Probabilistic Analysis of Solid Oxide Fuel Cell Based Hybrid Gas Turbine System

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ABSTRACT

The emergence of fuel cell systems and hybrid fuel cell systems requires the evolution of analysis strategies for evaluating thermodynamic performance. A gas turbine thermodynamic cycle integrated with a fuel cell was computationally simulated and probabilistically evaluated in view of the several uncertainties in the thermodynamic performance parameters. Cumulative distribution functions and sensitivity factors were computed for the overall thermal efficiency and net specific power output due to the uncertainties in the thermodynamic random variables. These results can be used to quickly identify the most critical design variables in order to optimize the design and make it cost effective. The analysis leads to the selection of criteria for gas turbine performance.

INTRODUCTION

The majority of global power is generated from the consumption of fossil fuels, which represent a finite source. The rapid depletion of these fossil fuel resources remains one of the most important problems facing future world power generation. This means that the way in which power is generated will have to change over the next century. The power harnessed in the fossil fuels must be used more efficiently until renewable sources can be developed to meet a significant proportion of the increasing energy requirements. The option of developing new technology such as fuel cells, which exceed the maximum efficiencies of gas turbines seems very lucrative.

A fuel cell is a simple means of converting chemical energy to electricity without ignition combustion and it is promising to revolutionize the power generation industry. The chemical energy to the fuel cell is supplied on a continuous basis in the form of a fuel such as natural gas or synthesis gas while the oxidant is also supplied continuously. The fuel cell is not constrained by the Carnot efficiency and therefore higher conversion efficiencies are achievable with a fuel cell. The intermediate step of conversion into heat as in a heat engine is eliminated in a fuel cell.

As governments around the world strive to meet their escalating energy demands under increasing pressure from environmental issues, there exists a need for clean and efficient energy sources. Even the most advanced gas turbine cycles have difficulty in reaching a thermal efficiency of 40 percent and have NO\textsubscript{x} emission problems due to their high operating temperatures. Standalone fuel cells have been manufactured with efficiencies of around 48 percent, producing negligible NO\textsubscript{2} or SO\textsubscript{2} and a reduced CO\textsubscript{2}, owing to the increased thermal efficiency. It has also been recognized that they could be symbiotically incorporated into gas turbine cycles in order to produce efficiencies estimated to be as high as 70 percent as mentioned in reference [1]. The most suitable fuel cell for this application is the solid oxide fuel cell (SOFC) which, on account of its all solid state, has the highest operating temperature. The fuel cell model was based on the principle of operation of the Westinghouse tubular SOFC design, involving internal reforming and preforming of methane. Its performance was examined under a wide range of operating conditions leading to exhaust temperatures in the range 370 to 935 °C. Fuel cell system efficiencies as high as 56 percent were predicted at low current densities (150 mA/cm\textsuperscript{2}), while the efficiency dropped to 38.7 percent at high current densities (600 mA/cm\textsuperscript{2}) owing to irreversibilities developed. Pressurized operation of the fuel cell was assessed in the range 1 to 25 atm; high pressure operation leads to an increased power output but produces a significant decrease in electrical efficiency. At full commercial size, the Westinghouse tubular SOFC has a diameter of 22 mm with an active total length of 1500 mm and a total active area of 834 cm\textsuperscript{2}. A 100 kW unit employs 1152 commercial size SOFCs in its cell stack. The stack is maintained at a sufficiently high temperature level which leads to approximately 100 percent reforming of the methane to pure hydrogen.

The compressor provides the SOFC with a preheated inlet flow from which it produces dc power while heating to its operating temperature of 1000 °C. This is similar to the required turbine entry temperature from which the turbine can
produce ac power. The solid constructions of SOFCs mean that they can withstand the high operating pressures of the system.

The complexity and performance over the past 50 years in gas turbine engines has increased and an array of safety nets was created to ensure against component failures in turbine engines. In order to reduce what is now considered to be excessive conservatism and yet maintain the same adequate margins of safety, there is a pressing need to explore methods of incorporating probabilistic design procedures into engine development. Probabilistic methods combine and prioritize the statistical distributions of each design variable, generate an interactive distribution and offer the designer a quantified relationship between robustness, endurance and performance.

Pangalis et. al [1] developed a detailed thermodynamic model of a fuel cell. This model predicted the performance of a gas turbine and a fuel cell system integrated together in hybrid generation cycles. Cunnell et al [2] modeled gas turbine cycles integrated with a fuel cell model. Rao and Samuelson [3] presented a description and application of an analysis for tubular SOFC based systems. Models for simulating fuel cell based plants were developed by Ferguson [4], Haynes [5], and Bessette [6]. These models were limited to systems consisting of ideal gases and pure steam or the models required for simulating many of the unit operations and processes that could make up a hybrid plant were not included.

A probabilistic design system was developed by Fox [7] at Pratt and Whitney for the purpose of integrating deterministic design methods with probabilistic design techniques. Here, two different approaches were used for estimating uncertainty. A Monte Carlo approach was used on design codes that were judged to run relatively quickly. For more computationally intensive design codes, a second order response surface model in conjunction with Box-Behnken design experiments was used and then a Monte Carlo simulation was executed. Several researchers at NASA Glenn Research Center have applied the probabilistic design approaches to turbine engines and related systems. Chamis [8] developed a Probabilistic Structural Analysis Method (PSAM) using different distributions such as the Weibull, normal, log-normal, etc. to describe the uncertainties in the structural and load parameters or primitive variables. Nagpal, Rubinstein, and Chamis [9] presented a probabilistic study of turbopump blades of the Space Shuttle Main Engine (SSME). They found that random variations or uncertainties in geometry have statistically significant influence on the response variable and random variations in material properties have statistically insignificant effects. Chamis [10] summarized the usefulness and importance of the probabilistic approach, especially for turbopumps. To cost effectively accomplish the design task, we need to formally quantify the effect of uncertainties (variables) in the design. Probabilistic design is one effective method to formally quantify the effect of uncertainties.

In the present paper, a probabilistic analysis is presented for the influence of a priori fixed parameter variations on the random variables for a gas turbine thermodynamic cycle integrated with a fuel cell. We focus on the integration of two major power generation devices, fuel cells and gas turbines. With the available energy resources becoming increasingly scarce and modern power generation as well as process plants becoming increasingly complex, the requirement of an efficient thermodynamic computation technique for design optimization is presently setting demanding challenges.

**PROBABILITY THEORY**

Let \(X_1, X_2, ..., X_n\) be a set of random variables defined on a (discrete) probability space \(\Omega\). The probability that the events \(X_i = x_1, X_2 = x_2, ..., X_n = x_n\) happen concurrently, is denoted by \(f(x_1, x_2, ..., x_n) = P(X_1 = x_1, X_2 = x_2, ..., X_n = x_n)\) for the set of desired solutions \(A \subseteq \Omega\). If the function \(f(x_1, x_2, ..., x_n)\) is discrete, it is called the joint probability mass function of \(X_1, X_2, ..., X_n\) and has the following properties:

\[
0 \leq f(x_1, x_2, ..., x_n) \leq 1
\]

\[
\sum_{(x_1, x_2, ..., x_n) \in \Omega} f(x_1, x_2, ..., x_n) = 1
\]

\[
P[(X_1, X_2, ..., X_n) \in A] = \sum_{(x_1, x_2, ..., x_n) \in \Omega} f(x_1, x_2, ..., x_n) = 1, A \subseteq \Omega
\]

If \(f(x_1, x_2, ..., x_n)\) is continuous it is called joint probability density function of \(X_1, X_2, ..., X_n\) and has the following properties:

\[
0 \leq f(x_1, x_2, ..., x_n)
\]

\[
\int_\Omega \int \int \int \cdots \int f(x_1, x_2, ..., x_n) dx_1 dx_2 \cdots dx_n = 1
\]

\[
P[(X_1, X_2, ..., X_n) \in A] = \int_A \int \int \cdots \int f(x_1, x_2, ..., x_n) dx_1 dx_2 \cdots dx_n, A \subseteq \Omega
\]

If the lower bound of \(A\), the set of desired solutions, is equal to the infimum of \(\Omega\) for all \(X_i\), i.e., if \(A = [\inf\Omega, \Omega]\), for all \(i = 1, 2, ..., n\), a function \(F(a_1, a_2, ..., a_n)\) can be defined, such that:

\[
F(a_1, a_2, ..., a_n) = P[(X_1, X_2, ..., X_n) \in A] = \sum_{(x_1, x_2, ..., x_n) \in A} f(x_1, x_2, ..., x_n) \]
\[ F(a_1, a_2, \ldots, a_n) = P[(X_1, X_2, \ldots, X_n) \in A] \]
\[ = \int_{A} \ldots \int_{A_n} f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n, \quad A \subseteq \Omega \]
\[ (f \text{ is continuous}) \] (4)

\[ F \] is called the joint cumulative probability distribution function. For \( \Omega = \mathbb{R}^n \) and a continuous function \( f \):
\[ F(a_1, a_2, \ldots, a_n) = P[(X_1, X_2, \ldots, X_n) \in (\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R})], \]
\[ (a_1, a_2, \ldots, a_n)] = \int_{-\infty}^{a_1} \ldots \int_{-\infty}^{a_n} f(x_1, x_2, \ldots, x_n) dx_1 dx_2 \ldots dx_n \] (5)

The common notation
\[ F(a_1, a_2, \ldots, a_n) = P(X_1 \leq a_1, X_2 \leq a_2, \ldots, X_n \leq a_n) \] will be used subsequently also.

The univariate probability function \( f_{X_i} \) for each criterion \( X_i \), obtained from the traditional probabilistic design process, can also be generated with the joint probability function \( f \). \( f_{X_i} \) is called marginal probability mass or density function of \( X_i \) and is defined by:
\[ f_{X_i} = \sum_{x_2, \ldots, x_n} \ldots \sum_{x_2, \ldots, x_n} f(x_2, \ldots, x_n) \quad \text{if \( f \) is discrete} \] (6)
\[ f_{X_i} = \int_{a_2}^{a_n} \ldots \int_{a_2}^{a_n} f(x_2, \ldots, x_n) dx_2 \ldots dx_n \quad \text{if \( f \) is continuous} \] (7)

The joint probability function, \( f_{XY}(x, y) \), creates the surface of a probability ‘hump’ in the \( x-y \)-probability space, characterized by rings of constant probabilities. The distribution curves over the \( x \)- and \( y \)-axes are the aforementioned marginal probability functions \( f_X(x) \) and \( f_Y(y) \), respectively. The last necessary concept to mention here for the development of a joint probabilistic formulation is the concept dependence of criteria. Two random variables \( X \) and \( Y \) are said to be independent, if \( f_{XY}(x, y) = f_X(x) \cdot f_Y(y) \). \( f_{XY}(x, y) \) otherwise \( X \) and \( Y \) are said to be dependent. This dependence is a mathematical notion and should not be confused with ‘casual dependence’. For here on, mathematical dependence will be referred to as \textit{correlation}. Correlation is measured by the covariance of two criteria, \( X \) and \( Y \), defined by
\[ \text{Cov}(X, Y) = E[X \cdot Y] - E[X] E[Y]. \] (8)

It is more convenient, however, to use a covariance normalized by the standard deviations, \( \sigma_X \) and \( \sigma_Y \), for both criteria, called \textit{correlation coefficient}.
\[ \rho = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y} \] (9)

The correlation coefficient is defined over the interval \([-1,1]\), indicating strongly positively correlated criteria at values close to 1 and strongly negatively correlated criteria at values close to \(-1\). The criteria are independent, if \( \rho = 0 \). In aerospace systems design \( \rho \) can be quite difficult to calculate by Eq. (9). It is much more effective to view the correlation coefficient differently for calculation purposes. Jointly collected data from a probabilistic or any other analysis can be thought of as vectors of numbers. The correlation coefficient measures the orthogonality, i.e., independence, of both vectors. \( \rho \) is simply the cosine of the angle between the two criterion vectors, indicating their alignment. For \( \rho = 1 \), vectors are parallel and point in same direction, for \( \rho = -1 \), vectors are parallel and point in opposite direction. For \( \rho = 0 \), vectors are orthogonal and the criteria are independent. The correlation coefficient plays a significant role in the formulation of joint probability distribution models as described in the next section.

**Probability Functions**

Attention is now directed to the implementation of this probabilistic formulation in the design process. The necessary transition from the mathematical formulation above to a probabilistic model that yields the information relevant for multivariate decision making is described in this section. There are two alternatives for this task.

**Joint Probability Model**

The first joint probability density function introduced here is an analytical probability model for criteria whose univariate distributions and their corresponding means and standard deviations are known. All necessary information for the model can be generated by the traditional probabilistic design process, using its output of univariate criterion distributions. A particular model for two criteria with normal distributions, represented by Eq. (10), has been introduced by Garvey and Tuah. Garvey further generated models for two criteria with combinations of normal and lognormal distributions, which are summarized in ref. [11].

\[ f_{XY}(x, y) = \frac{1}{2\pi \sigma_x \sigma_y \sqrt{1 - \rho^2}} \exp\left\{ -\frac{1}{2(1 - \rho^2)}(\frac{x - \mu_x}{\sigma_x})^2 - 2\rho(\frac{x - \mu_x}{\sigma_x})(\frac{y - \mu_y}{\sigma_y}) + (\frac{y - \mu_y}{\sigma_y})^2 \right\} \] (10)

Note that the only information needed for the Joint Probability Model consists of the means \( \mu_X \) and \( \mu_Y \), the standard deviations \( \sigma_X \) and \( \sigma_Y \), and the correlation coefficient \( \rho \) for the criteria \( X \) and \( Y \). The model variables, \( x \) and \( y \), are defined over the interval of all possible criterion values. The advantage of this model is the limited information needed, which makes it very flexible for use and application. For example, if only expert knowledge and no simulation/modeling is available in the early stages of design, educated guesses for the means, standard deviations, and the correlation coefficient can be used to execute the joint probability model. It also lends itself to use in combination with increasingly important fast probability integration (FPI) techniques.
Implementation of Probabilistic Procedure Using FPI

FPI is a probabilistic analysis tool that implements a variety of methods for probabilistic analysis. The procedure follows the steps given below:

1. Identify the independent and uncorrelated design variables with uncertainties.
2. Quantify the uncertainties of these design variables with probability distributions based on expert opinion elicitation, available data or benchmark testing.
3. It is required that there is a response function that defines the relationship between the response and the independent variables.
4. The FPI uses the responses generated to compute the cumulative distribution functions (CDF)/probability density functions (PDF) and the corresponding sensitivities of the response.

Several methods are available in the FPI to compute a probabilistic distribution. In addition to obtaining the CDF/PDF of the response, the FPI provides additional information regarding the sensitivity of the response with respect to the primitive variables. They provide valuable information in controlling the scatter of the response variable. The random primitive variable with the highest sensitivity factor will yield the biggest payoff in controlling the scatter in that particular response variable. Such information is very useful to the test/design engineer in designing or interpreting the measured data.

DISCUSSION OF RESULTS

This paper deals with modeling a gas turbine cycle integrated with a fuel cell model to produce hybrid cycle. The choice of hybrid configuration includes a regenerative gas turbine cycle with fuel cell ahead of the combustor as shown in Figure 1. The cycle achieved a thermal efficiency of 64.1 percent at a pressure ratio of 14. The specific power output was found to be 520 W/kg s. Figures 2 and 3 illustrate the variation of thermal efficiency and net specific power with compression ratio. The combustion temperature acts to ensure that the turbine inlet temperature reaches a maximum of 1300 K. The cycle is less dominated by the fuel cell, especially at higher compression ratios where the turbine contributes almost equally.

The probabilistic analysis of gas turbine field performance due to the uncertainties was applied to a solid oxide fuel cell based hybrid system. The thermodynamic random variables and their respective values used in this analysis are shown in Table 1. All the random variables were assumed to be independent. A scatter of ±5 percent was specified for all the variables. Normal distribution was assumed for all random variable scatters.

The overall thermal efficiency and net specific power output of the hybrid gas turbine system was determined from a control volume analysis using the first and second laws of thermodynamics. The cumulative distribution functions (CDF) and the sensitivity factors were evaluated for the overall thermal efficiency response. CDF for the overall thermal efficiency shown in Figure 4. The sensitivity factors for the overall thermal efficiency are plotted in Figures 5 to 7. From these figures, we observe that the inlet temperature of the compressor, cycle pressure ratio, output of the fuel cell, inlet temperature to the gas turbine, adiabatic efficiencies of the compressor and turbine and effectiveness of the regenerator have a lot of influence on the overall thermal efficiency. These thermodynamic random variables represent the most important indices for the gas turbine health determination. The adiabatic efficiencies of the compressor and turbine in the system are measures of irreversibilities or increase of entropy. The sensitivity factor for the compressor adiabatic efficiency is much larger than the corresponding value for the turbine. The sensitivity factors due to the adiabatic efficiency and inlet temperature of the compressor influence the most in the determination of the overall thermal efficiency of the system.

Figure 8 shows the CDF for the net specific output of the hybrid gas turbine cycle. The sensitivity factors for the net specific output are plotted in Figures 9 to 11. The sensitivity factor due to the turbine inlet temperature influences the most in the evaluation of the net specific power output in the cycle.

These results can be used to further optimize the design for cost effectiveness and also to assist in the health determination of the system. The prediction of degradation of system performance can be achieved from the results obtained.

CONCLUDING REMARKS

In this paper, a non-deterministic, non-traditional method has been developed to support reliability-based design. Probabilistic methods were applied to the thermodynamic analysis of a hybrid gas turbine system integrated with a fuel cell. The interconnection between the thermodynamic analysis and NESTEM codes was necessary to compute the probabilistic evaluation of a gas turbine field performance. Overall thermal efficiency and net specific power output of the gas turbine plant was evaluated using the thermodynamic random variables. Cumulative distribution functions and sensitivity factors were computed for the overall thermal efficiency and net specific power output due to the uncertainties in the thermodynamic random variables. Evaluating probability of risk and sensitivity factors will enable the identification of the most critical design variables in order to optimize the design, make it cost effective and assist in the health determination of the system.

REFERENCES


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<th>Table 1.—Random Variables</th>
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<tr>
<td>Compressor inlet pressure (P1)</td>
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<td>Compressor inlet temperature (T1)</td>
<td>288 K</td>
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<tr>
<td>Pressure ratio</td>
<td>14</td>
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<tr>
<td>Turbine inlet temperature</td>
<td>1300 K</td>
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<tr>
<td>Adiabatic efficiency of compressor</td>
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<tr>
<td>Adiabatic efficiency of turbine</td>
<td>0.88</td>
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<tr>
<td>Effectiveness of regenerator</td>
<td>0.90</td>
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</tr>
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</tr>
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<td>PRAT</td>
</tr>
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Figure 1.—Layout of the hybrid gas turbine system.

Figure 2.—Thermal efficiency versus compression ratio.

Figure 3.—Net specific power output versus compression ratio.
Figure 10.—Sensitivity factors versus random variables (probability = 0.1).

Figure 11.—Sensitivity factors versus random variables (probability = 0.999).
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**Subject Terms:**
- Thermodynamic analysis
- Gas turbines
- Probabilistic analysis
- Fuel cell

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