SERVICING AND DEPLOYMENT OF NATIONAL RESOURCES IN SUN-EARTH LIBRATION POINT ORBITS

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SERVICING AND DEPLOYMENT OF NATIONAL RESOURCES IN SUN-EARTH LIBRATION POINT ORBITS

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Abstract
Spacecraft travel between the Sun-Earth system, the Earth-Moon system, and beyond has received extensive attention recently. The existence of a connection between unstable regions enables mission designers to envision scenarios of multiple spacecraft traveling cheaply from system to system, rendezvousing, servicing, and refueling along the way. This paper presents examples of transfers between the Sun-Earth and Earth-Moon systems using a true ephemeris and perturbation model. It shows the $\Delta V$ costs associated with these transfers, including the costs to reach the staging region from the Earth. It explores both impulsive and low thrust transfer trajectories. Additionally, analysis that looks specifically at the use of nuclear power in libration point orbits and the issues associated with them such as inadvertent Earth return is addressed. Statistical analysis of Earth returns and the design of biased orbits to prevent any possible return are discussed. Lastly, the idea of rendezvous between spacecraft in libration point orbits using impulsive maneuvers is addressed.

Introduction
Satellite servicing has received a great deal of study and significant execution. Several satellites were designed for servicing using the Multi-Mission Modular Spacecraft design, including Solar Max Mission, Landsat IV & V, Upper Atmosphere Research Satellite, and Extreme Ultra-Violet Explorer. Rescue missions have been performed on geostationary satellites trapped in low earth orbit, such as WESTAR-IV, PALAPA-B, Intelsat-VI, and LEASAT/SYNCOM-IV. More routine human servicing work occurs(ed) at various space stations (International Space Station, Skylab, Salyut, and Mir). The servicing of the Hubble Space Telescope (HST) has become a successful landmark, allowing HST to become one of NASA's most productive missions.\(^1,2,3,4\)

Some obstacles to human servicing are the orbital mechanics, the cost of lifting mass, and the problems associated with travel time and thermal and instrument environmental conditions. As more ambitious missions are planned, such as the placement of the Next Generation Space Telescope (NGST) and several other missions into a Sun-Earth L\(_2\) (SEL\(_2\)) or SEL\(_1\) libration orbit, servicing by the Shuttle and the use of low-Earth orbits (LEOs) will be limited. Development of robotic satellite servicing capabilities, such as DARPA's Orbital Express, NASA's Robonaut, and the University of Maryland's Ranger, may provide for the possibility of robotic satellite servicing at various orbital locations in the near- or mid-range time frame.\(^5\)

An enabling set of circumstances for an expansion of satellite servicing would be the placement of humans and valuable robotic assets in close proximity to one another. A
space architecture that includes these conditions is a servicing facility in a lissajous or halo orbit about one of the Earth-Moon L₁ (EML₁), Earth-Moon L₂ (EML₂), or Earth-Moon L₃ (EML₃) co-linear libration points. From such an orbit, spacecraft have access to a wide variety of interesting orbits at a relatively lower ΔV cost. Use of the Earth-Moon stable L₄ and L₅ Lagrange regions provides additional scenarios. These servicing locations are also an excellent staging point for lunar surface and Earth-Moon orbital exploration. The orbits also have ready access to geostationary orbits and transfer back to LEO orbits. Table 1 provides a brief overview of Earth-Moon libration orbit staging node characteristics.

### Table 1. Earth/Moon Libration Orbit Overview

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Potential Uses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EML₁</strong></td>
<td></td>
</tr>
<tr>
<td>-Unstable orbit between earth and moon</td>
<td>-Assembly &amp; maintenance of s/c</td>
</tr>
<tr>
<td>-Halo: continuous view of the earth, moon, and sun</td>
<td>-Access to lunar surface</td>
</tr>
<tr>
<td>-Earth / moon always &gt;150° separation</td>
<td>-Low Δv access to SELₙ transfer manifolds</td>
</tr>
<tr>
<td><strong>EML₂</strong></td>
<td></td>
</tr>
<tr>
<td>-Unstable orbit far side of moon</td>
<td>-Assembly &amp; maintenance of s/c</td>
</tr>
<tr>
<td>-Halo: continuous view of earth, moon, sun</td>
<td>-Access to lunar surface</td>
</tr>
<tr>
<td>-At full moon, sun/earth/moon/ in alignment</td>
<td>-Comm relay to lunar far side,</td>
</tr>
<tr>
<td></td>
<td>-Low Δv access to SELₙ transfer manifolds</td>
</tr>
<tr>
<td><strong>EML₃</strong></td>
<td></td>
</tr>
<tr>
<td>-Unstable orbit opposite moon</td>
<td>-Assembly &amp; maintenance of s/c that are thermally sensitive</td>
</tr>
<tr>
<td>-Halo: continuous view of earth, sun</td>
<td>-Low Δv access to SELₙ transfer manifolds</td>
</tr>
<tr>
<td>-At new moon, sun/earth/moon/ in alignment</td>
<td></td>
</tr>
<tr>
<td><strong>EML₄₅</strong></td>
<td></td>
</tr>
<tr>
<td>-Stable orbit, low stationkeeping Δv, Varying sun/earth/moon geometry</td>
<td>-Minimum fuel loads</td>
</tr>
<tr>
<td></td>
<td>-Moderate transfer to SELₙ</td>
</tr>
</tbody>
</table>

The servicing of national resources in the Earth-Moon and the Sun-Earth regions is made difficult by the unstable dynamics of these regions. Trip times associated with achieving SEL₁ and SEL₂ orbits can vary from weeks to months while achieving EML₁ and EML₂ may vary from days to weeks, with both dependent upon many initial conditions such as energy and departure locations. The design of a trajectory used to achieve servicing mission requirements is intricate but can be facilitated by the use of unique orbits to achieve the proper flight time or to minimize the necessary fuel mass. The use of dynamical systems (such as invariant manifolds) and optimization plays a key role in designing transfer trajectories, as these tools afford the luxury of using natural dynamics where possible. Results demonstrate the viability of some unique transfers that meet the NASA Exploration Team (NEXT) goals. These goals focus on opening the human frontier beyond low-Earth orbit by building infrastructure robotically and with humans in-situ at strategic outposts - libration points, planetary moons, planets, etc. - and include a wide range of exploration tools (e.g., space planes, balloons, human-constructed and maintained observatories located at libration point, etc.).

A primary purpose of this paper is to detail mission planning scenarios for servicing of national resources and to contrast them to other transfer options. It explores the transfer from the Earth-Moon region to Sun-Earth libration orbits. The ΔV and trip time associated with such a mission is compared to a more traditional direct injection and the relevant differences highlighted. Analysis recently completed for NGST orbit trades are used where possible. The servicing options vary over a wide range, from low thrust direct transfers to transfers from elliptical orbits achieved using bi-propellant systems. This paper covers only several of the many topics analyzed, but it represents a general investigation of possibilities. The results of the analysis presented herein will be available to aid in the support of evaluating in-space
servicing options for mission studies in the GSFC Integrated Mission Design Center.

Software Applications and Assumptions
For this analysis, software applications that use high fidelity perturbation modeling and full ephemeris data were utilized. The models utilized include as a minimum:

- Earth potential, 4x4 or greater
- Point mass bodies based on full ephemeris (DE405)
- Solar radiation pressure
- Jacchia-Roberts atmospheric drag
- Spacecraft area and mass characteristics
- Spacecraft engine mass depletion and accelerations
- Runge-Kutta 8/9 integrator
- Inertial and rotating coordinate systems
- Differential correctors and optimization methods

The software used consists of GSFC's Swingby and Generator tools and MATLAB. Swingby has been used operationally by GSFC to support four libration and two lunar missions. In addition to using Swingby to model and target the solution set, software that allows robust modeling and optimization of the Sun-Earth and Earth-Moon dynamics is used. This utility allows the generation of the full phase space while including full perturbation models. The Generator code developed at Purdue University was used to compute invariant manifolds for EML₂ to SELₐ transfers. MATLAB using GSFC developed m-files and a multi-level iteration scheme were used to optimize these transfer trajectories.¹⁰

Transfers Between The Earth-Moon And Sun-Earth Libration Orbits
Transfers between the Earth-Moon and Sun-Earth libration points have been previously investigated in varying consideration and detail.¹¹,¹²,¹³,¹⁴,¹⁵ This work builds upon 25 years of GSFC libration orbit experience and analysis and new optimal transfer studies. A servicing scenario using co-linear Earth-Moon libration orbits was investigated first. This scenario allows a fast transfer of human and expendable resources for deployment or servicing at the EML₂ vicinity, with a round trip servicing time of approximately a week. The transfers presented here demonstrate feasibility and are not fully optimized.

End-To-End Transfer Scenarios
The following scenarios demonstrate a cis-lunar transfer into an EML₁ or EML₂ orbit where a human deployment or servicing of a national resource can occur. A subsequent transfer to and a return from the SEL₁ via EML₂ is then designed that would provide for a second servicing opportunity. The initial cis-lunar trajectory is modeled as injection from a 28.5° inclined 186-km circular parking orbit based on expendable launch vehicle parameters. To obtain the trajectories in this scenario a combination of targeting goals were used including: lunar B-plane components, rotating coordinate system position and velocities, orbital energy, and epochs and propagation duration. To achieve targets, deterministic ΔVs were varied at the parking orbit injection, orbit insertion, and trajectory mid-points.

Figures 1 and 2 present a transfer from LEO to EML₁, transfer to EML₂, transfer to SEL₁, and back to EML₂. The cis-lunar trajectory as shown in Figure 1 is in an earth-moon rotating coordinate system. This trajectory transfers the spacecraft from LEO orbit injection to EML₁ orbit insertion. From there, after EML₁ insertion and any stationkeeping, a small maneuver is required to achieve the EML₂ orbit. As EML₂ is a co-linear unstable location, it is straightforward to achieve a transfer trajectory that places the spacecraft on a departing invariant manifold. Figure 2 shows the EML₂ to SEL₁ transfer in a sun-earth rotating coordinate system. Following insertion into the SEL₁ lissajous orbit, one orbit revolution about the SEL₁ point was used. A small maneuver then places the spacecraft on an inbound trajectory that results in a re-capture into the EM system. The computation of the initial transfer orbit was not constrained to achieve any SEL₁ lissajous orbit conditions that are
advantageous to any return trajectory. In fact, the transfer and final SEL₁ orbit achieved may be considered a worst-case scenario since they were unconstrained and resulted in the maximum offset in timing for a return trajectory. The ΔVs and trip times associated with this transfer scenario are presented in Table 2.

As shown in Figures 3 and 4, a second return transfer was also achieved using the same initial LEO to SEL₁ outbound trajectory. The original outbound transfer results in a timing differential for a return lunar encounter of 14 days. Without a phase jump in the SEL₁ orbit, the return trajectory achieves an EML₃ intersection. Using the dynamics of the Earth-Moon region, a differential correction scheme can be used to compute a maneuver that allows a transfer to an orbit in the Earth-Moon region to achieve the EML₂ orbit. Figure 3 shows the transfer orbit in the Earth-Moon region while Figure 4 shows the continuous path from SEL₁ back to the Earth-Moon region.

Figures 5 and 6 show a similar transfer from or to the EML₁ region without a phase jump. These return transfers demonstrate that a wide variety of transfers are available for return servicing missions. Segments of this full scenario can be used to size the requirements for a returning spacecraft from SEL₁ and rendezvous from Earth at the EML₂ orbit. The impact of injection from an ISS-like inclination is not significant. A similar transfer to EML₁ using a higher inclination can be computed by timing the alignment of the outgoing velocity asymptote and the addition of a small deterministic maneuver. The maneuver ΔV required for insertion into the EML₁ or EML₂ libration orbit from the incoming SEL₁ libration orbit varies as a function of lissajous amplitude and the energy of the incoming transfer trajectory. For EML₁ and EML₃ insertions, ΔVs on the order of 50 to 300 m/s were observed. A typical maneuver insertion for EML₂ was 70 m/s. In all the above scenarios, the injection energy to reach EML₁ orbit from LEO remained constant at -2.13 km²/s² (ΔV ~3.14 km/s), similar in magnitude to a lunar mission such as Lunar Prospector.

Adjusting the Amplitude of the SEL₁₂ Orbit From EML₂

Achieving the above SEL₁ lissajous orbit was not dependent upon any mission constraints (such as y-amplitude). The size and shape of the orbit was unconstrained in order to show the feasibility of such transfers. The orientation and amplitude of the orbit will vary depending upon mission requirements and natural dynamics. Analysis was performed to determine the ΔV and impacts to temporal or spatial conditions for control of the lissajous characteristics in order to rendezvous with resources already in SELₙ libration orbits.

Reducing the lissajous y-amplitude

Trajectory optimization is a broad and

Table 2. Sample ΔV and Trip Times For Full Scenario

<table>
<thead>
<tr>
<th>Parameters</th>
<th>EML₁ Insertion at X-axis Crossing</th>
<th>Transfer to EML₂</th>
<th>SEL₁ Injection and Transfer</th>
<th>Insertion into SEL₁ and 1 Orbit</th>
<th>Transfer from SEL₁ Orbit to EML₂</th>
<th>Capture into EML₁₂₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔV (m/s)</td>
<td>423</td>
<td>0.2</td>
<td>None</td>
<td>None</td>
<td>32 to EML₁ 100 to EML₂</td>
<td>50-293</td>
</tr>
<tr>
<td>Flight Time Segment and Cumulative (days)</td>
<td>5</td>
<td>9</td>
<td>86</td>
<td>115</td>
<td>185 (includes 1 orbit)</td>
<td>300</td>
</tr>
<tr>
<td>Libration Amplitude (km)</td>
<td>X ~ 27000 (EML₁ orbit)</td>
<td>X ~ 27000 (EML₁ orbit)</td>
<td>---</td>
<td>X ~ 300000 (SEL₁ orbit)</td>
<td>Y ~ 800000 (EML₂ orbit)</td>
<td>---</td>
</tr>
</tbody>
</table>
complicated subject that has been studied extensively in the aerospace literature. Nevertheless, fundamentally any path can be discretized as a set of patch states that include time, position and velocity. At any given patch state, a before and after (-/+1) maneuver state is included. In this context, impulsive maneuvers — at any of the patch states — are modeled by fixing time and position while letting the velocity vary instantaneously. Once an initial guess is provided and carefully discretized, the first level of the Howell-Pernicka two level iteration scheme can be used to achieve position continuity. That is, the patch states are connected in a (numerical) Lambert type of scheme. The result of this
operation is the computation of the total $\Delta V$ required for the path. This total $\Delta V$ becomes the cost function. If one or more components (of one or more patch states) are allowed to vary, the total $\Delta V$ required for the path changes. As a result, a trajectory optimization scheme could vary the patch states and keep track of the total $\Delta V$ that is needed to "connect the points". This essentially allows the use of any optimization strategy.

As an example, the MATLAB optimization toolbox using the \textit{fmincon} function is utilized. A trajectory from the vicinity of the EML$_2$ point to a lissajous orbit of the SEL$_2$ point is examined. The ephemerides model is utilized with the Earth as the central body (with J2) and with the Sun and the Moon as point mass. Solar radiation pressure is also included. An initial estimate, with a total $\Delta V$ of about 700 m/s, is computed by numerical experimentation. This initial estimate is discretized into four patch states. The patch state's positions and times are selected as independent variables. When the optimizer converges, the total $\Delta V$ decreased to 180 m/s. As a comparison to the above solution of departure from the EML$_2$ point, two lissajous orbits with the same epoch were constructed that yield a transfer to two selected SEL$_1$ y-amplitude orbits. These lissajous characteristics were chosen based on the full transfer scenario. The control of the SEL$_1$ y-amplitude is dependent upon the selected transfer manifold and the initial conditions at the EML$_2$ libration orbit. Unstable manifolds from the SEL$_1$ orbit back to the vicinity of the Moon at a given epoch can be constructed.$^{10}$ Also, unstable manifolds of the dynamics of the Earth-Moon region can be constructed. The intersection of these two manifolds represents a locus of transfer points.

As shown in Figure 7, a smaller SEL$_1$ y-amplitude lissajous orbit can be achieved by using a slightly different initial EML$_2$ departure condition that inserts onto a different outbound unstable manifold. Table 3 provides sample $\Delta V$s and trip times for these transfers. The transfer trajectories to SEL$_1$ depart at the same epoch but have slightly different initial EML$_2$ states and energy. A deterministic $\Delta V$ performed several days after departure on the SEL$_1$ lissajous transfer manifold is required to acquire a smaller SEL$_1$ lissajous. Also shown in Figure 7 is a transfer into a small SEL$_2$ orbit. This transfer was achieved using an optimization process. The departure location was chosen to be the EML$_2$ libration point itself. Selection of the departing conditions as part of the overall mission design allows one to achieve any SEL$_2$ lissajous orbit.

<table>
<thead>
<tr>
<th>Achieved Lissajous</th>
<th>Starting EML$_2$</th>
<th>Required $\Delta V$</th>
<th>Trip Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-Amplitudes (km)</td>
<td>y-Amplitudes (km)</td>
<td>(m/s) and Departure</td>
<td>to first</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C3 (km/s$^2$ - Moon</td>
<td>SEL$_1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>centered)</td>
<td>x-axis</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>crossing</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(days)</td>
</tr>
<tr>
<td>SEL$_1$, Y-900,000</td>
<td>X: 20000</td>
<td>$\Delta V$: 0.18</td>
<td>115</td>
</tr>
<tr>
<td></td>
<td>Y: 75000</td>
<td>C3: 0.0002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SEL$_2$, Y-700,000</td>
<td>$\Delta V$: 16</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>X: 2000</td>
<td>C3: -0.05</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Y: 6000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SEL$_1$, Y-450,000</td>
<td>X: 2000</td>
<td>$\Delta V$: 127</td>
<td>82</td>
</tr>
<tr>
<td></td>
<td>Y: 5000</td>
<td>C3: 0.03</td>
<td></td>
</tr>
<tr>
<td>SEL$_2$, Y-200,000</td>
<td>X: 0</td>
<td>$\Delta V$: 180</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>Y: 0</td>
<td>C3: 0.75</td>
<td></td>
</tr>
</tbody>
</table>

| Table 3. EML$_2$ Departure Characteristics and $\Delta V$s Transfer |

Figure 7. Changing the SEL$_{1/2}$ Lissajous Y-Amplitude

**Low Thrust Options**

Another option for servicing and deployment is to utilize low thrust propulsion. Low thrust propulsion has the benefit of being more efficient than an impulsive type system. A transfer from LEO into either an EML$_1$ orbit...
or into a SEL₂ orbit was analyzed. As compared to the ΔV and time associated with a high-energy direct transfer, low thrust obviously takes much longer. The amount of available mass to final EML₁ orbit is increased but is limited by the launch capability. Figure 8 shows a low thrust trajectory to the EML₁ in an Earth-Moon Rotating Coordinate system. An example of a low thrust trajectory from Earth parking orbit to the SEL₂ orbit in a Sun-Earth rotating coordinate system is shown in Figure 9. The thrust direction is along the velocity vector. While usually called constant thrust, the trajectory includes coasting arcs that are used to ‘target’ to an unstable manifold, which places the spacecraft on a trajectory that will achieve a lissajous orbit. Little additional thrust is required after lunar orbit distance; the majority of the post-lunar trajectory in Figure 9 is a coast phase. This trajectory is similar to a direct transfer to SEL₂. Table 4 shows a number of different direct low thrust options considered for NGST, as well as for an EML₁ mission. Obviously a high thrust to mass ratio is required to minimize flight time. The information in the table assumed a launch using the STS into a 28.5° LEO.

Nuclear Energy Options
An option for some missions is the use of nuclear energy as a power source for low thrust missions. Once the mission is achieved a disposal plan must be enacted and analysis performed to determine the possible reentry conditions.¹⁸ For missions in the SEL₁ or SEL₂ region, this becomes a question of the probability of Earth impact if a perturbation places the spacecraft on an unstable manifold that returns it to the Earth environment.

Table 4. Sample Low Thrust Option Parameters

<table>
<thead>
<tr>
<th>Propulsion Information</th>
<th>S/C Mass (kg)</th>
<th>Total Launch Mass (kg)</th>
<th>Thrust to Mass Ratio</th>
<th>Transfer Time to SEL₂ or EML₁ (Days)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thrust (N), Isp(sec), Thruster # / Type</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.02, 3100, 22 DS-1 (NGST)</td>
<td>5633</td>
<td>16421</td>
<td>1.230e-4</td>
<td>672 to SEL₂</td>
</tr>
<tr>
<td>2.78, 1732, 2 Halls (NGST)</td>
<td>1408</td>
<td>16420</td>
<td>1.693e-4</td>
<td>365 to SEL₂</td>
</tr>
<tr>
<td>2.08, 2488, 2 Halls (NGST)</td>
<td>4198</td>
<td>16420</td>
<td>1.267e-4</td>
<td>642 to SEL₂</td>
</tr>
<tr>
<td>2.31, 3800, 14 XIPS (NGST)</td>
<td>7200</td>
<td>16419</td>
<td>1.407e-4</td>
<td>610 to SEL₂</td>
</tr>
<tr>
<td>0.50, 2500, 1 Hall (non-NGST)</td>
<td>744</td>
<td>1000</td>
<td>1.14e-3</td>
<td>150 to EML₁</td>
</tr>
<tr>
<td>2.08, 2488, 2 Halls (non-NGST)</td>
<td>3178</td>
<td>4198</td>
<td>4.955-3</td>
<td>138 to EML₁</td>
</tr>
</tbody>
</table>

Figure 8. Low Thrust Trajectory to EML₁

Figure 9. Low Thrust Trajectory from LEO to SEL₂

No-return trajectories
The issue of an unplanned return of a nuclear power plant in a Sun-Earth libration point orbit is a valid one. Periodic orbits about the SEL₁ and SEL₂ are unstable. Loss of control of a spacecraft will cause the spacecraft to depart the periodic orbit asymptotically. The direction of this divergence is locally determined by the unstable eigenvector of the monodromy matrix, or State Transition Matrix (STM) over one period. This eigenvector is then globalized (non-linear integration) to determine the trajectory path of the departing spacecraft. The energy of the
periodic orbit limits the space which the spacecraft can reach. This limit is called the zero velocity curve and is roughly a sphere about the Earth at low energy levels. At higher energy levels, specifically the energy levels of the SEL₁ and SEL₂ points themselves, the sphere opens up via corridors near SEL₁ and SEL₂ to reveal a region either inside the Earth's orbit about the Sun or outside it. The energy levels of the periodic orbits about SEL₁ and SEL₂ are higher than the points themselves and therefore spacecraft can depart the neighborhood of the Earth and SEL₁ / SEL₂ points. Once a spacecraft departs though the SEL₁ or SEL₂ corridors, its return is considered to be statistically irrelevant, although recent events indicate that an Apollo 3rd stage accomplished this. However, some of the unstable manifolds remain in the region for some time and pass between the SEL₁ and SEL₂ points with multiple Earth flybys. It is these trajectories that sometimes provide Earth impacts.

Generally, half of the entire manifold leaves the system directly though a corridor with no Earth encounters. The other half exhibits some sort of Earth encounter before the majority of them leave the EM system. Monte Carlo analysis has shown that approximately 0.7% of the time, a randomly perturbed spacecraft in a large periodic orbit about the SEL₁ or SEL₂ points will impact the Earth within two years. Departures from smaller quasi-periodic orbits have not shown any impacts in a similar analysis. This analysis is limited to Circular Restricted Three Body Problem modeling. Additional force modeling including lunar perturbations and solar radiation pressure would significantly affect the propagation of any single trajectory and could significantly increase the possibility of impact, such as with ISEE-1&2. However, it's not clear if those forces would change the overall statistical behavior of the trajectories. Figure 10 shows the minimum Earth radial distance for 150 equally spaced displaced points on a small periodic orbit over a two-year propagation. Figure 11 shows the same data from a large periodic orbit. Figure 12 shows the trajectory in views along three axes of one of the impact scenarios.
Biased Libration Orbits

Another method to ensure a non-return trajectory is one that models a stationkeeping strategy that incorporates a constant acceleration. It necessitates the control of an unstable orbit in a constrained direction so that an onboard propulsion failure would guarantee a drift away trajectory. As with all libration orbits about the unstable SEL and SEL2 points, stationkeeping is required. One can design a biased orbit through inclusions of deterministic AVs and accelerations that result in a trajectory that diverges on an outbound eigenvector direction.

A strategy for a non-return libration orbit accommodation can be found by including deterministic AVs in the nominal SEL lissajous orbit. This can be determined using the two-level differential scheme that is part of an overall dynamical systems approach. Expected gravitational and non-gravitational acceleration are modeled in the differential equations of motion. The deterministic maneuvers are pre-specified to be any appropriate value that accommodates accelerations and maintains the corrective maneuvers (i.e., stationkeeping) in a positive x direction. Without these maneuvers, the orbit departs into a drift away orbit.

Libration Orbit Rendezvous And Phase Jumps

In the above EM and SE transfer cases, the timing of the arrival into the final mission orbit was not considered. In some servicing options it may be required to rendezvous with the national resource to provide the propulsion to return it to the EML1 or EML2 regions, as was in the full transfer scenario.

The rendezvous problem has been extensively studied and performed in LEO for many years. Additionally, the rendezvous problem for libration point orbits has been solved and optimized using continuous thrust inputs by Marinescu, among others. However, future mission concepts now include multiple spacecraft that may require rendezvous and docking. Low thrust propulsion systems are only feasible for a small number of spacecraft with large masses and extensive power systems. The libration orbit rendezvous problem using high (impulsive) thrust has not been addressed significantly. This paper addresses some of the initial investigation into this problem.

The primary technique used here is a Lambert solution using a numerically calculated STM and a simple differential correction. The STM is obtained through the integration of the variational equations along with the equations of motion. A state vector is created consisting of the six state elements of the target spacecraft (B) and the relative state vector between the two spacecraft. The STM matrix provides the sensitivity of the relative position between the two spacecraft and the velocity of the maneuvering spacecraft (A):

\[
\begin{align*}
\frac{\partial r}{\partial v_A} \cdot \frac{\partial r}{\partial v_A}. \text{Given a time to rendezvous of } T, \text{ the calculated } \Delta V \text{ applied to A at time 0 is obtained from: } d\hat{V}_A(0) = \Phi_s^*(\hat{r}(T))
\end{align*}
\]

where \( \Phi_s = \frac{\partial r}{\partial v_A} \). This technique will provide a one-impulse solution to this rendezvous problem. There is no expectation that this technique will work for most rendezvous problems, however the applicability of this technique will show where enhanced techniques such as multiple impulses, weak stability boundary, target points, or Floquet Modes are required. An example case is shown below.

The target spacecraft is placed in a small Lissajous orbit about the SEL point. The maneuvering spacecraft is placed 7 days behind the target along the same trajectory. The time to rendezvous is set for 60 days. In this case, figure 13 shows the relative position closes to zero at a cost of 24 m/s in \( \Delta V \) of spacecraft A at time 0. However, the relative velocity at time T was 44 m/sec. As a comparison STS requires less than 0.3 m/s to rendezvous with ISS.
The process of jumping ahead or behind on the same periodic orbit is called “Phase Jumping”. This idea has been used previously for z-axis control to avoid the solar exclusion zone. The goal is to phase jump the maneuvering spacecraft to rendezvous with the target spacecraft at a later time. This initial phase error would be due to a number of different sources including launch errors, maneuver execution errors, or phasing of other systems/orbits. Obviously, two spacecraft coming from different orbits or launches would not end up in the same phase of the same orbit. Phase jumping is critical in order to close the distance between spacecraft to within the free-space approximation defined above. The following figures show the effect on the cost of phase jumping due to initial phase angle and time to rendezvous. Figure 14 shows the $1^{st}$ $\Delta V$, $2^{nd}$ $\Delta V$, and the total $\Delta V$ for a 6 day phase jump over 60 days (time to rendezvous) versus the initial location of the target spacecraft in the periodic orbit (or phase angle). Figure 14 shows that the total $\Delta V$ is fairly insensitive to the location. Total $\Delta V$s of 54 to 84 m/s are seen about one revolution of the orbit. Figure 15 fixes the initial location (approximately 0 deg initial phase angle) and shows total $\Delta V$ versus time of the phase jump for various $T$ (times to rendezvous).
Conclusions

We have demonstrated several concepts and trajectory designs in support of servicing Sun-Earth libration missions in Earth-moon libration orbits. Several critical areas regarding AVs, travel time, rendezvous, and orbit dynamics were addressed. It was found that the AVs are in a manageable range of less than 500m/s, similar to direct transfer mission requirements with small SEL, y-amplitudes. Timing associated with servicing is of importance and can drive the orbit mechanics and rendezvous opportunities. The timing requirement results in additional maneuvers, phase jumping, and targeting schemes. It is clear that algorithms and software tools for trajectory design in this regime must be further developed to incorporate better understanding of the solution space, and to improve the efficiency, and to expand the capabilities of current approaches.

References