dimensional Euler analysis is compared to an isolated blade row analysis (LINSUB [Whitehead, 1987]), and the coupled mode method of Hall & Silkowski (1997). Note almost exact agreement with previously derived multistage analysis. These solutions were computed using nine coupling modes.
Modelling of Unsteady Three-Dimensional Flows in Multistage Machines

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GUlde Consortium III, Annual Review
July 31 – August 1, 2002
Goal of research is to develop analysis tool for predicting unsteady aero-
dynamic forces acting on airfoils of multistage machine.

- Multistage effects.
- Viscous effects.
- Nonlinear effects.

Will bring together and further develop two technologies previously de-
developed at Duke University:

- Frequency domain (time-linearized) analysis of multistage flows.
- Nonlinear harmonic balance analysis of unsteady viscous flows in
  single blade row.

Final code will be able to model nonlinear, viscous, three-dimensional,
multistage flows much more efficiently than conventional time-marching
algorithms.
Description of Multistage Coupling Mechanism

- Neighboring blade rows are coupled via pressure, vorticity, and entropy waves (spinning modes).

- Original disturbance with a single frequency and interblade phase angle gives rise to response with multiple interblade phase angles (scattering) and frequencies (shifting).
Computational grid near mid-span of front stage (IGV/rotor/stator) of modern compressor.
Time-Linearized Multistage Modelling

- Generate single passage computational grid for each blade row and compute steady flow through multistage geometry.

- For each blade row, solve several unsteady linearized Navier-Stokes problems in parallel on each grid, one for each spinning mode.

- We use Ni's Lax-Wendroff "pseudo-time" time marching with local time stepping and multiple grid acceleration.

- At each iteration, pass inter-row wave information at far-field between various spinning mode solutions.

- Iterate until converged.

Method is computationally efficient, several orders of magnitude more efficient than time marching simulations.

Note spinning modes are only coupled at inter-row boundaries.
Work per cycle (aerodynamic damping) for blade rotor blade of IGV/rotor/stator configuration vibrating in first bending mode.
Solution of Harmonic Balance Euler Equations

In harmonic balance approach, assume unsteady periodic flow may be represented by Fourier series in time, i.e.

\[ \rho(x, y, t) = \sum_n R_n(x, y)e^{j\omega nt} \]

Harmonic balance equations then take the form

\[ \frac{\partial \tilde{U}}{\partial \tau} + \frac{\partial \tilde{F}(\tilde{U})}{\partial x} + \frac{\partial \tilde{G}(\tilde{U})}{\partial y} + \tilde{S}(\tilde{U}) = 0 \]

If \( n \) harmonics are kept in solution, then \( 2n + 1 \) coefficients are stored for each flow variable (1 for mean flow, \( 2n \) for real and imaginary parts of unsteady harmonics).

Note harmonics are coupled via nonlinearities in governing equations.
"Simultaneous Dual Time-Step" Form of Harmonic Balance

Computation of harmonic fluxes difficult and computationally expensive, especially for viscous flows. Alternatively, could store solution at $2n + 1$ equally spaced points in time over one temporal period.

\[ \tilde{U} = EU^* \]

\[ U^* = E^{-1}\tilde{U} \]

Where the matrices $E$ and $E^{-1}$ are discrete Fourier transform and inverse Fourier transform operators. Thus, pseudo-time harmonic balance equations become

\[ \frac{\partial EU^*}{\partial \tau} + \frac{\partial EF^*}{\partial x} + \frac{\partial EG^*}{\partial y} + j\omega NEU^* = 0 \]

Pre-multiplying by $E^{-1}$ gives

\[ \frac{\partial U^*}{\partial \tau} + \frac{\partial F^*}{\partial x} + \frac{\partial G^*}{\partial y} + j\omega E^{-1}NEU^* = 0 \]

\[ \frac{\partial}{\partial t} \]

\[ \partial/\partial t \]
“Simultaneous Dual Time-Step” Form of Harmonic Balance

\[
\frac{\partial U^*}{\partial \tau} + \frac{\partial F^*}{\partial x} + \frac{\partial G^*}{\partial y} + S^* = 0
\]

where

\[
S^* = j\omega [E^{-1}N][E] U^* \approx \frac{\partial U^*}{\partial t}
\]

Here we use spectral operator to compute time derivative. Using finite difference does not work well. Use of spectral difference operator allows for very coarse temporal discretization.

- Note that since only “steady-state” solution is desired, can use local time stepping, multiple-grid acceleration techniques, and residual smoothing to speed convergence.

- For 2D and 3D cascades, only a single blade passage is required, with complex periodicity conditions along periodic boundaries.

- Because we work in the frequency domain, essentially exact non-reflecting boundary conditions are available.
Example: Transport of Stator Wakes in Low-Speed Turbine Rotor

Hodson's low-speed turbine. $M = 0.146$, $\omega \ G/V = 8$, $\sigma = 270^\circ$, Gaussian velocity and density defect in stator wake. Three modes used in harmonic balance analysis. Single passage $129 \times 33$ computational grid.

Linearized Analysis

Harmonic Balance Analysis
Nonlinear Unsteady Flow Using Harmonic Balance

Unsteady pressure distribution for front stage compressor rotor airfoils vibrating in pitch with $\bar{\omega} = 1.0$ and $\sigma = 30^\circ$. Top: zeroth harmonic. Bottom: first harmonic.
Nonlinear Unsteady Flow Using Harmonic Balance

First harmonic of unsteady pitching moment for front stage compressor rotor airfoils vibrating in pitch with $\bar{\omega} = 1.0$. 
First harmonic of unsteady pitching moment for front stage compressor rotor airfoils vibrating in pitch with $\sigma = 30^\circ$. 
Convergence history for steady flow solver (left) and harmonic balance flow solver (right) with \( \sigma = 30^\circ, \bar{\omega} = 1.0, \) and \( \Delta \alpha = 1.0^\circ. \)
Statement of Work

The following tasks will be performed over a four-year period:

1. We will use the existing and newly developed three-dimensional linearized Euler and Navier–Stokes multistage analysis codes developed under the AFOSR grant and this program to investigate and characterize the influence of vane clocking on the forced response and flutter of representative multistage machines.

2. We will use the existing and newly developed three-dimensional linearized Euler and Navier–Stokes multistage analysis codes to investigate the propagation of low-engine order excitations through multiple stages.

3. We will further develop the existing multistage linearized multistage analysis. Specifically, we will develop a block-structured version of the analysis (the existing method uses only H-grids). The block-structured version will improve the accuracy of the analysis, and eventually enable tip clearances and other flow path features to be modelled.
The following tasks will be performed over a four-year period:

4. We will further develop the unsteady linearized block-structured version of the multistage analysis, converting the code into a nonlinear harmonic balance analysis, which will be capable of analyzing nonlinear unsteady flows for the special case of vane/rotor or vane/rotor/vane problems with equal vane counts.

5. We will further develop both the linearized and nonlinear multistage analyses, converting the Euler codes into Navier–Stokes codes.
### Timeline

#### Project Timeline

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Steering committee recommended changes to this timeline, pushing linearized Navier-Stokes work up.
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Timeline
Namba Test Case: $\sigma = 94.7$ deg.
Namba Test Case: $\sigma = 180$ deg.
Namba Test Case: $\sigma = 94.7$ deg.
Namba Test Case

![Graph showing the decrease in residual with increasing number of iterations.](image-url)
C1 Compressor. H-O-H computational grid
C1 Compressor. Mach Number Distribution.
C1 Compressor. Velocity Vectors.
During first year, have most of pieces in place for multistage linearized Navier-Stokes analysis.

- We have “cleaned up” original H-grid Euler code developed in earlier project. We use this code for “boot strap” verification of H-O-H codes.

- We have developed H-O-H time-linearized Euler multistage code. Still debugging. Final debugging in progress.

- We have developed H-O-H Navier-Stokes solver. Development well underway (single blade row). Have steady code well in hand. Currently working on time-linearized version.

- Will in the next quarter work to couple multiple stages in H-O-H Navier-Stokes code.

Have had minor setback. Ph.D. student working on this project is stuck in Ukraine with visa problems.
In Year 2, we will begin development of nonlinear multistage analysis, and will use linearized analysis to investigate problems of interest to industry.

- We will complete development of linearized multistage Navier-Stokes analysis.
- Will develop harmonic balance form of multistage analysis.
- We will begin to use linearized analysis to examine low-engine order excitations in engines (long wavelength disturbances can travel long distances through machine).
- We will begin to investigate the influence of blade row spacing on flutter and forced response.
This report describes progress made on NASA Glenn Research Center Grant NAG3–2627 for the period April 2001 through September 2002. This grant was funded by NASA as part of the GUIde Consortium III program. This report details work to date, as well as plans for the upcoming reporting period.