AERODYNAMIC SHAPE OPTIMIZATION USING AN EVOLUTIONARY ALGORITHM

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Abstract

A method for aerodynamic shape optimization based on an evolutionary algorithm approach is presented and demonstrated. Results are presented for a number of model problems to access the effect of algorithm parameters on convergence efficiency and reliability. A transonic viscous airfoil optimization problem—both single and two-objective variations—is used as the basis for a preliminary comparison with an adjoint-gradient optimizer. The evolutionary algorithm is coupled with a transonic full potential flow solver and is used to optimize the inviscid flow about transonic wings including multi-objective and multi-discipline solutions that lead to the generation of pareto fronts. The results indicate that the evolutionary algorithm approach is easy to implement, flexible in application and extremely reliable.
AERODYNAMIC SHAPE OPTIMIZATION
USING EVOLUTIONARY ALGORITHMS

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PRESENTATION OUTLINE

▶ EVOLUTIONARY ALGORITHMS--GENERAL
▶ SINGLE OBJECTIVE RESULTS
▶ MULTI-OBJECTIVE ALGORITHM CHARACTERISTICS--PARETO FRONTS
▶ COMPARISON OF RESULTS FROM AN EVOLUTIONARY ALGORITHM AND AN ADJOINT GRADIENT BASED ALGORITHM
▶ ADDITIONAL COMPUTATIONAL RESULTS
▶ CONCLUSIONS
GENERAL CHARACTERISTICS:
SINGLE-OBJECTIVE EVOLUTIONARY ALGORITHMS
EVOLUTIONARY ALGORITHMS—GENERAL

EVOLUTIONARY ALGORITHMS (EA) are search algorithms based on natural selection. "They combine survival of the fittest with structured yet randomized information exchange..." GOLDBERG (1989)

- EA optimization has many advantages:
  - Simplicity
  - Robustness
  - Wide applicability
  - Embarrassingly parallel implementation

- EA optimization works for design spaces that are
  - Function discontinuous
  - Derivative discontinuous
  - Multi-modal
  - Multi-objective

- EAs typically require more function evaluations than other methods especially gradient-based methods
EVOLUTIONARY ALGORITHM CHARACTERISTICS

▶ ENCODING (DESIGN SPACE PARAMETERIZATION)
▶ Each problem being optimized must be *representable* as a set of parameters called GENES, e.g., geometric parameters used in aerodynamic shape optimization. One set of genes is called a CHROMOSOME.
▶ Chromosomes are constructed in one of two ways:
  ▶ Bit strings
  ▶ Real number strings

▶ FITNESS
▶ A FITNESS FUNCTION is used to evaluate figure of merit for each chromosome, e.g., pressure integration to obtain lift

▶ SELECTION
▶ SELECTION operation is used to determine which chromosomes will be carried forward to the next generation
▶ More fit individuals are always favored in the selection process
EVOLUTIONARY ALGORITHM—SELECTION

TWO SELECTION ALGORITHMS HAVE BEEN STUDIED

- Multiple pass selection ("greedy selection")
  - FIRST PASS: Select all chromosomes ranked 1
  - SECOND PASS: Select all chromosomes ranked 1 and 2
  - THIRD PASS: Select all chromosomes ranked 1, 2 and 3
  - And so on until NC chromosomes have been selected

- Tournament selection
  - Select the NOB chromosomes with the highest fitness in each objective
  - Select three chromosomes at random and compare rankings
  - Retain the highest ranking (in case of ties, retain the first selected)
  - Repeat until NC chromosomes have been selected
New Generation is Finalized Using Various Modification Operators

- **PASSTHROUGH (Controlled by \( P_1 \))**
  - Small number of chromosomes with highest rankings included without modification (ELITISM)

- **CROSSOVER (Controlled by \( P_2 \))**
  - Two chromosomes (PARENTS) are chosen at random from new generation
  - Genes are combined using an averaging operator to produce a CHILD with shared characteristics from each PARENT

- **MUTATION**
  - Random gene chosen from random chromosome in new generation
  - Using a small probability the chosen gene is randomly modified
  - Two types of mutation used
    - **PERTURBATION MUTATION**: Changes are small (Controlled by \( P_3 \))
    - **Standard MUTATION**: Changes are large (Controlled by \( P_4 \))
  - **MODIFICATION OPERATOR USAGE CONTROLED BY P-VECTOR**\( \Sigma P_i = 1.0 \)

- CROSSOVER is generally viewed as most important operation for producing a rapid search or exploration.

- MUTATION adds randomness, ensuring that no part of design space is neglected.
SAMPLE RESULTS--SINGLE OBJECTIVE

- HILL CLIMBING PROBLEM
  - TWO GENES
  - MULTI-MODAL (MULTIPLE HILLS AND VALLEYS)

- TRANSONIC WING OPTIMIZATION
  - LIFT-TO-Drag MAXIMIZATION
  - AERODYANMIC FUNCTION EVALUATIONS
    - TRANSONIC OVERSET POTENTIAL SOLVER (TOPS)
    - CHIMERA ZONAL GRID APPROACH
    - HYPGEN USED FOR WING VOLUME GRID GENERATION
  - WING PARAMETERIZATION
    - HICKS-HENNE BUMP FUNCTIONS USED (UPPER SURFACE ONLY)
    - LEADING EDGE, TRAILING EDGE AND LOWER SURFACE FIXED
    - FOUR BUMPS AT TWO STATIONS (ROOT AND TIP) + TWIST >> TEN GENES (GEOMETRIC DECISION VARIABLES)
    - LINEAR LOFTING BETWEEN ROOT AND TIP
    - FIXED PLANFORM
HILL CLIMBING PROBLEM

ISOMETRIC VIEW OF FUNCTION
USED IN HILL CLIMBING PROBLEM

SAMPLE EA CONVERGENCE
$\beta=0.01$, $\text{CONV}=10^{-5}$, $\text{NC}=20$

- $P = (0.1, 0.2, 0.3, 0.4)$
- $P = (0.4, 0.3, 0.2, 0.1)$

NUMBER OF FUNCTION EVALUATIONS
EA CONVERGENCE—HILL CLIMBING PROBLEM

**EFFECT OF $\beta$ ON CONVERGENCE**

$\text{CONV} = 10^{-5}$, $P = (0.1, 0.2, 0.3, 0.4)$

**EFFECT OF $P$ ON CONVERGENCE**

$\text{CONV} = 10^{-5}$, $\beta = 0.01$
$M_\infty = 0.84$
$\alpha = 4^\circ$
$TR = 0.333$
$AR = 6.0$
$\Lambda_{LE} = 36.65^\circ$
$RMAX < 10^{-6}$
$NG = 10$
$NC = 20$
$\beta = 0.3$
$P = (0.1, 0.2, 0.3, 0.4)$

$$\text{OBJ} = \frac{1}{(C_D/C_L + (C_L-0.45)^2)}$$
MACH NUMBER CONTOURS—WING OPTIMIZATION

$M_\infty = 0.84$, $\alpha = 4^\circ$, $R\text{MAX} < 10^{-6}$, $NG = 10$, $\beta = 0.3$, $P = (0.1, 0.2, 0.3, 0.4)$

BASELINE SOLUTION

OPTIMIZED SOLUTION
EA CONVERGENCE—WING OPTIMIZATION

EFFECT OF POPULATION SIZE ON GA CONVERGENCE

$M_\infty = 0.82, \alpha = 4^\circ, RMAX < 10^{-6}, NG = 55, \beta = 0.3, P = (0.1, 0.3, 0.4, 0.2)$

![Graph showing the effect of population size on GA convergence.](image-url)
MULTI-OBJECTIVE
EVOLUTIONARY ALGORITHM
CHARACTERISTICS
EAs are useful for multi-objective optimization, e.g., max L/D and min weight.
The $i^{th}$ gene in the $j^{th}$ chromosome of the $n^{th}$ EA generation is indicated by

$$X_{i,j}^n$$

The $j^{th}$ chromosome within the $n^{th}$ generation composed of NG genes

$$X_j^n = (X_{1,j}^n, X_{2,j}^n, \ldots, X_{i,j}^n, \ldots, X_{NG,j}^n)$$

The fitness vector associated with the $j^{th}$ chromosome and the $n^{th}$ generation

$$F_j^n = [f_1^n(X_j^n), f_2^n(X_j^n), \ldots, f_{NOB}^n(X_j^n)]$$

where NOB is the number of objective functions.
MULTIPLE OBJECTIVE OPTIMIZATION
PARETO FRONT DEFINITIONS

► PARETO OPTIMAL SET or PARETO FRONT:
  ► The optimal result of a multi-objective optimization

► Membership in the Pareto Optimal Set determined using the concept of DOMINANCE:

\[
\text{Chromosome } X_a \text{ dominates chromosome } X_b \\
\text{iff } f_{a,k} \geq f_{b,k} \text{ for all } k \text{ with } f_{a,k} > f_{b,k} \text{ for at least one } k
\]

► Chromosome rank tied to dominance.
  ► Several ranking algorithms available:
    ► Goldberg ranking
    ► Fonseca and Fleming ranking
    ► Others
MULTI-OBJECTIVE OPTIMIZATION
RANKING

Goldberg ranking using maximization for two objectives
MUTLIPLE OBJECTIVE OPTIMIZATION
ACTIVE AND ACCUMULATION FILES

▶ ACTIVE FILE:
  ▶ Current collection of chromosomes (nth population)

▶ ACCUMULATION FILE:
  ▶ Collection of all #1 ranked chromosomes discovered during EA iteration

▶ ACCUMULATION FILE development and use:
  ▶ Add all newly discovered #1 ranked chromosomes
  ▶ Cull old individuals that lose dominance
  ▶ Increases in size with EA iteration
  ▶ Used in active file ranking
  ▶ Not used in the EA selection/crossover/mutation process (Some variations do use accumulation file in selection)
COMPARISON OF ADJOINT GRADIENT AND EVOLUTIONARY ALGORITHM APPROACHES
COMPARISON OF EVOLUTIONARY AND ADJOINT GRADIENT METHODS

ADJOINT GRADIENT (AG) METHOD
- ADJOINT METHOD USED TO DETERMINE DESIGN SPACE GRADIENTS
- BFGS QUASI-NEWTON APPROACH USED FOR GRADIENT OPTIMIZATION
- WEIGHTED OBJECTIVE FUNCTION (WOF) USED FOR "MULTI-OBJECTIVE" OPTIMIZATIONS, i.e., $\text{OBJ}_{\text{NEW}} = W \cdot \text{OBJ}_1 + (1-W) \cdot \text{OBJ}_2$

EVOLUTIONARY ALGORITHM (EA)
- WOF AND DOMINANCE PARETO FRONT (DPF) APPROACHES BOTH USED

MULTI-OBJECTIVE VISCOUS AIRFOIL OPTIMIZATION:
- ALL FUNCTION EVALUATIONS PERFORMED USING ARC2D
  - STEADY STATE SOLUTIONS TO NAVIER-STOKES EQUATIONS
  - SPALART-ALMARAS TURBULENCE MODEL
- B-SPLINE REPRESENTATION OF AIRFOIL USED
  - FIVE SPLINE KNOTS ON EACH SURFACE PLUS $\alpha$ -- TOTAL OF 11 GENES (DECISION VARIABLES)

PARETO FRONT COMPARISONS

\[ M_\infty = 0.7, \, Re = 9 \times 10^6, \, C_l^* = 0.55, \, C_d^* = 0.0095 \]
COMPARISON OF AG-WOF AND EA-DPF RESULTS

\[ M_\infty = 0.7, \quad \text{Re} = 9 \times 10^6, \quad C_l^* = 0.55, \quad C_d^* = 0.0095 \]

\[ W = 0.2 \quad \text{W} = 0.5 \]
ALL METHODS PRODUCED CONSISTENT PARETO FRONTS

AG-WOF RESULTS ARE MORE TIGHTLY CONVERGED THAN EA-BASED RESULTS

AG-WOF APPROACH INVOLVES A SIGNIFICANT AMOUNT OF CODING FOR EACH IMPLEMENTATION WHEREAS THE TWO EA APPROACHES DO NOT

SPEED COMPARISONS:
  - AG-WOF ~ 30 TIMES FASTER THAN EA-WOF FOR SINGLE-OBJECTIVE OPTIMIZATION
  - AG-WOF ~ 4 TIMES FASTER THAN EA-DPF FOR TWO-OBJECTIVE OPTIMIZATION
    - AG-WOF 15 POINTS ON PARETO FRONT POINTS
    - EA-DPF 500 POINTS ON THE PARETO FRONT
EA RESULTS IN THREE DIMENSIONS

CASAS PRESENTED

► SINGLE-OBJECTIVE DRAG MINIMIZATION
► TWO-OBJECTIVE SINGLE-DISCIPLINE MINIMIZATION
► TWO-OBJECTIVE MULTI-DISCIPLINE MINIMIZATION
WING PARAMETERIZATION

- Wing defined using N airfoil defining stations
- Each airfoil defined using Sobieczky parameterization (see definition below)
- Twist angle added to each defining station >> total number of parameters = 11N
- Linear lofting used between each defining station

\[ z = \sum_{n=1}^{6} a_n \cdot x^{n-1/2} \]
FUNCTION EVALUATIONS

- AERODYNAMIC FUNCTION EVALUATIONS
  - TOPS (TRANSONIC OVERSET POTENTIAL SOLVER)

- TWO STATIONS (ROOT AND TIP) USED, I.E., NUMBER OF GENES (NG) IS 22

- WEIGHT FUNCTION EVALUATIONS
  - SIMPLE BOX BEAM MODEL
  - USES AERODYNAMIC LOADS TO ESTIMATE WEIGHT SO THAT MAX STRESS*FOS NOT EXCEEDED
  - SHEAR AND BENDING INCLUDED BUT NOT TORSION
SINGLE-OBJECTIVE WING OPTIMIZATION

\[ M_\infty = 0.84, \quad C_L = 0.45, \quad RMAX < 10^{-6}, \quad NG = 22, \quad NC = 20 \]
GA CONVERGENCE CHARACTERISTICS
DRAG MINIMIZATION

\[ M_\infty = 0.84, \quad C_L = 0.45, \quad RMAX < 10^{-6}, \quad NG = 22 \]

**EFFECT OF POPULATION SIZE ON GA CONVERGENCE**

**GA OPERATOR EFFECTIVENESS**

**NC = 20**

- **CROSSOVER**
- **PERTABATION-MUTATION**
- **MUTATION**

**NUMBER OF FUNCTION EVALUATIONS THAT INCREASE MAX FITNESS**

**NUMBER OF FUNCTION EVALUATIONS**

**GENERATION NUMBER**
GA CONVERGENCE CHARACTERISTICS
TWO-OBJECTIVE, SINGLE DISCIPLINE OPTIMIZATION

\[ M_\infty = 0.84, \quad C_L = 0.45, \quad \text{RMAX} < 10^{-6}, \quad \text{NG} = 22 \]
GA CONVERGENCE CHARACTERISTICS
TWO-OBJECTIVE, TWO-DISCIPLINE OPTIMIZATION

\[ M_\infty = 0.84, \ C_L = 0.45, \ RMAX < 10^{-6}, \ NG = 22 \]
CONCLUDING REMARKS

EVOLUTIONARY ALGORITHMS REPRESENT AN ATTRACTIVE ALTERNATIVE FOR FINDING OPTIMAL SOLUTIONS IN ENGINEERING DESIGN

Strengths include:
- Robustness
- Flexibility
- Ease of implementation
- Embarrassingly parallel (ideal for heterogeneous distributed computing)
- Amenable to multi-modal design spaces
- Ability to work for multi-objective cases (pareto fronts)

Weaknesses include:
- Potentially expensive
- Difficult to know when convergence is reached

Future focus on:
- Efficiency improvements especially for multi-objective cases
- Parallel implementation (load balancing)
- Application to other problems