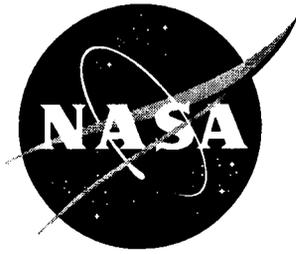


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Estimation of Complex Permittivity of Composite Multilayer Material at Microwave Frequency Using Waveguide Measurements

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May 2003

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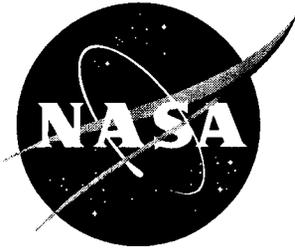
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Symbols

a	x-dimension of rectangular waveguide
b	y-dimension of rectangular waveguide
\bar{e}_i, \bar{h}_i	i^{th} waveguide modal vector functions
$\bar{E}_{i0}, \bar{E}_{i1}, \dots, \bar{E}_{iN}$	Transverse electric fields at z_0, z_1, \dots, z_N interfaces
\bar{E}_0, \bar{H}_0	Transverse electric and magnetic fields in $z \leq 0$ region
ER_1	$\text{Real}(S_{11c} - S_{11m})$
ER_2	$\text{Im ag.}(S_{11c} - S_{11m})$
ER_3	$\text{Real}(S_{21c} - S_{21m})$
ER_4	$\text{Im ag.}(S_{21c} - S_{21m})$
ER_5	$\text{Real}(S_{12c} - S_{12m})$
ER_6	$\text{Im ag.}(S_{12c} - S_{12m})$
ER_7	$\text{Real}(S_{22c} - S_{22m})$
ER_8	$\text{Im ag.}(S_{22c} - S_{22m})$
ER_T	$\sqrt{ER_1 + ER_2 + \dots + ER_8}$
FSS	Frequency Selective Surfaces
j	$\sqrt{-1}$
N	Total Number Dielectric Layers
n	n^{th} layer
MIC	Microwave Integrated Circuits
MMIC	Monolithic Microwave Integrated Circuit

$S_{11}, S_{21}, S_{12}, S_{22}$	Two port S-parameters
$S_{11m}, S_{21m}, S_{12m}, S_{22m}$	Measured values of S-parameters
$S_{11c}, S_{21c}, S_{12c}, S_{22c}$	Calculated values of S-parameters
TE_{10}	Transverse electric dominant mode
T_{nj}	Amplitude of j^{th} mode at $z = z_n$ transverse plane
Y_i^0	i^{th} modal admittance for the $z \leq 0$ region
Y_i^n	i^{th} modal admittance for the $z_{n-1} \leq z \leq z_n$ region
Y_i^{N+1}	i^{th} modal admittance for for the $z \geq z_N$ region
z_{n-1}, z_n	Location of planes for n^{th} layer
Δ_n	Thickness of n^{th} layer
(ϵ_m, μ_m)	Relative permittivity and permeability of n^{th} layer
β_i^0	i^{th} mode propagation constant for the $z \leq 0$ region
β_i^n	i^{th} mode propagation constant for the $z_{n-1} \leq z \leq z_n$ region
β_i^{N+1}	i^{th} mode propagation constant for the $z \geq z_N$ region
Δ_n	Thickness of n^{th} layer

Abstract

A simple method is presented to estimate the complex dielectric constants of individual layers of a multilayer composite material. The multilayer composite material sample is loaded in an X-band rectangular waveguide and its two port *S*-parameters are measured as a function of frequency using the Hewlett-Packard (HP) *8510 Network Analyzer*. Also, by applying the mode matching technique, expressions for the *S*-parameters of the composite material as a function of electric properties of individual layers are developed. Using the MatLab Optimization Tools simple MatLab scripts are written to search for electric properties of individual layers so as to match the measured and calculated *S*-parameters.

A single layer composite material formed by using materials such as Bakelite, Nomex Felt, Fiber Glass, Woven Composite B and G, Nano Material #0, Cork, Garlock, of different thicknesses are tested using the present approach. The dielectric constants of these materials estimated using the present approach (assuming the thicknesses are known) are in good agreement with their true values. Assuming the thicknesses of samples unknown, the present approach is shown to work well in estimating the dielectric constants and the thicknesses.

A number of two layer composite materials formed by various combinations of above individual materials are tested using the present approach. The values of dielectric constants of individual layers estimated (assuming the thickness of individual layers are known) using the present approach are in good agreement with their true values. However, the present approach could not provide estimate values close to their true values when the thicknesses of individual layers were assumed to be unknown. This is attributed to the difficulty in modelling the presence of airgaps between the layers while doing the measurement of *S*-parameters. A few example of three layer composite are also presented.

I. Introduction

Multilayer substrates are used for many practical applications such as Microwave Integrated Circuits (MIC), Monolithic MIC (MMIC) [1], radomes for protection of antennas from external environment, spatial filters for antenna beam shaping [2], and Frequency Selective Surfaces (FSS) [3-4]. With proper choices of individual layers in a multilayer composite substrates it is possible to achieve a composite material with altogether new properties that were otherwise not found in the individual layers. Exact knowledge of the material properties such as permittivity and

permeability of individual layers in a multilayer substrate is essential for designing spatial filters, radomes and composite materials for FSS applications. However, the present practice in the estimation of electric properties of composite materials with multi-layers emphasizes only the determination of overall effective properties. In this report an attempt is made to provide a procedure to estimate the electric properties of individual layers of a composite material.

Permittivity and permeability of these composite multi-layer substrates can be estimated using one of the following methods: 1) free-space techniques; 2) resonant cavity perturbation techniques; and 3) transmission line methods. Each method has its own advantages and limitations. For measurement of electric properties over a wide frequency range, transmission line or waveguide methods [5-8] are more popular even though they are less accurate due to unavoidable measurement errors. In the waveguide measurement method, a sample of composite multilayer substrate is placed in a waveguide and two port S-parameters are measured using a *hp-8510 Network Analyzer*. In the earlier work [7-8], assuming that an equivalent homogeneous material occupies the sample space, S-parameters are estimated as a function of effective electric properties of the equivalent material. Using the inverse procedure, constituent parameters of the equivalent material are determined by matching the estimated and measured S-parameters. In these methods electric properties of individual layers are not determined. However, for some applications such as radomes and FSS, a knowledge of electric properties of individual layers is required. In this report, a waveguide measurement method is presented to estimate the electric properties of individual layers of a composite multilayer substrate.

The remainder of this report is organized as follows. The mode matching formulation of a waveguide loaded with multilayer composite material is developed in section II. Also in section II, the expressions are developed to estimate 2-port S-parameters of composite material slab. Numerical and measured results on the S-parameters of numerous composite material are presented in section III for the direct problem where electrical properties of individual layers are assumed to be known. Also in section III, using the MatLab Codes given in Appendices, estimated values of dielectric constants of one, two, and three layer composite material are presented. The report concludes in section IV with remarks on the validity and usefulness of the present method.

II. Theory

In this section, the method of moments is used to determine the S-parameters of a rectangular waveguide loaded with a multilayer dielectric substrate as shown in figure

1.

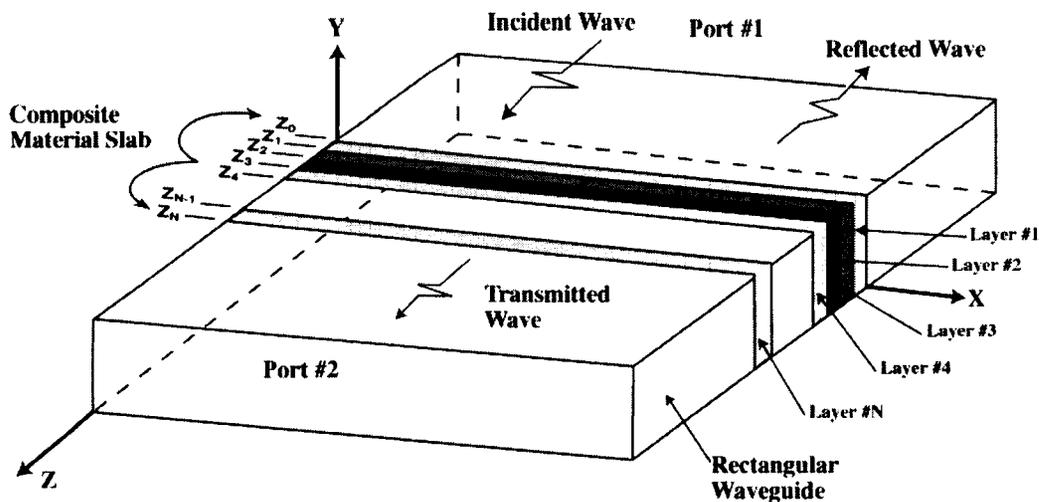


Figure 1: Geometry of rectangular waveguide loaded with multilayer composite material.

The multilayer substrate consists of N layers, where the n^{th} layer has parameters (ϵ_m, μ_m) and is located between the transverse planes at $z = z_{n-1}$ and $z = z_n$.

S_{11} and S_{21} Parameters:

To estimate the S_{11} and S_{21} components of 2-port parameters, it assumed that the TE_{10} mode of unit amplitude is incident on the interface at $z = 0$ from the region $z \leq 0$.

If $\vec{E}_{t0}, \vec{E}_{t1}, \dots, \vec{E}_{tN}$ are the transverse electric fields on the interfaces at $z = 0, z_1, z_2, \dots, z_N$, respectively, then the transverse electric fields in the various regions of waveguide are obtained as

$$\vec{E}_0 = -2j\vec{e}_0 \sin(\beta_0^0 z) + \sum_{i=0}^{\infty} \left(\iint_{z=0} \vec{E}_{t0} \cdot \vec{e}_i ds \right) \vec{e}_i e^{j\beta_i^0 z} \quad (1)$$

$$\vec{H}_0 = 2Y_0^0 \vec{h}_0 \cos(\beta_0^0 z) - \sum_{i=0}^{\infty} \left(\iint_{z=0} \vec{E}_{t0} \cdot \vec{e}_i ds \right) Y_i^0 \vec{h}_i e^{j\beta_i^0 z} \quad (2)$$

for $z \leq 0$,

$$\vec{E}_n = \sum_{i=0}^{\infty} \left[\frac{\sin(\beta_i^n (z_n - z)) \iint_{z=z_{n-1}} \vec{E}_{t(n-1)} \cdot \vec{e}_i ds + \sin(\beta_i^n (z - z_{n-1})) \iint_{z=z_n} \vec{E}_m \cdot \vec{e}_i ds}{\sin(\beta_i^n z_1)} \right] \vec{e}_i \quad (3)$$

$$\vec{H}_n = \sum_{i=0}^{\infty} \left[\frac{\cos(\beta_i^n (z_n - z)) \iint_{z=z_{n-1}} \vec{E}_{t(n-1)} \cdot \vec{e}_i ds - \cos(\beta_i^n (z - z_{n-1})) \iint_{z=z_n} \vec{E}_m \cdot \vec{e}_i ds}{j \sin(\beta_i^n \Delta)} \right] Y_i^n \vec{h}_i \quad (4)$$

for $z_{n-1} \leq z \leq z_n$, and

$$\vec{E}_{N+1} = \sum_{i=0}^{\infty} \left(\iint_{z=z_N} \vec{E}_{tN} \cdot \vec{e}_i ds \right) \vec{e}_i e^{j\beta_i^{N+1} (z_N - z)} \quad (5)$$

$$\vec{H}_{N+1} = \sum_{i=0}^{\infty} \left(\int_{z=z_N} \int \vec{E}_{tN} \cdot \vec{e}_i ds \right) Y_i^{N+1} \vec{h}_i e^{j\beta_i^{N+1}(z_N-z)} \quad (6)$$

for $z \geq z_N$.

For uniqueness, the tangential magnetic fields at each interface must be continuous.

Hence

$$2Y_0 \vec{h}_0 = \sum_{i=0}^{\infty} \left[\left[Y_i^0 + Y_i^1 \frac{\cos(\beta_i^1 \Delta_1)}{j \sin(\beta_i^1 \Delta_1)} \right] \int_{z=z_{n-1}} \int \vec{E}_{t0} \cdot \vec{e}_i ds - \frac{Y_i^1}{j \sin(\beta_i^1 \Delta_1)} \int_{z=z_n} \int \vec{E}_{t1} \cdot \vec{e}_i ds \right] \vec{h}_i \quad (7)$$

for the interface located at $z = 0$,

$$\begin{aligned} 0 &= \sum_{i=0}^{\infty} \left[\frac{\int_{z=z_{n-1}} \int \vec{E}_{t0} \cdot \vec{e}_i ds}{j \sin(\beta_i^1 \Delta_1)} \right] Y_i^1 \vec{h}_i + \sum_{i=0}^{\infty} \left[\left[Y_i^1 \frac{\cos(\beta_i^1 \Delta_1)}{j \sin(\beta_i^1 \Delta_1)} + Y_i^2 \frac{\cos(\beta_i^2 \Delta_2)}{j \sin(\beta_i^2 \Delta_2)} \right] \int_{z=z_1} \int \vec{E}_{t1} \cdot \vec{e}_i ds \right] \vec{h}_i \\ &\quad + \sum_{i=0}^{\infty} \left[\frac{-Y_i^2 \int_{z=z_2} \int \vec{E}_{t2} \cdot \vec{e}_i ds}{j \sin(\beta_i^2 \Delta_2)} \right] \vec{h}_i \\ 0 &= \sum_{i=0}^{\infty} \left[\frac{\int_{z=z_1} \int \vec{E}_{t1} \cdot \vec{e}_i ds}{j \sin(\beta_i^2 \Delta_2)} \right] Y_i^2 \vec{h}_i + \sum_{i=0}^{\infty} \left[\left[Y_i^2 \frac{\cos(\beta_i^2 \Delta_2)}{j \sin(\beta_i^2 \Delta_2)} + Y_i^3 \frac{\cos(\beta_i^3 \Delta_3)}{j \sin(\beta_i^3 \Delta_3)} \right] \int_{z=z_2} \int \vec{E}_{t2} \cdot \vec{e}_i ds \right] \vec{h}_i \\ &\quad + \sum_{i=0}^{\infty} \left[\frac{-Y_i^3 \int_{z=z_2} \int \vec{E}_{t2} \cdot \vec{e}_i ds}{j \sin(\beta_i^3 \Delta_3)} \right] \vec{h}_i \\ 0 &= \sum_{i=0}^{\infty} \frac{Y_i^n \vec{h}_i}{j \sin(\beta_i^n \Delta_n)} \int_{z=z_{n-1}} \int \vec{E}_{t(n-1)} \cdot \vec{e}_i ds + \sum_{i=0}^{\infty} \vec{h}_i \left[Y_i^n \frac{\cos(\beta_i^n \Delta_n)}{j \sin(\beta_i^n \Delta_n)} + Y_i^{n+1} \frac{\cos(\beta_i^{n+1} \Delta_{n+1})}{j \sin(\beta_i^{n+1} \Delta_{n+1})} \right] \int_{z=z_n} \int \vec{E}_m \cdot \vec{e}_i ds \\ &\quad + \sum_{i=0}^{\infty} \frac{-Y_i^{n+1} \vec{h}_i}{j \sin(\beta_i^{n+1} \Delta_{n+1})} \int_{z=z_{n+1}} \int \vec{E}_{t(n+1)} \cdot \vec{e}_i ds \end{aligned} \quad (8)$$

for the interface located at $z = z_n$, and

$$0 = \sum_{i=0}^{\infty} \frac{Y_i^N \bar{h}_i}{j \sin(\beta_i^N \Delta_N)} \iint_{z=z_{N-1}} \bar{E}_{t(N-1)} \cdot \bar{e}_i ds + \sum_{i=0}^{\infty} \bar{h}_i \left[Y_i^N \frac{\cos(\beta_i^N \Delta_N)}{j \sin(\beta_i^N \Delta_N)} + Y_i^{N+1} \right] \iint_{z=z_N} \bar{E}_{tN} \cdot \bar{e}_i ds \quad (9)$$

for the interface located at $z = z_N$.

The transverse electric fields over the interfaces can be expressed in terms of vector modal expansion functions as:

$$\bar{E}_{t0} = \sum_{j=0}^{J_0} T_{0j} \bar{e}_j, \quad \bar{E}_{t1} = \sum_{j=0}^{J_1} T_{1j} \bar{e}_j, \quad \bar{E}_m = \sum_{j=0}^{J_m} T_{mj} \bar{e}_j, \quad \dots, \quad \bar{E}_{tN} = \sum_{j=0}^{J_N} T_{Nj} \bar{e}_j \quad (10)$$

where $T_{0j}, T_{1j}, \dots, T_{Nj}$ are the complex unknown coefficients. Substitution of (10) into (7)-

(9) yields

$$2Y_0^0 \bar{h}_0 = \sum_{j=0}^{J_0} T_{0j} \bar{h}_j \left[Y_j^0 + Y_j^1 \frac{\cos(\beta_j^1 \Delta_1)}{j \sin(\beta_j^1 \Delta_1)} \right] - \sum_{j=0}^{J_1} T_{1j} \bar{h}_j \left[\frac{Y_j^1}{j \sin(\beta_j^1 \Delta_1)} \right] \quad (11)$$

$$0 = \sum_{j=0}^{J_{n-1}} T_{(n-1)j} \frac{Y_j^n \bar{h}_j}{j \sin(\beta_j^n \Delta_n)} + \sum_{j=0}^{J_n} T_{nj} \bar{h}_j \left[Y_j^n \frac{\cos(\beta_j^n \Delta_n)}{j \sin(\beta_j^n \Delta_n)} + Y_j^{n+1} \frac{\cos(\beta_j^{n+1} \Delta_{n+1})}{j \sin(\beta_j^{n+1} \Delta_{n+1})} \right] + \sum_{j=0}^{J_{n+1}} \frac{-T_{(n+1)j} Y_j^{n+1} \bar{h}_j}{j \sin(\beta_j^{n+1} \Delta_{n+1})} \quad (12)$$

$$0 = \sum_{j=0}^{J_{N-1}} T_{(N-1)j} \frac{Y_j^N \bar{h}_j}{j \sin(\beta_j^N \Delta_N)} + \sum_{j=0}^{J_N} T_{Nj} \bar{h}_j \left[Y_j^N \frac{\cos(\beta_j^N \Delta_N)}{j \sin(\beta_j^N \Delta_N)} + Y_j^{N+1} \right] \quad (13)$$

Equations (11)–(13) are the required integral equations to be used to determine the complex amplitudes $T_{0j}, T_{1j}, \dots, T_{Nj}$. Selecting \vec{h}_k as a testing function and using the Galerkin's procedure, the equations (11)-(13) are converted into a set of simultaneous equations:

$$2Y_0^0 = T_{00} \left[Y_0^0 + Y_0^1 \frac{\cos(\beta_0^1 \Delta_1)}{j \sin(\beta_0^1 \Delta_1)} \right] - T_{10} \left[\frac{Y_0^1}{j \sin(\beta_0^1 \Delta_1)} \right] \quad (14a)$$

$$0 = T_{01} \left[Y_1^0 + Y_1^1 \frac{\cos(\beta_1^1 \Delta_1)}{j \sin(\beta_1^1 \Delta_1)} \right] - T_{11} \left[\frac{Y_1^1}{j \sin(\beta_1^1 \Delta_1)} \right] \quad (14b)$$

.....

$$0 = T_{0j_0} \left[Y_{j_0}^0 + Y_{j_0}^1 \frac{\cos(\beta_{j_0}^1 \Delta_1)}{j \sin(\beta_{j_0}^1 \Delta_1)} \right] - T_{1j_0} \left[\frac{Y_{j_0}^1}{j \sin(\beta_{j_0}^1 \Delta_1)} \right] \quad (14c)$$

obtained from equation (11). From the continuity of magnetic field at $z = z_1$ we get

$$0 = \frac{T_{00} Y_0^1}{j \sin(\beta_0^1 \Delta_1)} + T_{10} \left[Y_0^1 \frac{\cos(\beta_0^1 \Delta_1)}{j \sin(\beta_0^1 \Delta_1)} + Y_0^2 \frac{\cos(\beta_0^2 \Delta_2)}{j \sin(\beta_0^2 \Delta_2)} \right] - T_{20} \frac{Y_0^2}{j \sin(\beta_0^2 \Delta_2)} \quad (15a)$$

$$0 = \frac{T_{01} Y_1^1}{j \sin(\beta_1^1 \Delta_1)} + T_{11} \left[Y_1^1 \frac{\cos(\beta_1^1 \Delta_1)}{j \sin(\beta_1^1 \Delta_1)} + Y_1^2 \frac{\cos(\beta_1^2 \Delta_2)}{j \sin(\beta_1^2 \Delta_2)} \right] - \frac{T_{21} Y_1^2}{j \sin(\beta_1^2 \Delta_2)} \quad (15b)$$

.....

$$0 = \frac{T_{0J_1} Y_{J_1}^1}{j \sin(\beta_{J_1}^1 \Delta_1)} + T_{1J_1} \left[Y_{J_1}^1 \frac{\cos(\beta_{J_1}^1 \Delta_1)}{j \sin(\beta_{J_1}^1 \Delta_1)} + Y_{J_1}^2 \frac{\cos(\beta_{J_1}^2 \Delta_2)}{j \sin(\beta_{J_1}^2 \Delta_2)} \right] - \frac{T_{2J_1} Y_{J_1}^2}{j \sin(\beta_{J_1}^2 \Delta_2)} \quad (15c)$$

Likewise, using the continuity of magnetic fields at $z = z_2, z = z_3, \dots, z = z_{N-1}$, similar sets of simultaneous equations are obtained. From the continuity of magnetic fields at $z = z_N$ we get

$$0 = \frac{T_{(N-1)0} Y_0^N}{j \sin(\beta_0^N \Delta_N)} + T_{N0} \left[Y_0^N \frac{\cos(\beta_0^N \Delta_N)}{j \sin(\beta_0^N \Delta_N)} + Y_0^{N+1} \right] \quad (16a)$$

$$0 = \frac{T_{(N-1)1} Y_1^N}{j \sin(\beta_1^N \Delta_N)} + T_{N1} \left[Y_1^N \frac{\cos(\beta_1^N \Delta_N)}{j \sin(\beta_1^N \Delta_N)} + Y_1^{N+1} \right] \quad (16b)$$

.....

$$0 = \frac{T_{(N-1)J_N} Y_{J_N}^N}{j \sin(\beta_{J_N}^N \Delta_N)} + T_{NJ_N} \left[Y_{J_N}^N \frac{\cos(\beta_{J_N}^N \Delta_N)}{j \sin(\beta_{J_N}^N \Delta_N)} + Y_{J_N}^{N+1} \right] \quad (16c)$$

Due to the orthogonal nature of vector modal functions it can be shown that the complex amplitudes $T_{0j}, j = 1, 2, 3, \dots, J_0, T_{1j}, j = 1, 2, 3, \dots, J_1, \dots, T_{Nj}, j = 1, 2, 3, \dots, J_N$ are all zeros.

Hence equations (14)-(16) can be simplified as

$$2Y_0^0 = T_{00} \left[Y_0^0 + Y_0^1 \frac{\cos(\beta_0^1 \Delta_1)}{j \sin(\beta_0^1 \Delta_1)} \right] - T_{10} \left[\frac{Y_0^1}{j \sin(\beta_0^1 \Delta_1)} \right] \quad (17a)$$

$$0 = \frac{T_{00} Y_0^1}{j \sin(\beta_0^1 \Delta_1)} + T_{10} \left[Y_0^1 \frac{\cos(\beta_0^1 \Delta_1)}{j \sin(\beta_0^1 \Delta_1)} + Y_0^2 \frac{\cos(\beta_0^2 \Delta_2)}{j \sin(\beta_0^2 \Delta_2)} \right] - T_{20} \frac{Y_0^2}{j \sin(\beta_0^2 \Delta_2)} \quad (17b)$$

$$0 = \frac{T_{10} Y_0^2}{j \sin(\beta_0^2 \Delta_2)} + T_{20} \left[Y_0^2 \frac{\cos(\beta_0^2 \Delta_2)}{j \sin(\beta_0^2 \Delta_2)} + Y_0^3 \frac{\cos(\beta_0^3 \Delta_3)}{j \sin(\beta_0^3 \Delta_3)} \right] - T_{30} \frac{Y_0^3}{j \sin(\beta_0^3 \Delta_3)} \quad (17c)$$

.....

$$0 = \frac{T_{(N-1)0} Y_0^N}{j \sin(\beta_0^N \Delta_N)} + T_{N0} \left[Y_0^N \frac{\cos(\beta_0^N \Delta_N)}{j \sin(\beta_0^N \Delta_N)} + Y_0^{N+1} \right] \quad (17d)$$

solution of above $(N + 1)$ equations gives an estimate of complex amplitudes

$T_{00}, T_{10}, T_{20}, \dots, T_{N0}$ from which S_{11} and S_{21} are determined as

$$S_{11} = T_{00} - 1 \quad \text{and} \quad S_{21} = T_{N0} e^{j\beta_0^0 2z_N} \quad (18)$$

S_{22} and S_{12} Parameters:

The port 2 parameters, S_{22} and S_{12} can be determined by following the procedure used for estimation of S_{11} and S_{21} , and reversing the locations of the layers as shown in figure 2. Note that the S_{22} calculated using the reference planes shown in figure 2 and S_{22} measured using the *hp-8510* network analyzer differ by phase $e^{j\beta_0^0 2z_N}$.

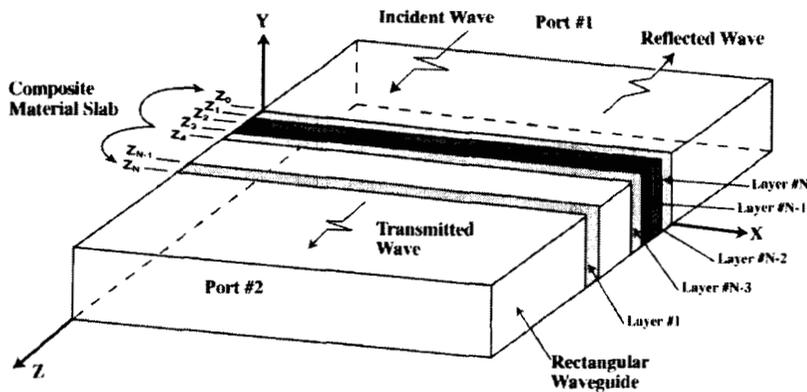


Figure 2: Geometry of rectangular waveguide loaded with a composite material for estimation of S_{22} and S_{12}

III. Numerical Results

A: Estimation of S-Parameters(Direct/Forward Problem)

A simple MatLab code (Appendix A) is written to solve the simultaneous equations given in equations (17) and determine all four S -parameters of a composite material slab placed in a rectangular waveguide. In this section, assuming the properties of individual layers of a composite material known, the S -parameters for various composite slabs are computed (using the MatLab code) as a function of frequency and compared with the measured S -parameters.

Single Layer Composite Material:

Figure 3 shows S_{11} and S_{21} parameters of a composite material consisting of a Garlock single layer as a function of frequency. In this case a single layer of Garlock with thickness $\Delta_1 = 0.17\text{cm}$ and electric properties $\epsilon_r = 7.5 - j0.001$ is used to form a composite material. Excellent agreement between measured and estimated

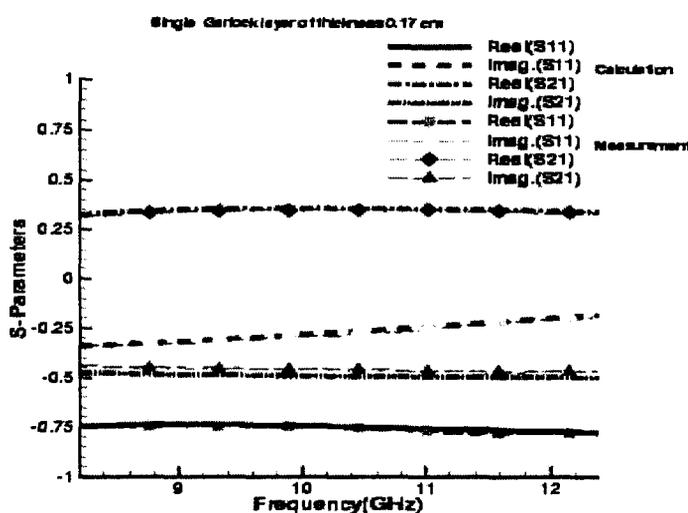


Figure 3: Measured and estimated S -parameters of single Garlock slab. Thickness $\Delta_1 = 0.17\text{cm}$, $\epsilon_r = 7.5 - j0.001$, $\mu_r = 1.0 - j0.0$

values of the S -parameters validates the MatLab code. Figure 4 shows the S_{11} and S_{21} parameters of a composite material formed by a single layer of Teflon material ($\epsilon_r = 2.03 - j0.001$, $\mu_r = 1.0 - j0.0$) of thickness $\Delta_1 = 0.635\text{cm}$. A good agreement between the measured and estimated values of S -parameters confirms validity of the present method. Note that the parameters S_{22} and S_{12} are expected and found to be identical to S_{11} and S_{21} , respectively.

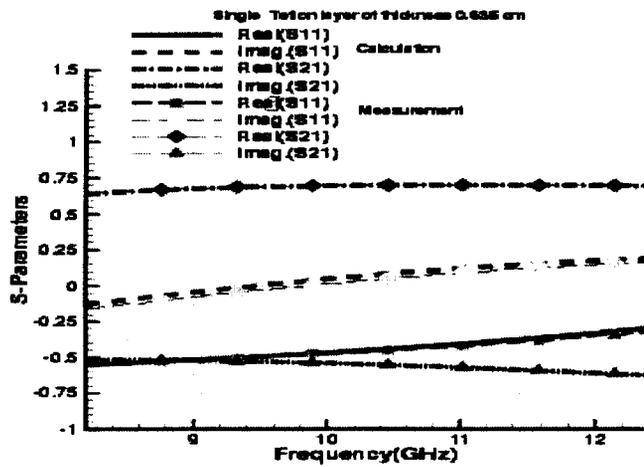


Figure 4: Measured and estimated S -parameters of single teflon slab. Thickness $\Delta_1 = 0.635\text{cm}$, $\epsilon_r = 2.03 - j0.001$, $\mu_r = 1.0 - j0.0$

Two Layer Composite Material:

For further validation of the present method and MatLab code, the S -parameters of composite material formed by various combination of two layers are fabricated and tested. A first sample considered consists of Bakelite and Teflon layers.

The composite slab is formed by placing the bakelite layer with $\Delta_1 = 0.33cm$, $z_0 = 0$, $z_1 = 0.33cm$ and the Teflon layer with $\Delta_2 = 0.635cm$, $z_1 = 0.33cm$, $z_2 = 0.965cm$. Measured and estimated S_{11} and S_{21} parameters as a function of frequency are shown in figure 5. For the composite material described in figure 5, the measured and estimated S_{22} and S_{12} values are shown in figure 6.

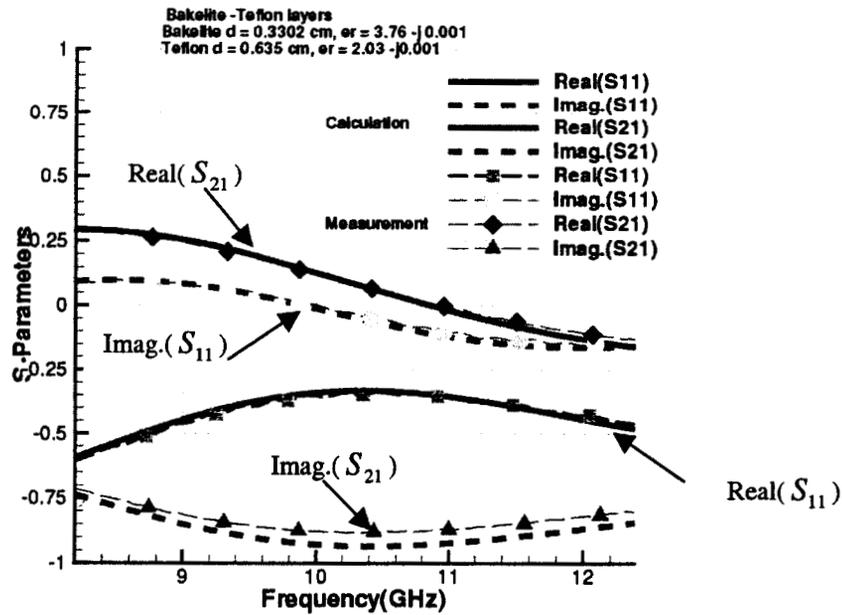


Figure 5: Measured and estimated S -parameters of two layers composite material (Bakelite-Teflon). Bakelite: $\Delta_1 = 0.33cm$, $z_0 = 0$, $z_1 = 0.33cm$, $\epsilon_r = 3.76 - j0.001$, $\mu_r = 1.0$, Teflon: $\Delta_2 = 0.635cm$, $z_1 = 0.33cm$, $z_2 = 0.965cm$, $\epsilon_r = 2.03 - j0.001$, $\mu_r = 1$

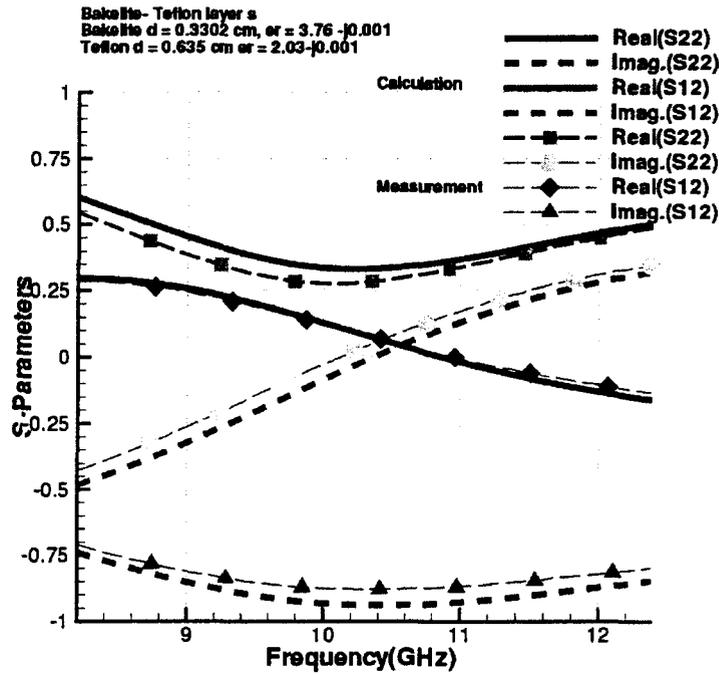


Figure 6: Measured and estimated values of S_{22} and S_{12} for composite slab (parameters as described in figure 5)

A second sample of composite material considered consists of Garlock and Nomex materials. The composite slab is formed by placing first a Garlock slab of thickness $\Delta_1 = 0.17\text{cm}$, $\epsilon_r = 7.5 - j0.001$, $\mu_r = 1.0$ at $z_0 = 0$ and $z_1 = 0.17\text{cm}$ and then a Nomex slab of thickness $\Delta_2 = 0.33\text{cm}$, $\epsilon_r = 1.2 - j0.001$, $\mu_r = 1.0$ at $z_1 = 0.17\text{cm}$ and $z_2 = 0.5\text{cm}$. Measured and estimated S_{11} , S_{21} , S_{12} , and S_{22} parameters for Garlock-Nomex composite slab are shown in figures 7 and 8.

From figures 7-8, estimated values of S -parameters for the Garlock-Nomex composite slab agrees well with measured values of the S -parameters. However, for the Bakelite-Teflon combination there is small disagreement between measured and estimated S_{22} and S_{12} parameters. This disagreement may be attributed to the presence of an air gap between the slabs of Bakelite and Teflon.

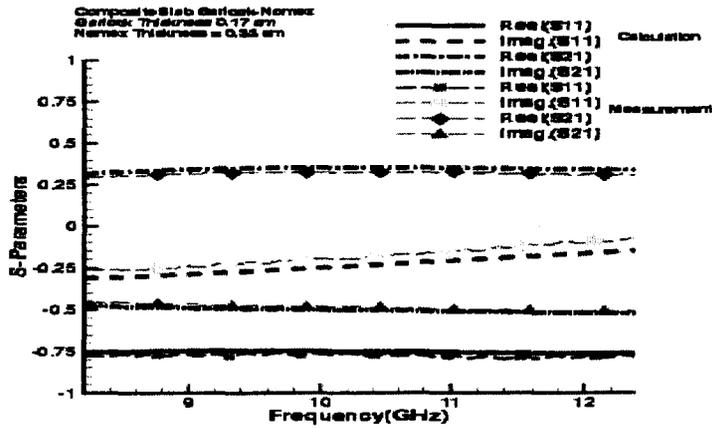


Figure 7: Measured and estimated S -parameters of two layers composite material (Garlock-Nomex). Galock: $\Delta_1 = 0.17\text{cm}$, $z_0 = 0$, $z_1 = 0.17\text{cm}$, $\epsilon_r = 7.5 - j0.001$, $\mu_r = 1.0$, Nomex $\Delta_2 = 0.33\text{cm}$, $z_1 = 0.17\text{cm}$, $z_2 = 0.50\text{cm}$, $\epsilon_r = 1.2 - j0.001$, $\mu_r = 1$

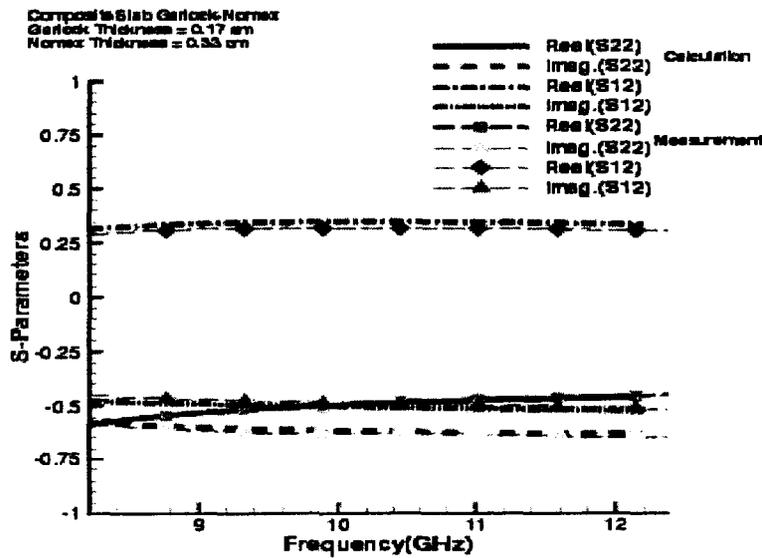


Figure 8: Measured and estimated S -parameters of two layers composite material (Garlock-Nomex) with dimensions as shown in Figure 7

Three Layer Composite Material:

A third sample of composite material considered consists of Nano Material # 0, Garlock and Garlock slabs. The composite slab is formed by placing first a Nano Material #0 slab of thickness $\Delta_1 = 0.31cm$, $\epsilon_r = 2.5 - j0.001$, $\mu_r = 1.0$ at $z_0 = 0$ and $z_1 = 0.31cm$ and then a Garlock slab of thickness $\Delta_2 = 0.17cm$, $\epsilon_r = 7.5 - j0.001$, $\mu_r = 1$, at $z_1 = 0.31cm$, and $z_2 = 0.48cm$. Another Garlock slab of thickness $\Delta_1 = 0.17cm$, $\epsilon_r = 7.5 - j0.001$, $\mu_r = 1.0$ is placed at $z_2 = 0.48cm$ and $z_3 = 0.65cm$. Figures 9-10 show measured and estimated S -parameters of composite slab consisting of Nano_Mat #0-Garlock-Garlock combination.

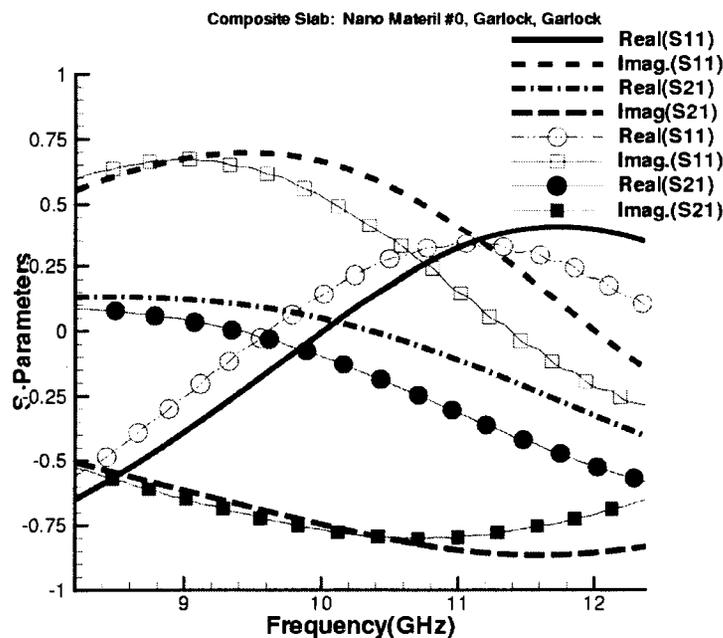


Figure 9: Measured and estimated S_{11} and S_{21} parameters of composite slab consisting of Nano-Material#0, Garlock, and Garlock(Dimensions as described in text)

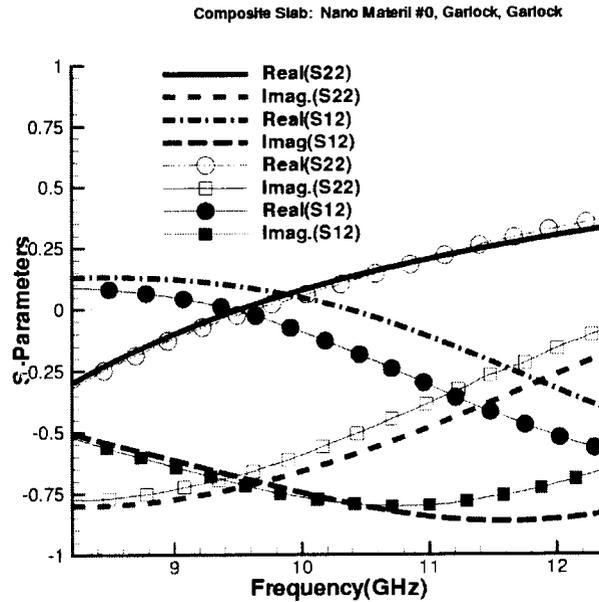


Figure 10: Measured and estimated S_{22} and S_{12} parameters of composite slab consisting of Nano-Material#0, Garlock, and Garlock(Dimensions as described in text)

From figures 9-10, the trend in the values of S -parameters predicted by the computation is similar to the trend observed in the measured values. However, significant differences are observed between the measured and estimated values of S -parameters. This may be attributed to the sample preparation where the presence of air gaps between the individual slabs is unavoidable. Also even though these individual slabs may be in physical contact with each other, electrical contact may still not be insured. It is also noticeable that the disagreement between the measured and estimated S -parameters gets worse as more individual slabs are used to form a composite slab.

B: Estimation of Dielectric Constants(Inverse Problem):

Single Layer Composite Material:

In this section a procedure is described for the computation of the complex dielectric constant of a given composite sample from the two port measured data. For a given composite material sample, let the two port S -parameters be $(S_{11m}, S_{21m}, S_{12m}, S_{22m})$, measured using the *hp-8510 Network Analyzer*. With a prior knowledge of the thickness of composite slabs, the two port S -parameters can be estimated using the MatLab Code given in Appendix A as a function of ϵ_r . Let the estimated values of S -parameters be $S_{11c}(\epsilon_r), S_{21c}(\epsilon_r), S_{12c}(\epsilon_r), S_{22c}(\epsilon_r)$. The errors in estimated and measured S -parameters can then be written as $ER_1 = real(S_{11c} - S_{11m})$, $ER_2 = imag.(S_{11c} - S_{11m})$, $ER_3 = real(S_{21c} - S_{21m})$, $ER_4 = imag.(S_{21c} - S_{21m})$, $ER_7 = real(S_{22c} - S_{22m})$, $ER_8 = imag.(S_{22c} - S_{22m})$. The total mean squared error or the objective function to be minimized as a function of ϵ_r can be written as

$$ER_T = \sqrt{(ER_1^2 + ER_2^2 + ER_3^2 + ER_4^2 + ER_5^2 + ER_6^2 + ER_7^2 + ER_8^2)} \quad (19)$$

A simple MatLab Code given in Appendix B minimizes the objective function in (19) to estimate the unknown value of dielectric constant ϵ_r .

Examples (When Thickness of Sample Is Known):

Garlock Slab: A single layer of Garlock slab of size (2.29 x 1.02 x 0.17)cm was placed in an X-band rectangular waveguide. After proper calibration of the *hp 8510 Network Analyzer*, two port S-parameters of the Garlock slab were measured as a function of frequency. The measured data is stored in a file FT_GRLK.60. Using the MatLab Code

given in the Appendix B, the dielectric constant of the Garlock slab is estimated and shown in figure 11.

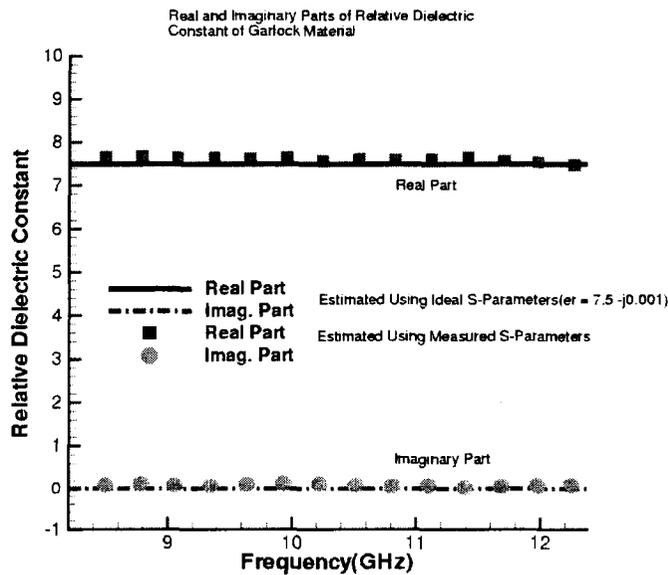


Figure 11: Relative dielectric constant (real and imaginary) of Garlock material estimated using measured and ideal S-parameters.

In figure 11 solid and dash-dot lines are the estimates of dielectric constant using the ideal or noise free S-parameters calculated assuming the dielectric constant is known and equal to $\epsilon_r = 7.5 - j0.001$. The estimates shown in figure 11 were obtained by considering the thickness of the slab equal to 0.170cm. To check the level of confidence in these estimates, from the estimated values of dielectric constant (estimated using the measured data), the difference between measured and computed values of S-parameters is plotted in Figure 12.

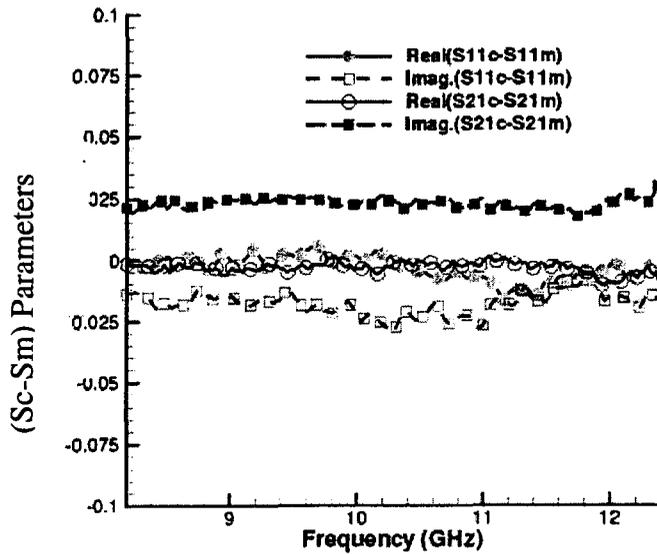


Figure 12: Differences between computed and measured values of S-parameters. Calculated S-parameters are determined using estimated dielectric constant

The differences between the computed and measured S-parameters are within the limits set in the optimizer. Ideally, the difference between the computed and measured S-parameters must be close to zero. However, to achieve ideal results, the optimizer would take longer time.

Number of single layer composite materials of various material and thickness were constructed and their S-parameters were measured over the X-band frequency range. From these measured values of S-parameters and using the MatLab Code given in Appendix B, the dielectric constants of these single layer composite were estimated and presented in figures 13-25.

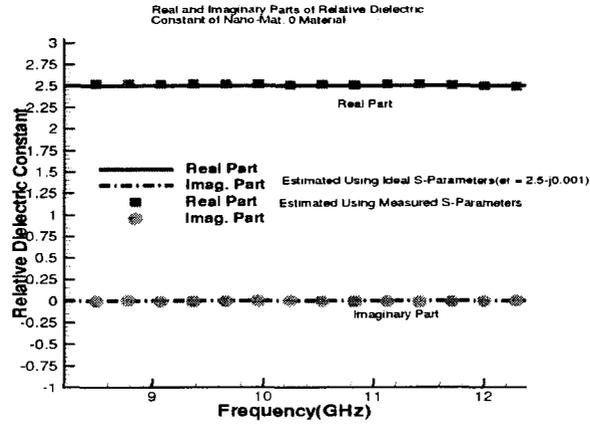


Figure 13: Relative dielectric constant (real and imaginary) of Nano material #0 estimated using measured and ideal S-parameters (thickness = 0.3099 cm)

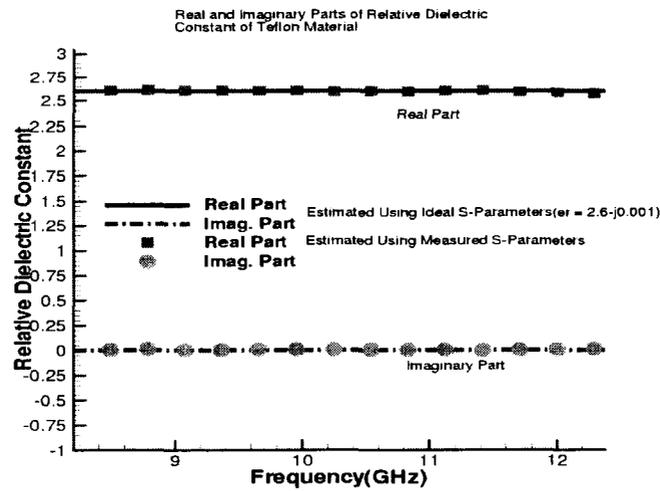


Figure 14: Relative dielectric constant (real and imaginary) of Teflon material estimated using measured and ideal S-parameters (thickness = 0.9398)

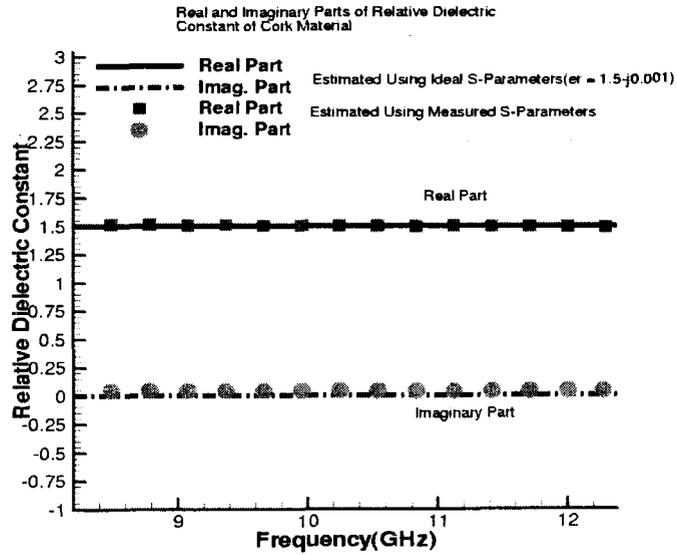


Figure 15: Relative dielectric constant (real and imaginary) of Cork material estimated using measured and ideal S-parameters (thickness = 0.3048cm)

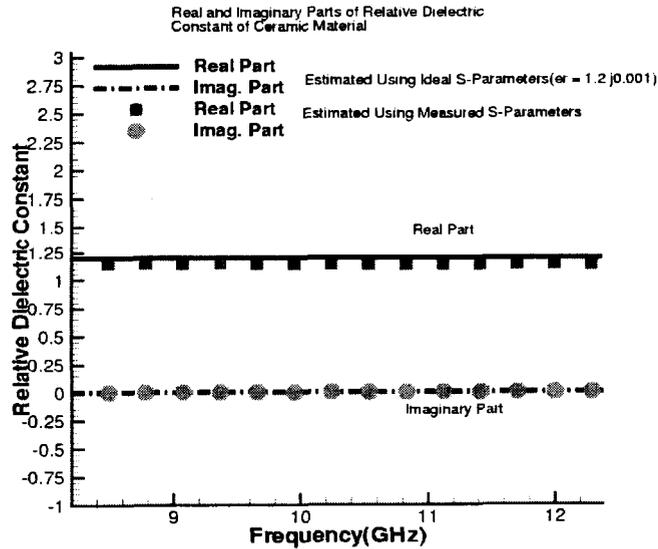


Figure 16: Relative dielectric constant (real and imaginary) of Ceramic material estimated using measured and ideal S-parameters (thickness = 0.2845 cm)

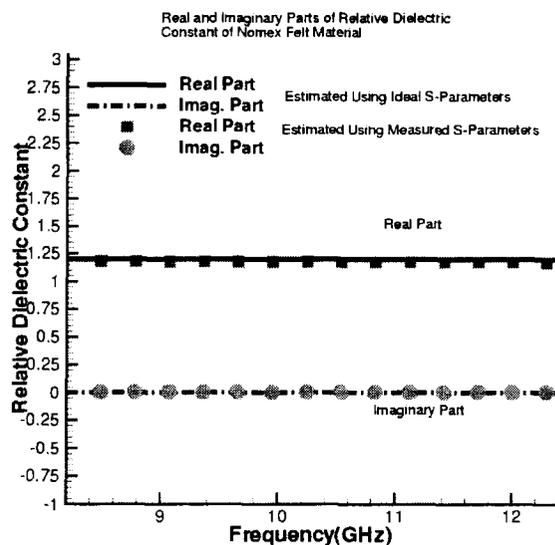


Figure 17: Relative dielectric constant (real and imaginary) of Nomex Felt material estimated using measured and ideal S-parameters (thickness = 0.33cm)

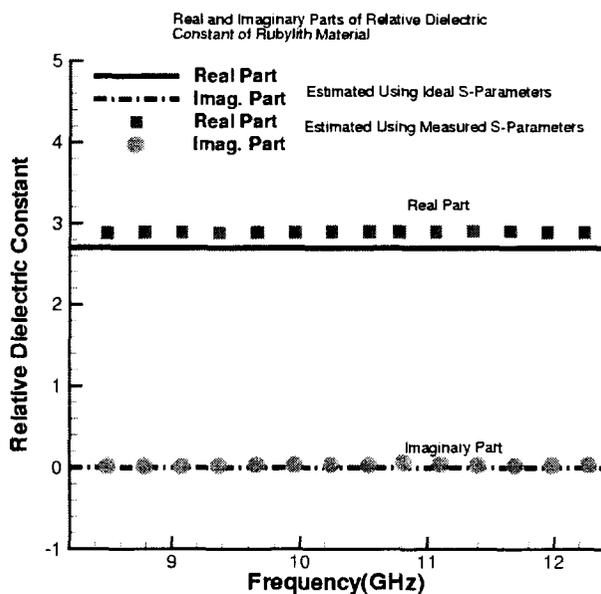


Figure 18: Relative dielectric constant (real and imaginary) of Rubylith material estimated using measured and ideal S-parameters (thickness = 0.3683 cm)

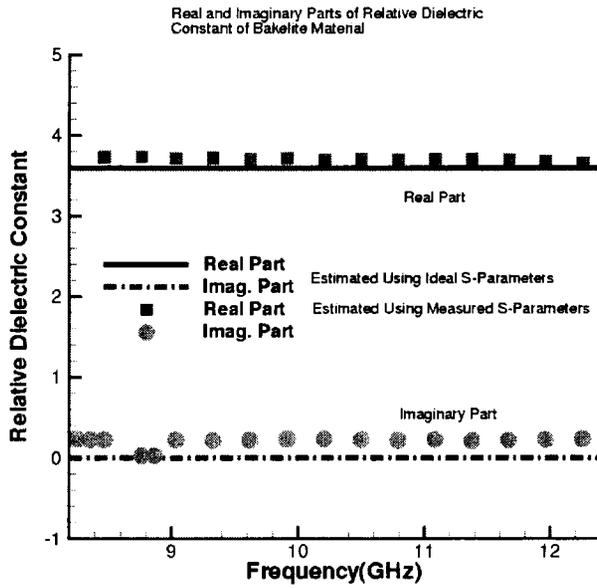


Figure 19: Relative dielectric constant (real and imaginary) of Bakelite material estimated using measured and ideal S-parameters (thickness = 0.33cm)

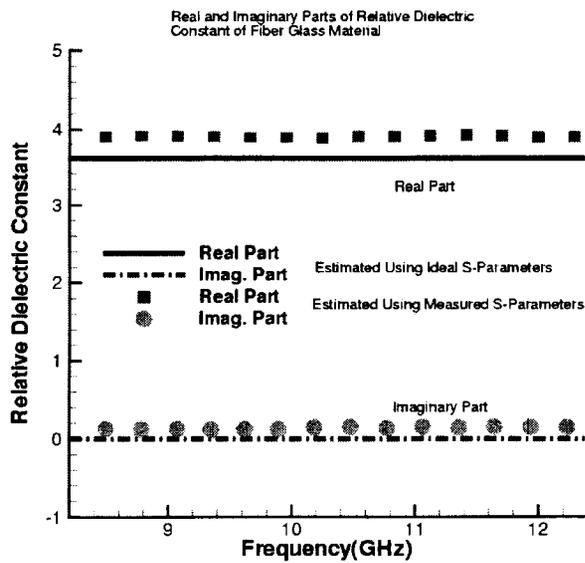


Figure 20: Relative dielectric constant (real and imaginary) of Fiber Glass material estimated using measured and ideal S-parameters (thickness = 0.0533cm)

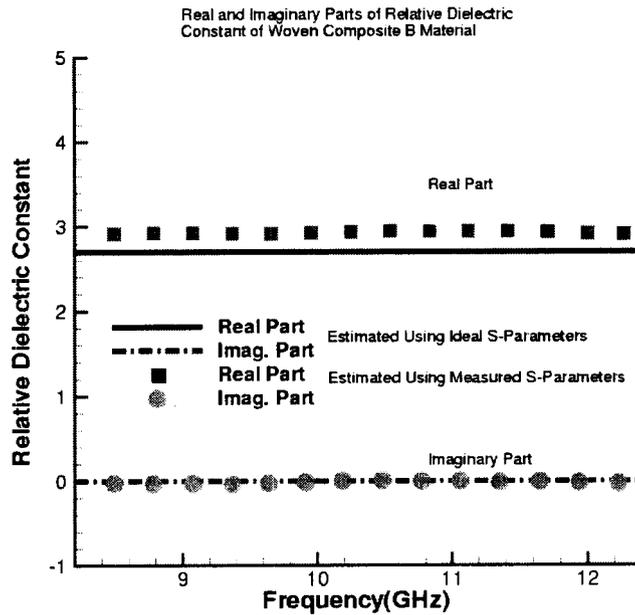


Figure 21: Relative dielectric constant (real and imaginary) of Woven Composite B material estimated using measured and ideal S-parameters (thickness = 0.1625cm)

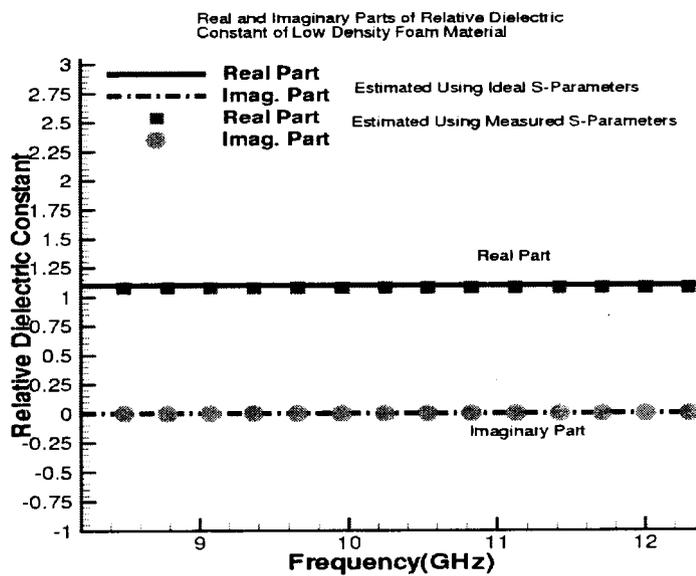


Figure 22: Relative dielectric constant (real and imaginary) of low density Foam material estimated using measured and ideal S-parameters (thickness = 0.2921cm)

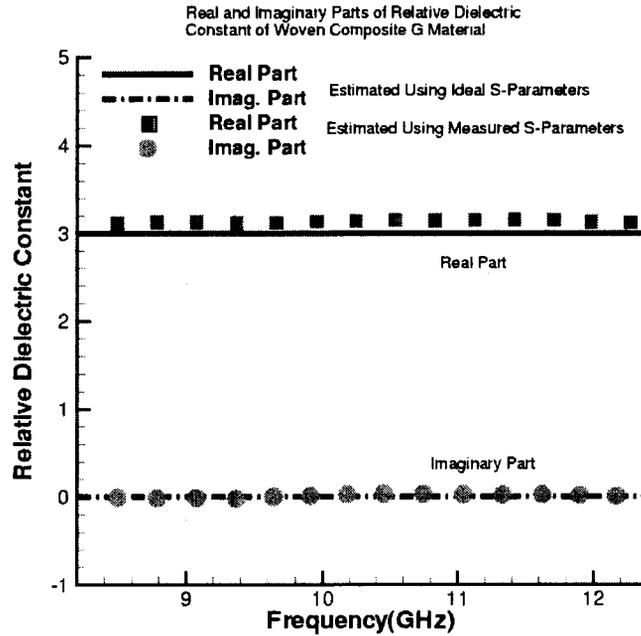


Figure 23: Relative dielectric constant (real and imaginary) of Woven Composite G material estimated using measured and ideal S-parameters (thickness = 0.2083 cm)

Examples (When Thickness of Sample Is Unknown):

In the previous section it was assumed that the thickness of the sample is known apriori. These thicknesses are measured in the laboratory using an electronic micrometer. However, for compressible and thin samples the accuracy of these measurements is questionable. In this section it is assumed that the thickness of the sample is unknown and the optimizer is asked to estimate the thickness along with the dielectric constant of a single layer composite material slab. The MatLab Code used for estimation of sample thickness as well as dielectric constant is given in Appendix B.

Garlock Slab: The measured data stored in a file FT_GRLK.60 is used to estimate the thickness of Garlock slab and dielectric constants. Using the MatLab Code given in the Appendix B, the dielectric constant of the Garlock slab and its thickness are estimated and shown in figure 24.

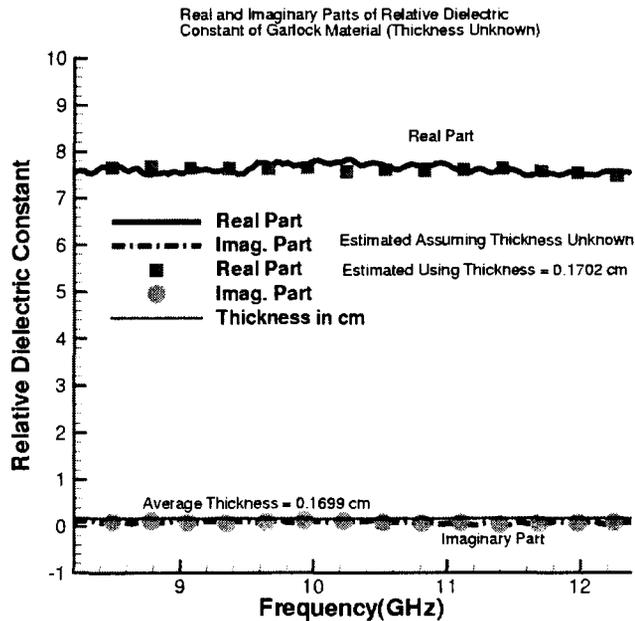


Figure 24: Relative dielectric constant (real and imaginary) of Garlock material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.17 cm. Thin solid line indicate estimated value of sample thickness.

In figure 24, estimated values of the dielectric constant assuming sample thickness unknown are in close agreement with the dielectric constant estimated assuming known value of sample thickness. The average value of estimated sample thickness is 0.1699 cm which is very close to the actual measured thickness of 0.1702cm. This validates the idea that apriori knowledge of sample thickness is not necessary for the inverse problem. In fact the thickness of the sample can be considered as one of the unknown variables along with the dielectric constants to optimize the error function defined in (19). Using the

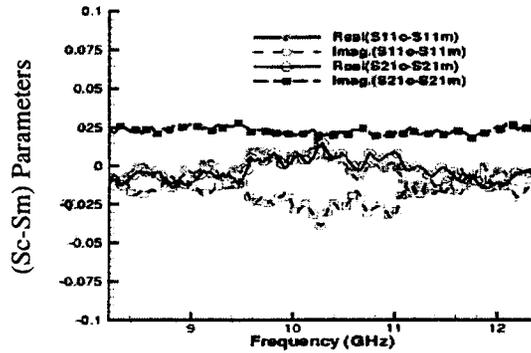


Figure 25: Differences between computed and measured values of S-parameters. Calculated S-parameters are determined using estimated dielectric constant and thickness of the slab

estimated values of dielectric constants and the sample thickness, the difference between measured and computed values of S-parameters is plotted in Figure 25.

The differences between the computed and measured S-parameters are within the limits set in the optimizer. Ideally, the difference between the computed and measured S-parameters must be close to zero. However, to achieve ideal results, the optimizer would take a longer time.

From the measured values of the S-parameters for a variety of single layer composite materials (Nano Material #0, Cork, Ceramic, Nomex Felt, Rubylith, Bakelite, Teflon, Fiber Glass, Woven Composite B and G, Nano Material # 0) the dielectric constants and thicknesses of the material are estimated using the MatLab Code given in Appendix B. These estimated values are shown in figures 26-36. For comparison, the estimated values of dielectric constants using MatLab Code given in Appendix B are also plotted in figures 26-36. The estimated values of dielectric constants assuming the thickness unknown are in good agreement with the estimates obtained using known

values of dielectric constants. From the figures 26-36 it may be concluded that the thickness of a single layer slab can be treated as one of the unknown variables along with the dielectric constants.

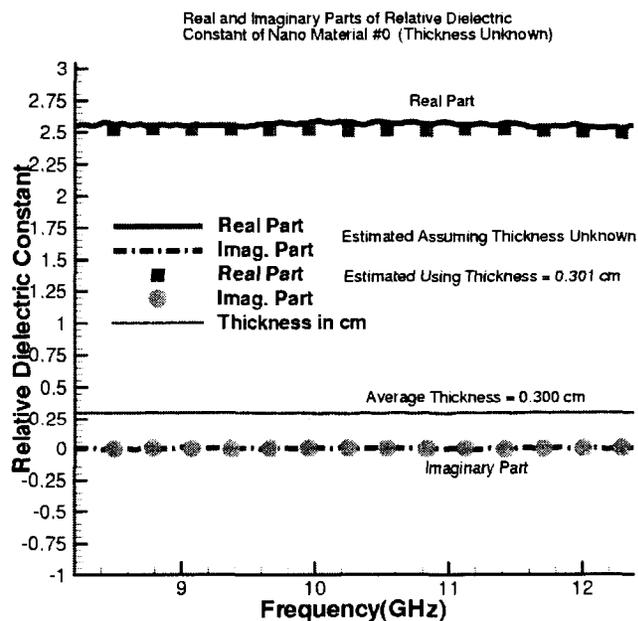


Figure 26: Relative dielectric constant (real and imaginary) of Nano material #0 estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.301 cm. Thin solid line indicate estimated value of sample thickness.

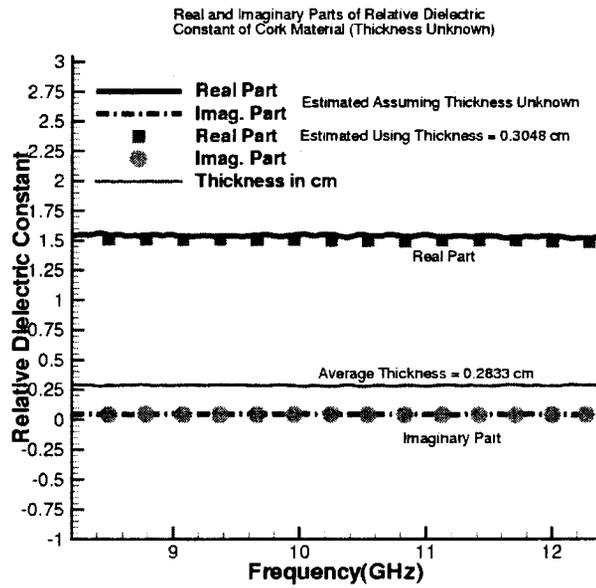


Figure 27: Relative dielectric constant (real and imaginary) of Cork material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.3048 cm. Thin solid line indicate estimated value of sample thickness.

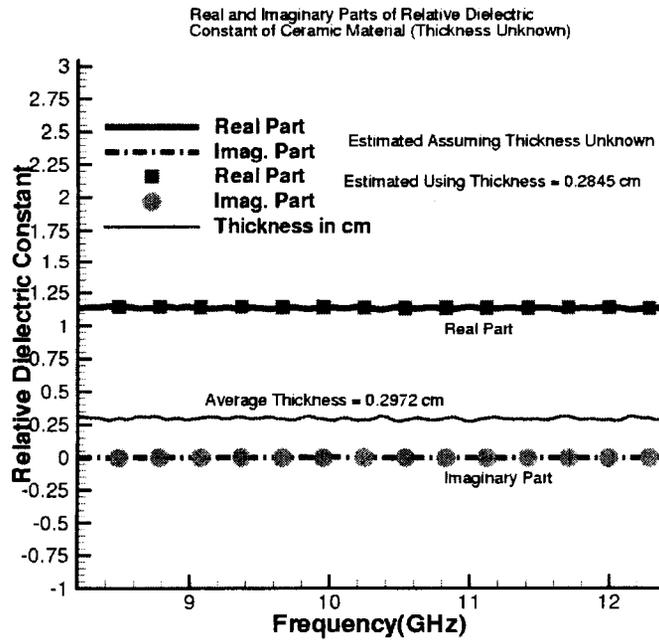


Figure 28: Relative dielectric constant (real and imaginary) of Ceramic material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.2845 cm. Thin solid line indicate estimated value of sample thickness.

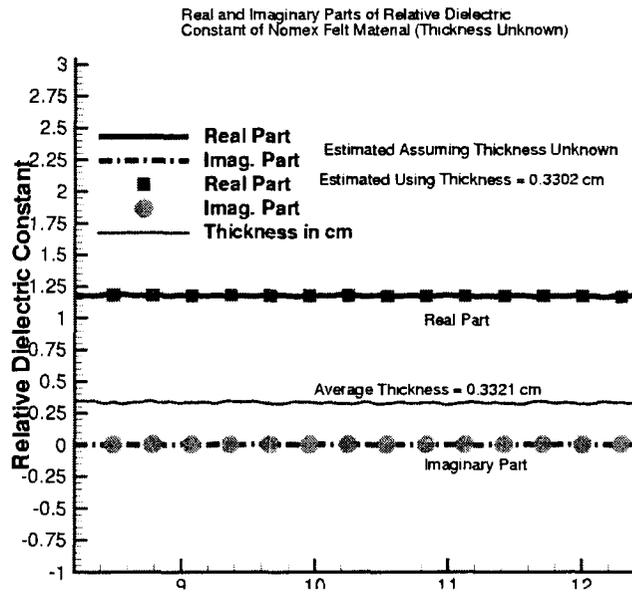


Figure 29: Relative dielectric constant (real and imaginary) of Nomex Felt material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.3302 cm. Thin solid line indicate estimated value of sample thickness.

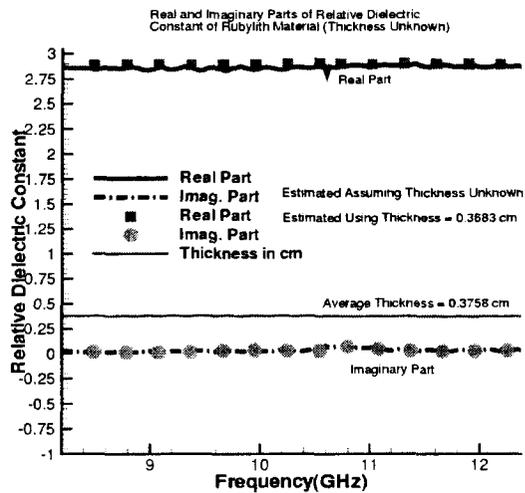


Figure 30: Relative dielectric constant (real and imaginary) of Rubylith material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.3683 cm. Thin solid line indicate estimated value of sample thickness.

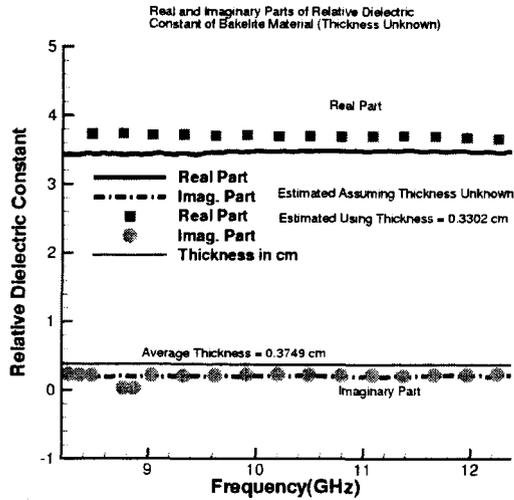


Figure 31: Relative dielectric constant (real and imaginary) of Bakelite material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.3302 cm. Thin solid line indicate estimated value of sample thickness.

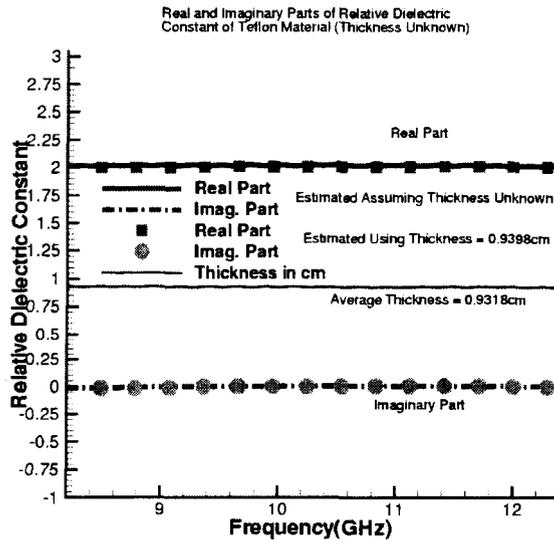


Figure 32: Relative dielectric constant (real and imaginary) of Teflon material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.9398 cm. Thin solid line indicate estimated value of sample thickness.

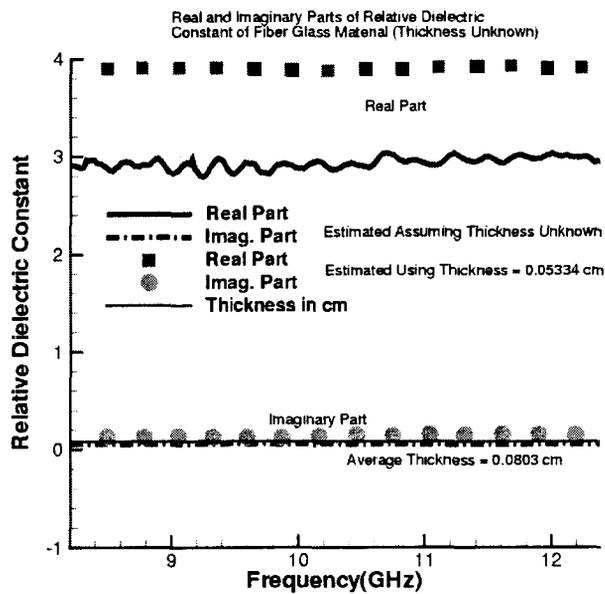


Figure 33: Relative dielectric constant (real and imaginary) of Fiber Glass material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.0533 cm. Thin solid line indicate estimated value of sample thickness.

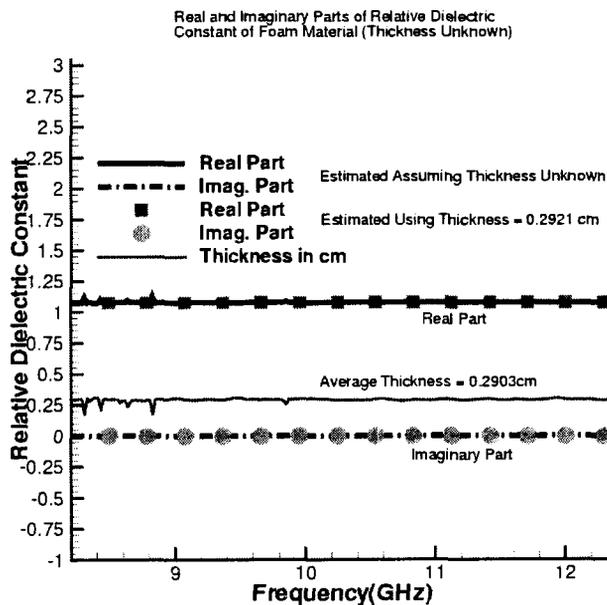


Figure 34: Relative dielectric constant (real and imaginary) of Foam material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.2921 cm. Thin solid line indicate estimated value of sample thickness.

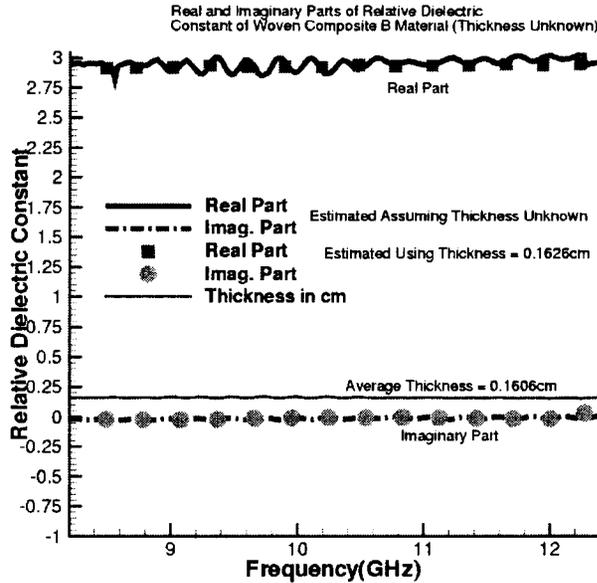


Figure 35: Relative dielectric constant (real and imaginary) of Woven Composite B material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.1626 cm. Thin solid line indicate estimated value of sample thickness.

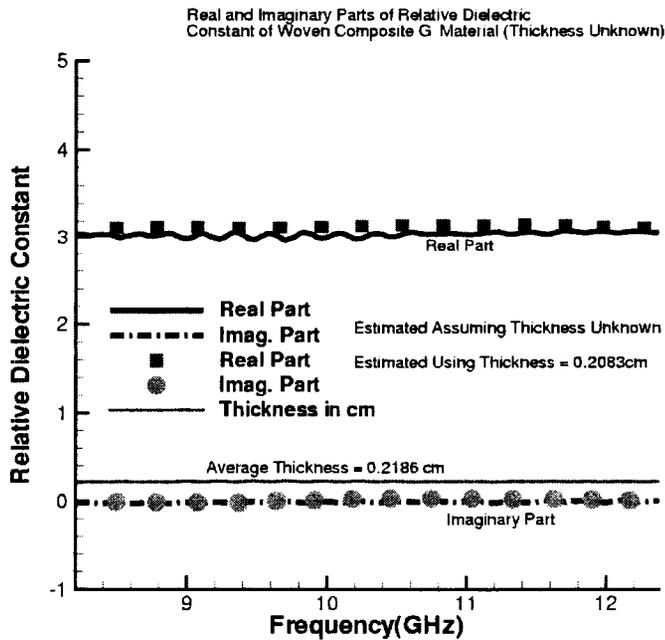


Figure 36: Relative dielectric constant (real and imaginary) of Woven Composite G material estimated assuming thickness unknown. Symbols indicate estimated values of dielectric constant using thickness = 0.2083 cm. Thin solid line indicate estimated value of sample thickness.

Most of estimated dielectric constants determined assuming thickness unknown agree well with the dielectric constants estimated using measured value of slab thickness. However, in figure 33, the two results disagree significantly. This is because the thickness of the slab is very small. In fact, the S-parameters calculated using estimated thickness and the dielectric constants agree very well with the measurements as shown in figure 37.

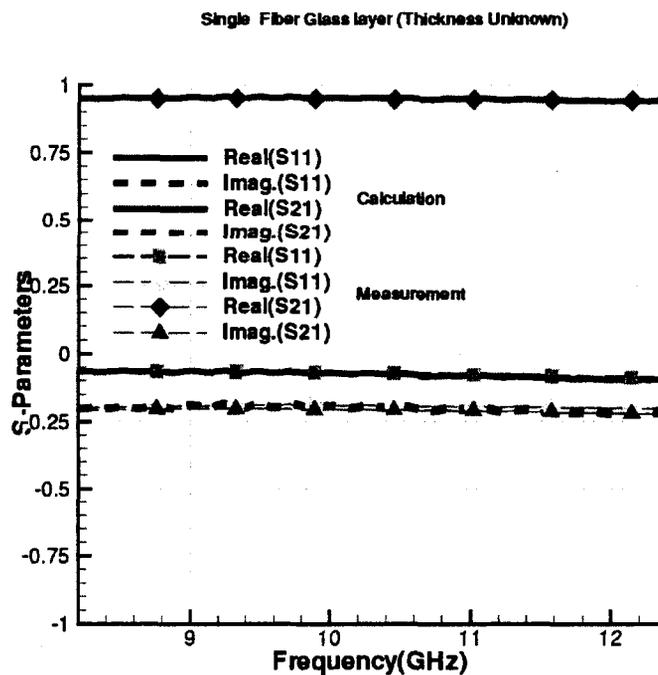


Figure 37: Measured and computed S-parameters for Fiber Glass Material. Computed S-parameters are determined using estimated values of dielectric constant and thickness of Fiber Glass.

Two Layer Composite Material:

In this section measured S-parameters of composite material consisting of two material layers are used to estimate the dielectric constants of individual layers. The error or objective function used for this purpose is identical to equation (19). The MatLab Code

given in Appendix B, with proper input variables is used to estimate the dielectric constants of two layers.

Examples:(Thickness of Layers Known):

A two layer composite material consisting of Bakelite and Teflon was formed by placing first the Bakelite layer of thickness $\Delta_1 = 0.33cm$ between the $z_0 = 0$ and $z_1 = 0.33cm$ planes. A Teflon layer of thickness $\Delta_2 = 0.635cm$ was then placed between $z_1 = 0.33cm$ and $z_2 = 0.965cm$. The measured values of S-parameters of the Bakelite-Teflon as a function of frequency was stored in a file FT_BKLTEF.60. Using the MatLab Code given in Appendix B, the dielectric constants of the two layers were estimated and plotted in Figure 38.

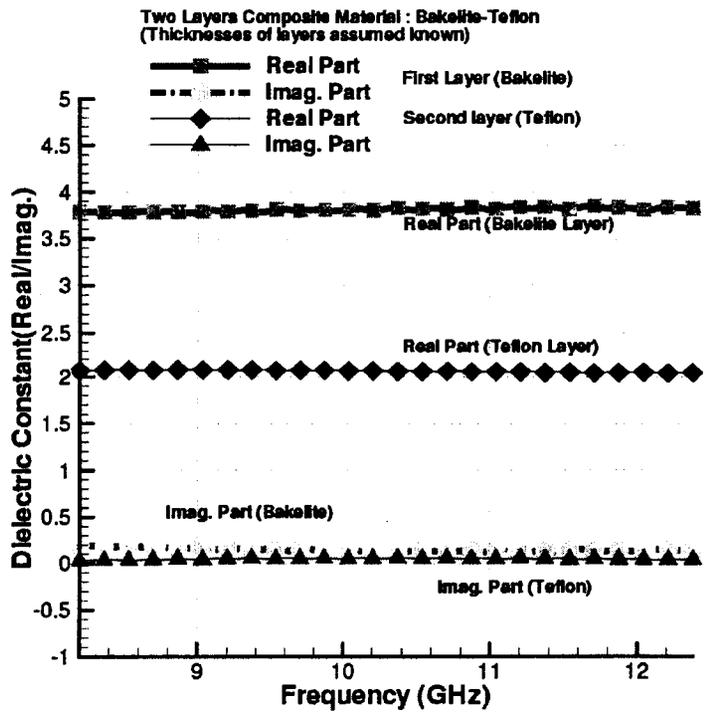


Figure 38: Relative dielectric constant (real and imaginary) of Bakelite-Teflon Composite material estimated using measured S-parameters (assuming thickness known)

Figure 39 shows estimated values of dielectric constants of Bakelite-Teflon Composite material using the ideal S-parameter values computed using the MatLab Code given in Appendix A. Figure 40 shows the error involved in the estimations of dielectric constants using measured and ideal values of S-parameters. From figure 40 it is clear that the error in estimation using the measured S-parameters is higher than the error involved in the estimation using ideal S-parameters. This may be attributed to the noise present in the measured data and inability of the analytical model to take into account potential air gaps present between the two layers.

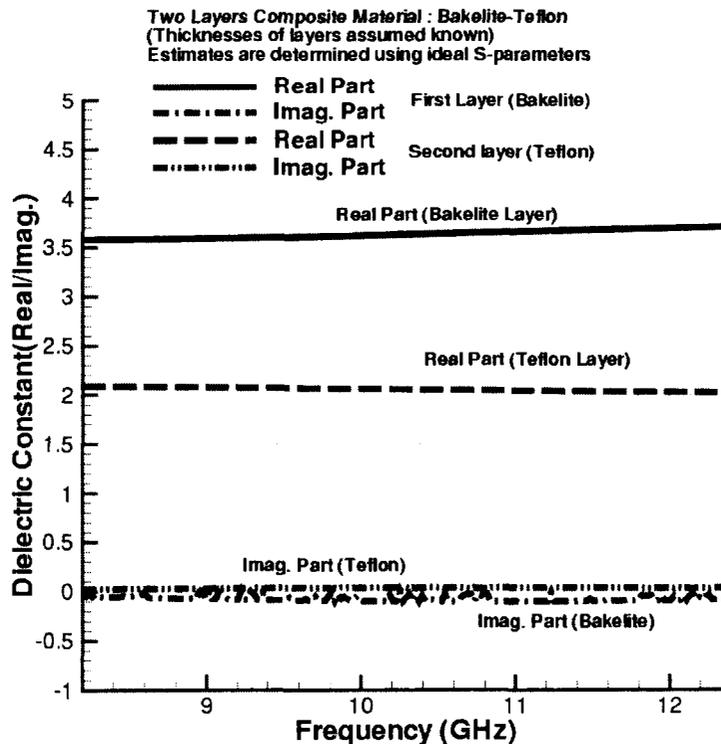


Figure 39: Relative dielectric constant (real and imaginary) of Bakelite-Teflon Composite material estimated using ideal S-parameters (assuming thickness known)

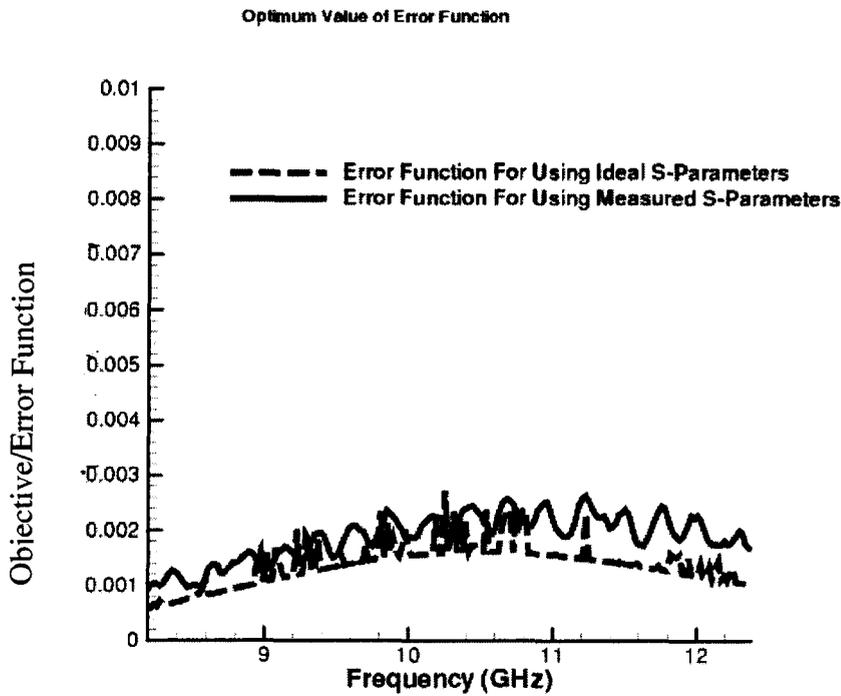


Figure 40: Value of error function given in (19) in estimating the dielectric constants of Bakelite-Teflon Composite using both measured and ideal S-parameters

For further verifications of the present approach for a two layer composite material the following samples were considered:

- 1) Garlock-Nomex Felt: $\Delta_1 = 0.17cm$, $\Delta_2 = 0.33cm$, $z_0 = 0.0$, $z_1 = 0.17cm$,
 $z_2 = 0.5cm$
- 2) Garlock-Ceramic: $\Delta_1 = 0.17cm$, $\Delta_2 = 0.2845cm$, $z_0 = 0.0$, $z_1 = 0.17cm$,
 $z_2 = 0.4545cm$
- 3) Garlock-Nano Material #0 : $\Delta_1 = 0.17cm$, $\Delta_2 = 0.3099cm$, $z_0 = 0.0$,
 $z_1 = 0.17cm$, $z_2 = 0.4799cm$

4) Garlock-Woven Composite G: $\Delta_1 = 0.17\text{cm}$, $\Delta_2 = 0.2083\text{cm}$, $z_0 = 0.0$,

$$z_1 = 0.17\text{cm}, z_2 = 0.3783\text{cm}$$

5) Garlock-Bakelite $\Delta_1 = 0.17\text{cm}$, $\Delta_2 = 0.33\text{cm}$, $z_0 = 0.0$, $z_1 = 0.17\text{cm}$,

$$z_2 = 0.50\text{cm}.$$

For these samples the relative dielectric constants of individual layer were estimated from the measured and ideal S-parameters. Estimated values of relative dielectric constants are shown in figures 41-56.

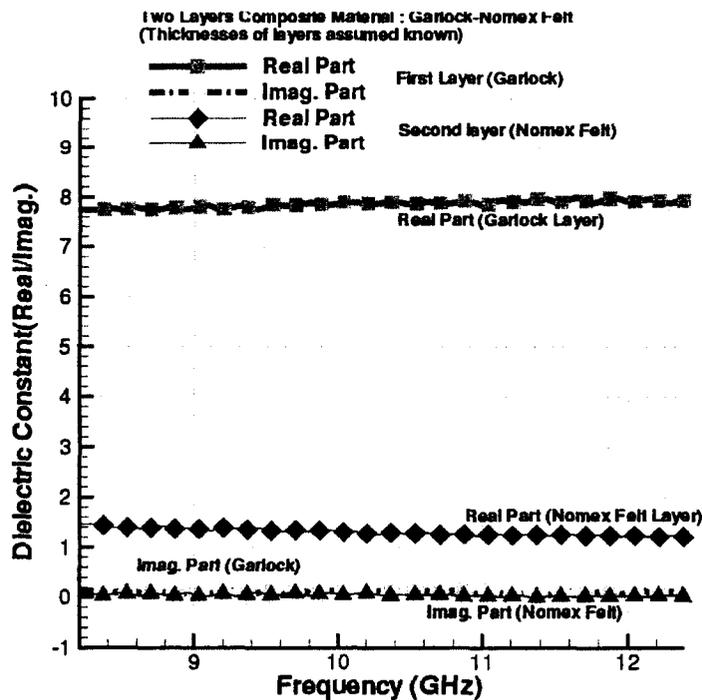


Figure 41: Relative dielectric constant (real and imaginary) of Garlock-Nomex Felt Composite material estimated using measured S-parameters (assuming thickness known)

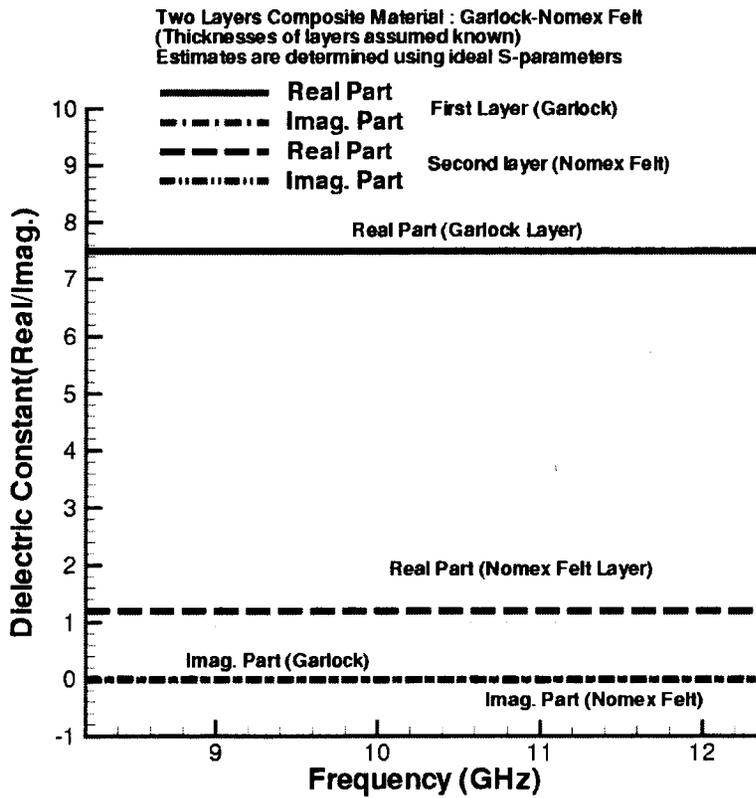


Figure 42: Relative dielectric constant (real and imaginary) of Garlock – Nomex Felt Composite material estimated using ideal S-parameters (assuming thickness known)

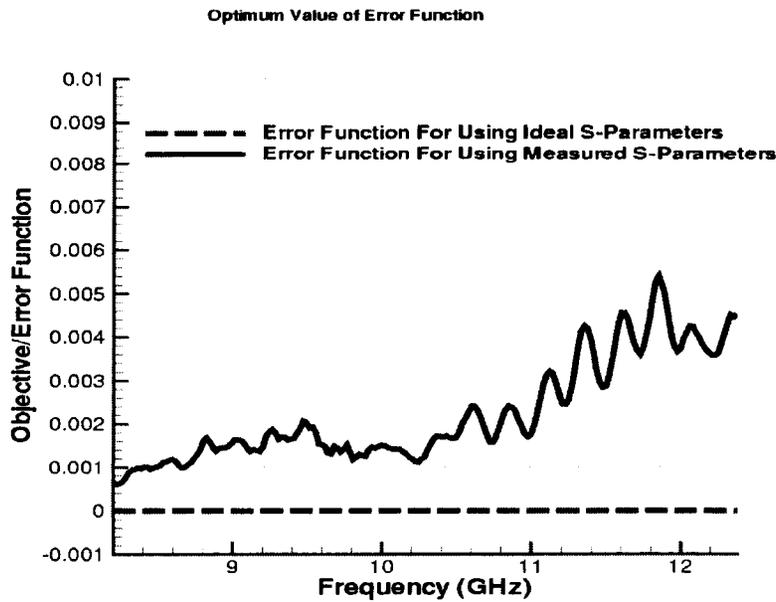


Figure 43: Value of error function given in (19) in estimating the dielectric constants of Garlock –Nomex Felt Composite using both measured and ideal S-parameters.

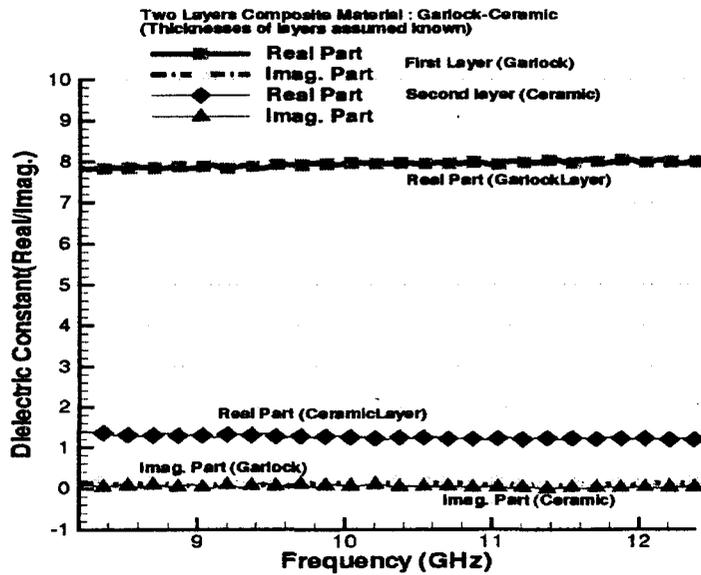


Figure 44: Relative dielectric constant (real and imaginary) of Garlock-Ceramic Composite material estimated using measured S-parameters (assuming thickness known)

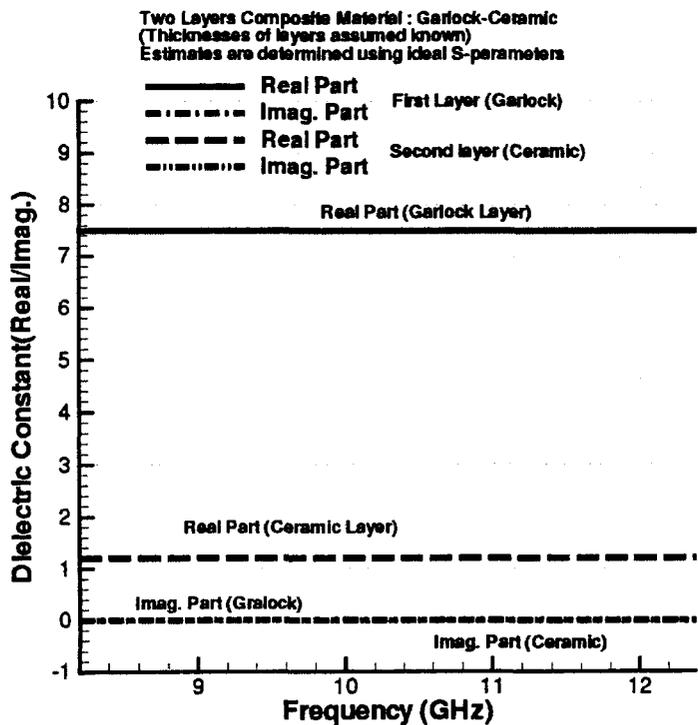


Figure 45: Relative dielectric constant (real and imaginary) of Garlock - Ceramic Composite material estimated using ideal S-parameters (assuming thickness known)

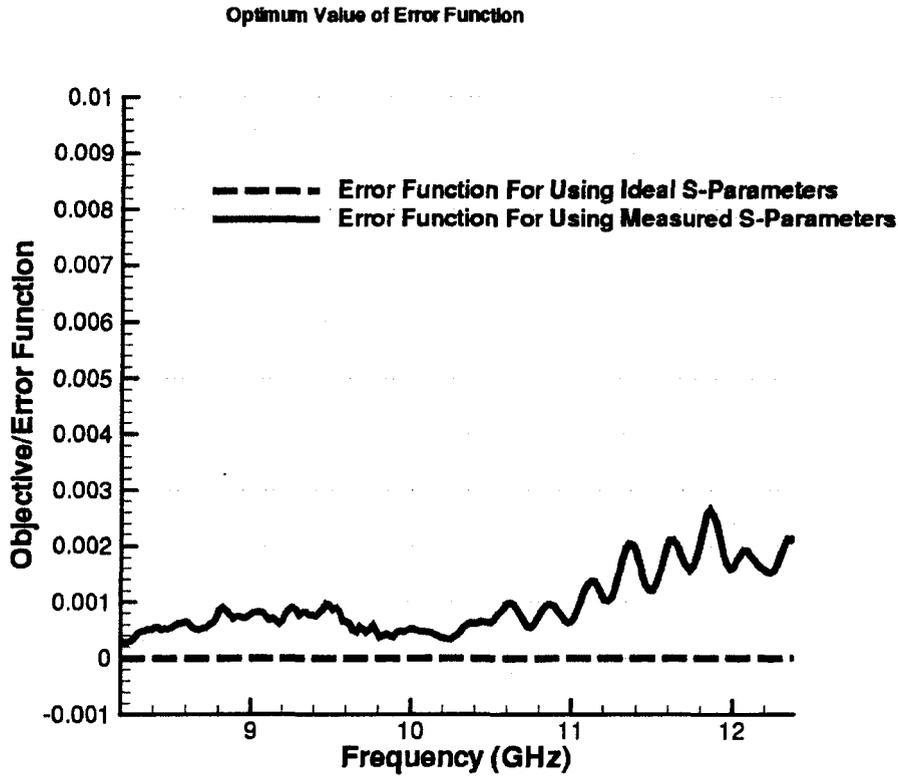


Figure 46: Value of error function given in (19) in estimating the dielectric constants of Garlock –Ceramic Composite using both measured and ideal S-parameters.

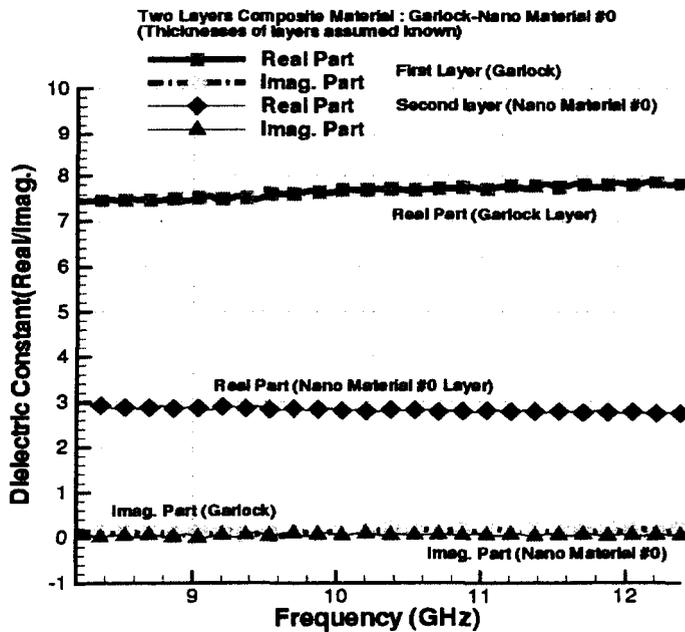


Figure 47: Relative dielectric constant (real and imaginary) of Garlock-Nano Material #0 Composite material estimated using measured S-parameters (assuming thickness known)

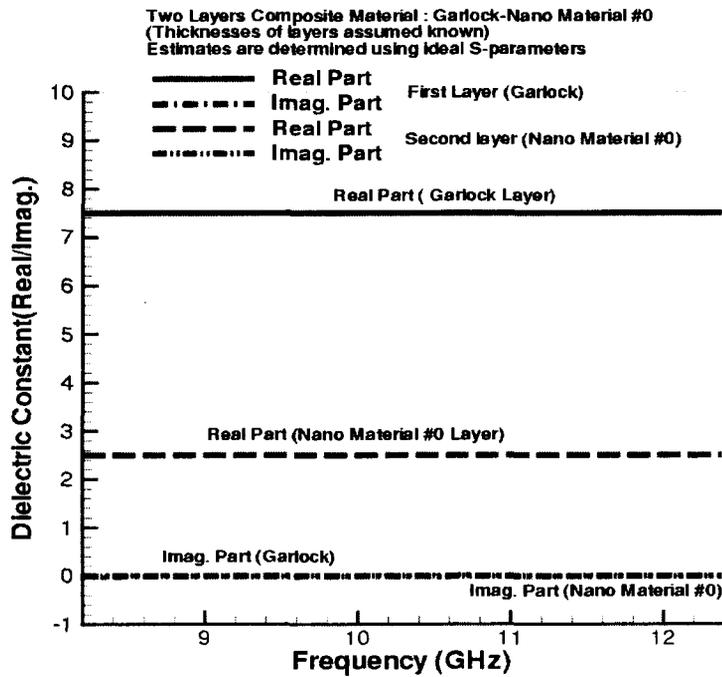


Figure 48: Relative dielectric constant (real and imaginary) of Garlock – Nano Material #0 Composite material estimated using ideal S-parameters (assuming thickness known)

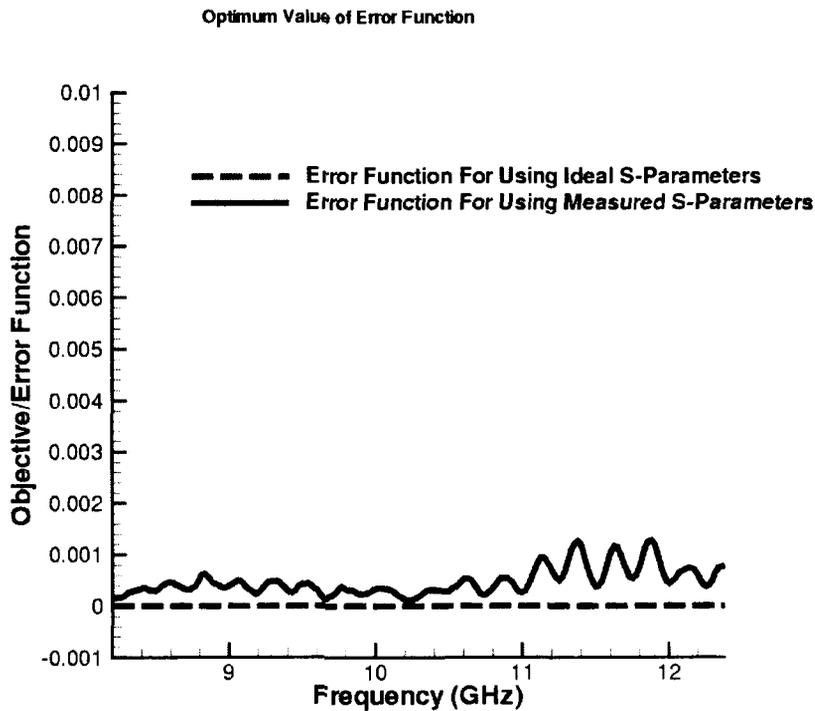


Figure 49: Value of error function given in (19) in estimating the dielectric constants of Garlock –Nano Material #0 Composite using both measured and ideal S-parameters.

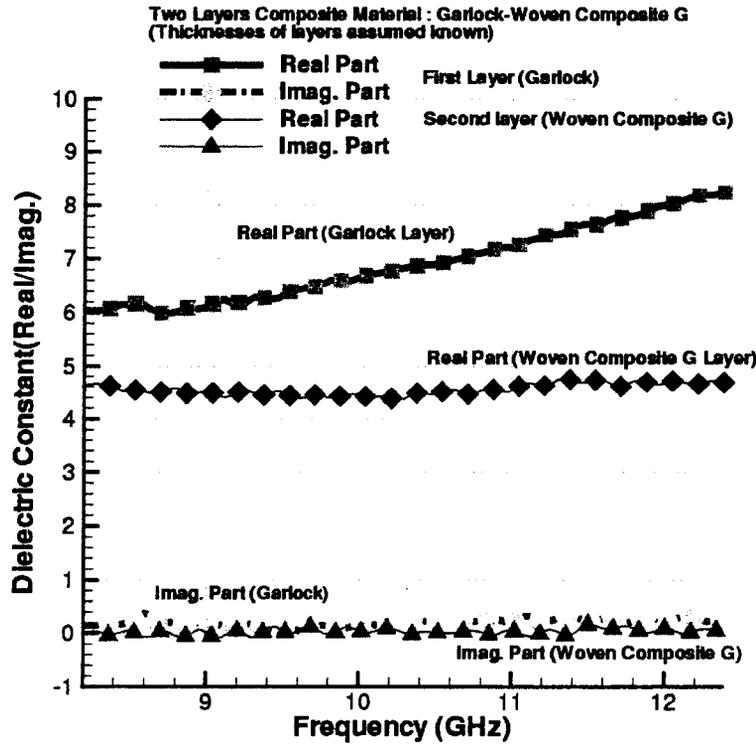


Figure 50: Relative dielectric constant (real and imaginary) of Garlock-Woven Composite G material estimated using measured S-parameters (assuming thickness known)

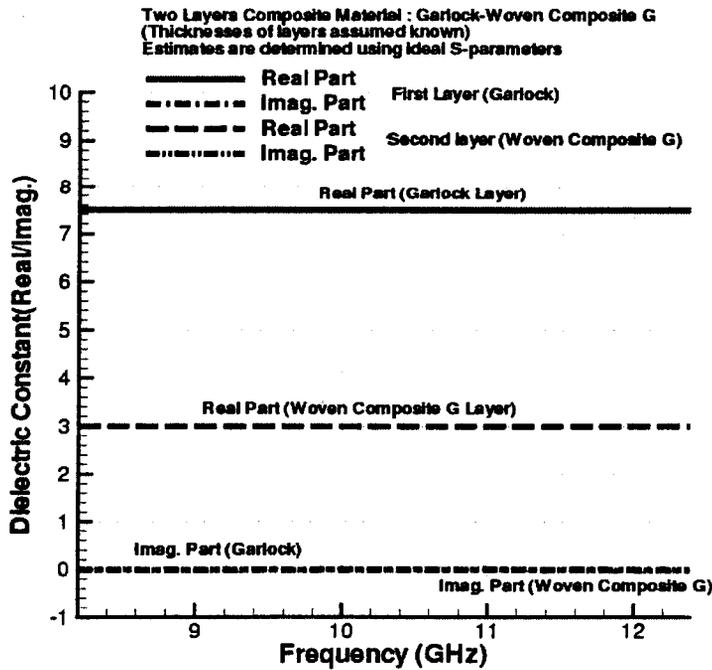


Figure 51: Relative dielectric constant (real and imaginary) of Garlock - Woven Composite G material estimated using ideal S-parameters (assuming thickness known)

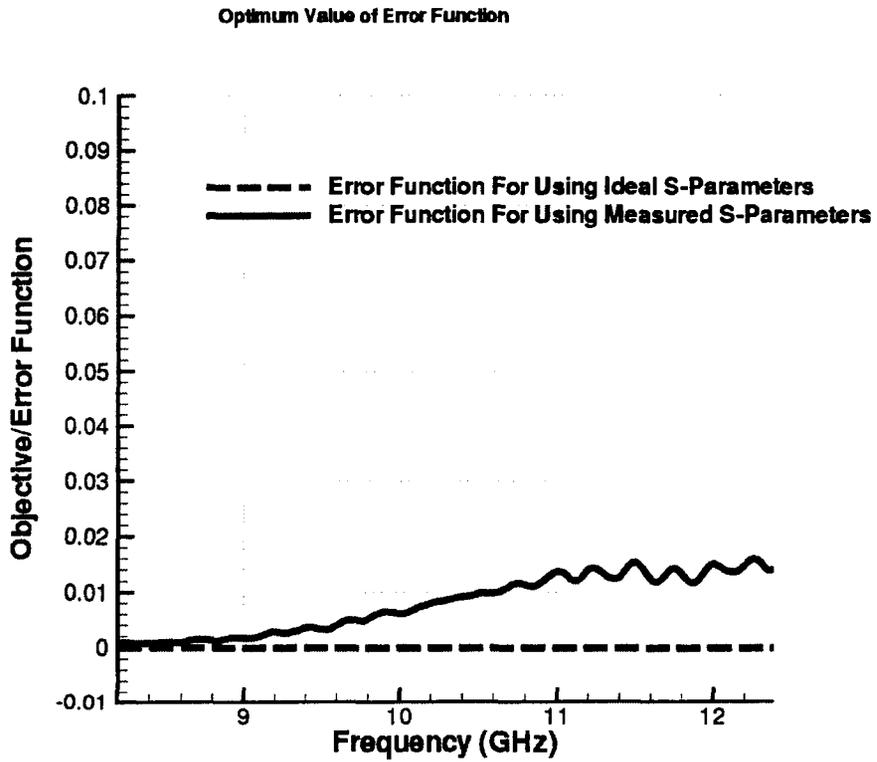


Figure 52: Value of error function given in (19) in estimating the dielectric constants of Garlock –Woven Composite G material using both measured and ideal S-parameters.

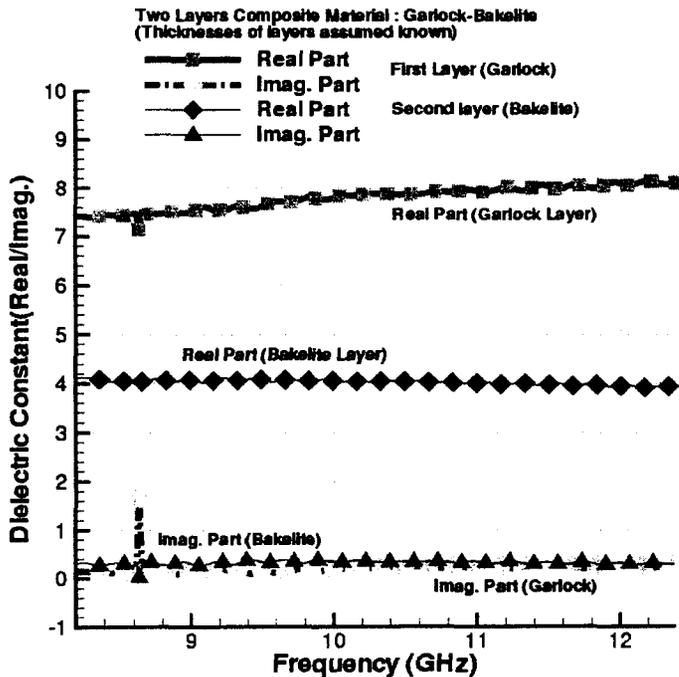


Figure 53: Relative dielectric constant (real and imaginary) of Garlock-Bakelite Composite material estimated using measured S-parameters (assuming thickness known)

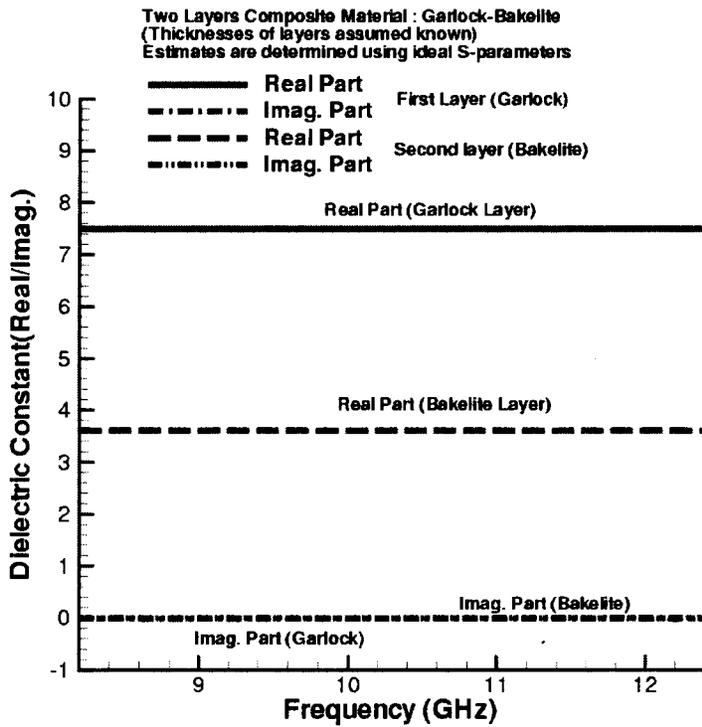


Figure 54: Relative dielectric constant (real and imaginary) of Garlock – Bakelite Composite material estimated using ideal S-parameters (assuming thickness known)

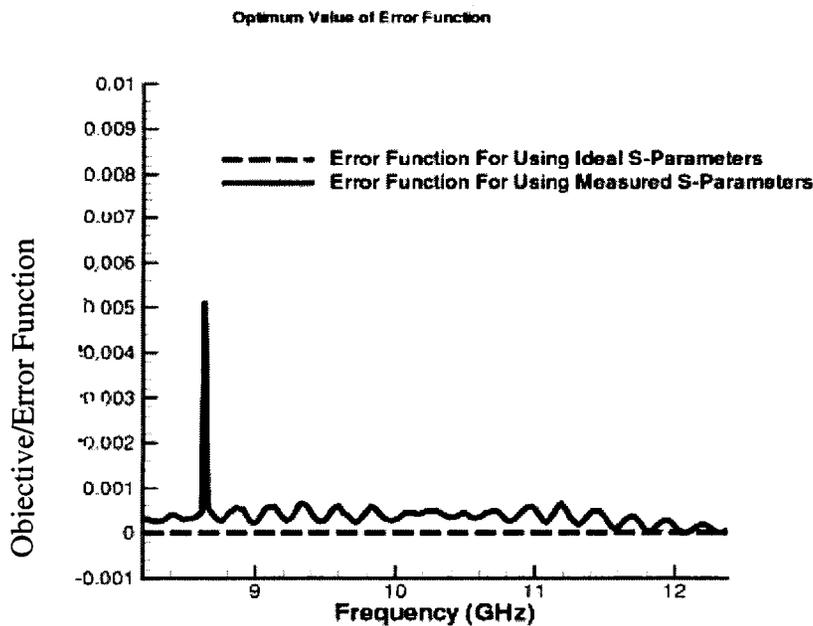


Figure 55: Value of error function given in (19) in estimating the dielectric constants of Garlock –Bakelite Composite material using both measured and ideal S-parameters.

From the results shown in figures 41-55, it may be concluded that the method presented in this report can successfully estimate dielectric constants of two layer composite material with apriori knowledge of the thickness of individual layers. An attempt was made to estimate thicknesses of individual layers along with their dielectric constants using the MatLab Code given in Appendix B. However, the optimizer did not estimate these values correctly and hence the results for such cases are not presented.

Three Layer Composite Material:

In this section measured S-parameters of composite material consisting of three material layers are used to estimate dielectric constants of individual layers. The error or objective function used for this purpose is identical to equation (19). The MatLab Code given in Appendix B, with proper input variables is used to estimate the dielectric constants of two layers.

Examples:(Thickness of Layers Known):

A three layer composite material consisting of layers of Nano Material #0, Garlock, and Garlock was formed by placing the Nano Material #0 layer of thickness

$\Delta_1 = 0.3099cm$ between $z_0 = 0$ and $z_1 = 0.3099cm$ planes, the Garlock layer of

thickness $\Delta_2 = 0.17cm$ between $z_1 = 0.3099cm$ and $z_2 = 0.4799cm$ planes, and

Garlock layer of thickness $\Delta_3 = 0.17cm$ between $z_2 = 0.4799cm$ and $z_3 = 0.6499cm$

planes. The measured values of S-parameters stored in FT_NANGRKGK.60 were used

to estimate dielectric constants of each individual layers using the MatLab code given in the Appendix B. The estimated values of dielectric constants are shown in Figure 56.

The values of dielectric constants of each these material when estimated using single

layer measured data were found to be $\epsilon_r = 2.5 - j0.00$ for the Nano Material #0,

$\epsilon_r = 7.5 - j0.00$ for the Garlock material. However, when the individual dielectric constants are estimated using the measured data for the three layer composite material, these estimates come out to be little different for the earlier estimates. This may be attributed to the presence of airgaps between the layers.

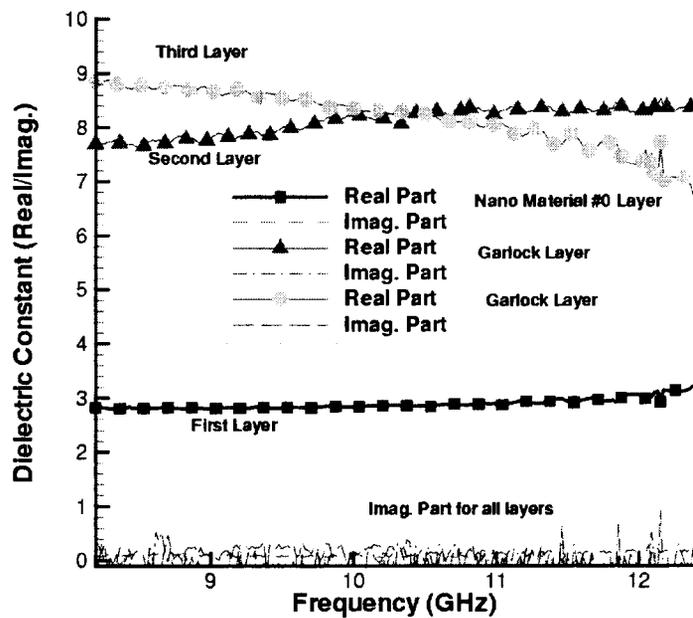


Figure 56: Estimated values of dielectric constants of individual layers from measured values of S-parameters of three layer composite slab (Nano Material #0-Garlock-Garlock)

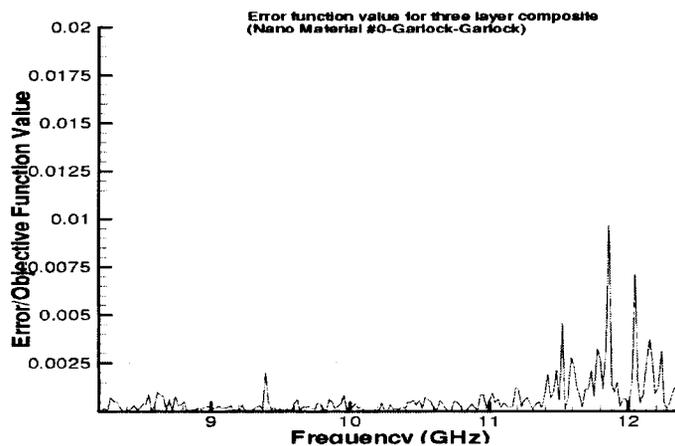


Figure 57: Final values of error function in estimation of dielectric constants of individual layers shown in Figure 56

The final value of error function or objective function for the estimation shown in Figure 56 is shown in Figure 57.

A second example of three layer composite material considered was formed by three layers of the same Garlock slabs. The first Garlock slab of thickness $\Delta_1 = 0.17\text{cm}$ was placed between the $z_0 = 0$ and $z_1 = 0.17\text{cm}$ planes, second Garlock layer of thickness $\Delta_2 = 0.17\text{cm}$ was placed between $z_1 = 0.17\text{cm}$ and $z_2 = 0.34\text{cm}$ planes, and the third Garlock layer of thickness $\Delta_3 = 0.17\text{cm}$ between $z_2 = 0.34\text{cm}$ and $z_3 = 0.51\text{cm}$ planes.

The measured values of S-parameters stored in FT_GRKGRKGRK.60 were used to estimate dielectric constants of each individual layers using the MatLab code given in Appendix B. The estimated values of dielectric constants are shown in Figure 58. The values of estimated dielectric constants of each individual layers must be close to $\epsilon_r = 7.5 - j0.00$. However, the results shown in the Figure 58 are widely spread around the correct value of the dielectric constant. This discrepancy in the estimated values derived using the measured S-parameters of multilayer composite material is attributed to the measurement errors caused by the airgaps between the layers. The error function or objective function at the final values of dielectric estimates is shown in Figure 59.

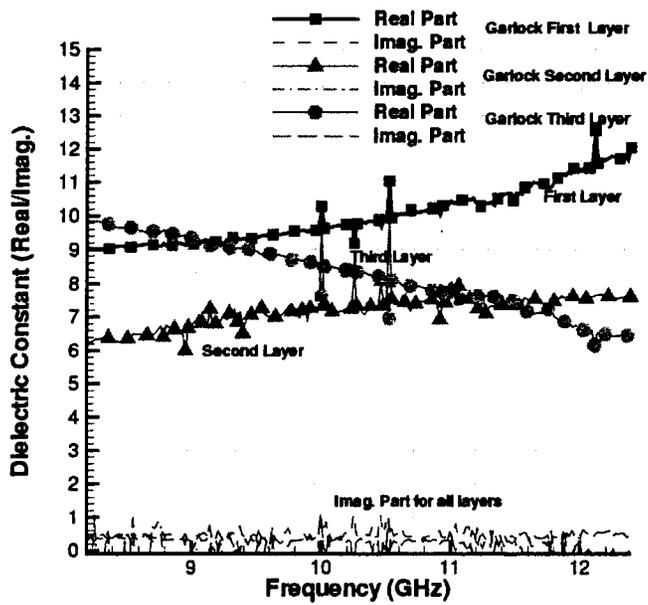


Figure 58: Estimated values of dielectric constants of individual layers from measured values of S-parameters of three layer composite slab (Garlock-Garlock-Garlock)

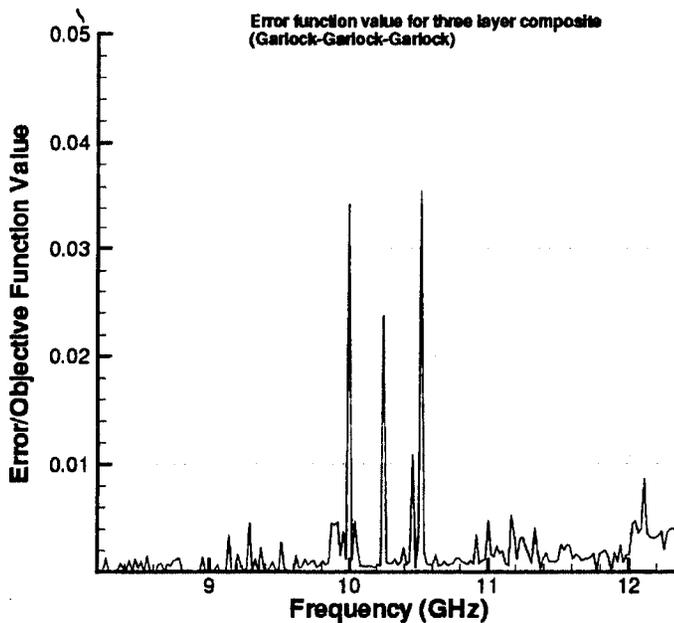


Figure 59: Final values of error function in estimation of dielectric constants of individual layers shown in Figure 58

IV Conclusion

A simple waveguide mode matching method in conjunction with the *fminsearch* MatLab optimization search function has been presented to estimate the complex permittivity of multi-layer composite material. A multi-layer composite material is placed in an X-band rectangular waveguide and its two-port S-parameters are measured over the X-band using *hp-8510 Network Analyzer*. For the same composite material using the simple mode matching technique the two port S-parameters are calculated as a function of complex dielectric constants and thicknesses of each layer. The *fminsearch* function available in the MatLab Optimization Tools is then used to determine complex dielectric constants of each layers assuming the thicknesses of each layer are unknown.

The present approach has been validated using number of composite material configurations. The composite material formed by single layers of Garlock, Cork, Ceramic, Nomex Felt, Woven Composite B and G, Rubylith, Bakelite, Teflon, Fiber Glass, Foam, Nano Material have been tested using the present method. The estimated values of dielectric constants and their thicknesses have been found to agree well with the true values supplied by the manufacturers. The composite materials formed by placing two layers of the above basic materials have been tested using the present method. The dielectric constants of individual layers estimated have been found to be in good agreement with their values specified by manufacturers. However, for the composite material consisting of more than one layer it was assumed that the thicknesses of individual layers were known. It has been observed that the present method could not estimate the dielectric constants of individual layers correctly when three or more layers were used to form a composite material. This is attributed to the fact that the airgap

present in the measurement samples could not be accurately modelled in the estimation model leading to an incorrect estimation of dielectric constants

Acknowledgement

The authors would like to acknowledge the assistance of Mr. Nima Pirzadeh, NASA, LARSS , 2001 summer student, in performing extensive 2-port S-parameter measurement on various composite materials using the *hp-8510 Network Analyzer*.

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Appendix A: MatLab code for estimation of S-parameters of multilayer substrate placed in a rectangular waveguide.

```
% This program calculates S-parameters of  
% N-layer composite material placed in an  
% X-band rectangular waveguide as a function  
% of frequency. Waveguide dimensions (2.29 cm x 1.02 cm)
```

```
%Select 201 frequency points over 8.2-12.4 GHz  
%Frequency band.  
delf = (12.4-8.2)/201;  
for j = 1:201  
    fr(j) = 8.2 +(j-1)*delf;  
end  
%Input number of layers  
n1 = input('n1 = ');  
%Input dielectric constants  
for k = 1:2*n1  
    %First value is real part of dielectric constant  
    %second value is imaginary part of dielectri constant  
    x0(k) = input('x0(k) = ');  
end  
%Input location of interfaces  
for k = 1:n1+1  
    zn(k) = input('zn(k) = ');  
end  
  
% Set dielectric constants depending upon  
% number of layers. Only 4 layers are  
% included. If n1 > 4 then add condition  
% for given n1  
  
if n1 == 1  
    er(1) = x0(1) -i*x0(2);  
  
end  
if n1 == 2  
    er(1) = x0(1) -i*x0(2);  
    er(2) = x0(3) -i*x0(4);  
end  
  
if n1 == 3  
    er(1) = x0(1) -i*x0(2);  
    er(2) = x0(3) -i*x0(4);  
    er(3) = x0(5)-i*x0(6);  
end
```

```

if n1 == 4
    er(1) = x0(1) -i*x0(2);
    er(2) = x0(3) -i*x0(4);
    er(3) = x0(5) -i*x0(6);
    er(4) = x0(7) -i*x0(8);
end

nlayer = n1; % number of layers redefined
n = nlayer+1; % Matrix order
t=zeros(n,1); % Right handside column matrix initilized
s=zeros(n); % Admittance matrix initilized
ak=zeros(n,1); % propagation column matrix

%Set dielectric constants redefined

%const is set to zeros (3 by 3)
const = zeros(n);
% Dimensions of x-band waveguide in cm
aa = 2.29;
bb = 1.02; %guide dimensions

% Free space dielectric constant
e0 = 1.e-9/(36*pi);
%f = 8.2e9 %frequency of microwave signal
c = 3.e10; % velocity of light in meters/second

u0 =4*pi*1.e-7; % Magnetic permeability of free space
n0 = (u0/e0)^.5; % Free space impedance
for ifr = 1:201
freq = fr(ifr); % Frequency in GHz

ak0 =2*pi*freq/30; % wave number in free space
% This loop calculates propagation
% constants in layers
for k1 = 1:n1
const(k1) = ak0*ak0*(er(k1))-(pi/aa)^2;
if (real(const(k1)) < 0.0) ak(k1) = -i*(-const(k1))^.5;
else ak(k1) = (const(k1))^.5;
end
end

% ak00 and ak11 are propagation constants
% for the free space regions of waveguide
const0 = ak0^2-(pi/aa)^2;

```

```

if (real(const0) < 0.0) ak00 = -i*(-const0)^.5;
else ak00 = (const0)^.5;

end
ak11 = ak00;

for k1 = 1:n1
    addm(k1) = ak(k1)/(ak0*n0);
end

addm00 = ak00/(ak0*n0);
addm11 = addm00;

% Initalize s matrix
for k1= 1:n1+1
    for k2 = 1:n1+1
        s(k1,k2) = 0.0;
    end
end

% LOOP TO CALCULATE IMPEDANCE MATRIX
% EXCLUDING FIRST ROW AND LAST ROW
for k1 = 2:n1

    k2 = k1-1;
    arg1 = ak(k2)*(zn(k1)-zn(k1-1));
    s(k1,k2) = -addm(k2)/(i*sin(arg1));
    k2 = k1;
    arg1 = ak(k2-1)*(zn(k1)-zn(k1-1));
    arg2 = ak(k1)*(zn(k1+1)-zn(k1));
    s(k1,k2) = addm(k2-1)*cos(arg1)/(i*sin(arg1)) + addm(k1)*cos(arg2)/(i*sin(arg2));
    k2 = k1+1;
    arg2 = ak(k1)*(zn(k1+1)-zn(k1));
    s(k1,k2) = -addm(k1)/(i*sin(arg2));
end

% FIRST ROW OF IMPEDANCE MATRIX

s(1,1) = addm00 + cos(ak(1)*zn(2))/(i*sin(ak(1)*zn(2)))*addm(1);
s(1,2) = -addm(1)/(i*sin(ak(1)*zn(2)));

% LAST ROW OF IMPEDANCE MATRIX

s(nlayer+1,nlayer) = -addm(nlayer)/(i*sin(ak(nlayer) * ...
*(zn(nlayer+1)-zn(nlayer))));
s(nlayer+1,nlayer+1) = addm(nlayer)*cos(ak(nlayer)* ...

```

```

(zn(nlayer+1)-zn(nlayer)))/(i*sin(ak(nlayer))* ...
(zn(nlayer+1)-zn(nlayer))) + addm11;

% ONLY FIRST ELEMENT OF RIGHT HAND SIDE
% IS NON ZERO
for j = 1:nlayer+1
    t(j) = 0.0 +i*0.0;
end

t(1) =2*addm00;

% INVERT S AND MULTIPLY BY t

T = inv(s)*t;

% FIRST ELEMENT OF T - 1 IS REFLECTION
% COEFFICIENT AND LAST VALUE OF T is TRANSMISSION
% COEFFICIENT
% SINCE MEASURED TRANSMISSION IS REFERENCED TO
% INPUT PLANE IT IS MULTIPLIED BY PHASE CORRECTION

s11c(ifr) =T(1,1)-1;
s21c(ifr) =T(n1+1,1)*exp(i*ak00*zn(n1+1));

end

% CALCULATING S22 AND S12

% Inverse the position of interfaces

if n1 == 1
    zn(1) = zn(1);
    zn(2) = zn(2);
    er(1) =x0(1) -i*x0(2);

end
if n1 == 2
    zn(1) = -zn(3) + zn(3);
    zn(2) = -zn(2) + zn(3);

```

```

    zn(3) = -zn(1) + zn(3);
    er(1) = x0(3) -i*x0(4);
    er(2) = x0(1) - i*x0(2);
end

if n1 == 3
    zn(1) = -zn(4) + zn(4);
    zn(2) = -zn(3) + zn(4);
    zn(3) = -zn(2) + zn(4);
    zn(4) = -zn(1) + zn(4);

    er(1) = x0(5) -i*x0(6);
    er(2) = x0(4) -i*x0(3);
    er(3) = x0(1) -i*x0(2);
end

if n == 4
    zn(1) = -zn(5) + zn(5);
    zn(2) = -zn(4) + zn(5);
    zn(3) = -zn(3) + zn(5);
    zn(2) = -zn(2) + zn(5);
    zn(1) = -zn(1) + zn(5);
    er(1) = x0(7) -i*x0(8);
    er(2) = x0(5) -i*x0(6);
    er(3) = x0(3) -i*xo(4);
    er(4) = x0(1) - i*x0(2);
end
nlayer = n1; %number of layers redifined
n = nlayer+1; % Matrix order
t=zeros(n,1); % Right handside column matrix initilized
s=zeros(n); % Admittance matrix initilized
ak=zeros(n,1); % propagation column matrix
%Set dielectric constants redifined

%const is set to zeros (3 by 3)
const = zeros(n);
aa = 2.29;
bb = 1.02; %guide dimensions
% Free space dielectric constant
e0 = 1.e-9/(36*pi);
%f = 8.2e9 %frequency of microwave signal
c = 3.e10; % velocity of light in meters/second

u0 =4*pi*1.e-7; % Magnetic permeability of free space
n0 = (u0/e0)^.5; % Free space impedance
for ifr = 1:201

```

```

freq = fr(ifr); % Frequency in GHz
ak0 =2*pi*freq/30; % wave number in free space
% This loop calculates propagation
% constants in layers
for k1 = 1:n1
const(k1) = ak0*ak0*(er(k1))-(pi/aa)^2;
if (real(const(k1)) < 0.0) ak(k1) = -i*(-const(k1))^.5;
else ak(k1) = (const(k1))^.5;
end
end

% ak00 and ak11 are propagation constants
% for the free space regions of waveguide
const0 = ak0^2-(pi/aa)^2;
if (real(const0) < 0.0) ak00 = -i*(-const0)^.5;
else ak00 = (const0)^.5;

end
ak11 = ak00;

% calculate admittance for layers:
for k1 = 1:n1
addm(k1) =ak(k1)/(ak0*n0);
end

addm00 = ak00/(ak0*n0);
addm11 = addm00;

% Initalize s matrix
for k1= 1:n1+1
for k2 = 1:n1+1
s(k1,k2) = 0.0;
end
end

% LOOP TO CALCULATE IMPEDANCE MATRIX
% EXCLUDING FIRST ROW AND LAST ROW
for k1 = 2:n1

k2 = k1-1;
arg1 = ak(k2)*(zn(k1)-zn(k1-1));
s(k1,k2) = -addm(k2)/(i*sin(arg1));
k2 = k1;
arg1 = ak(k2-1)*(zn(k1)-zn(k1-1));
arg2 = ak(k1)*(zn(k1+1)-zn(k1));
s(k1,k2) = addm(k2-1)*cos(arg1)/(i*sin(arg1)) + addm(k1)*cos(arg2)/(i*sin(arg2));

```

```

k2 = k1+1;
arg2 = ak(k1)*(zn(k1+1)-zn(k1));
s(k1,k2) = -addm(k1)/(i*sin(arg2));
end

% FIRST ROW OF IMPEDANCE MATRIX

s(1,1) = addm00 + cos(ak(1)*zn(2))/(i*sin(ak(1)*zn(2)))*addm(1);
s(1,2) = -addm(1)/(i*sin(ak(1)*zn(2)));

% LAST ROW OF IMPEDANCE MATRIX

s(nlayer+1,nlayer) = -addm(nlayer)/(i*sin(ak(nlayer) ...
*(zn(nlayer+1)-zn(nlayer))));
s(nlayer+1,nlayer+1) = addm(nlayer)*cos(ak(nlayer)* ...
(zn(nlayer+1)-zn(nlayer)))/(i*sin(ak(nlayer)* ...
(zn(nlayer+1)-zn(nlayer)))) + addm11;

% ONLY FIRST ELEMENT OF RIGHT HAND SIDE
% IS NON ZERO
for j = 1:nlayer+1
t(j) = 0.0 -i*0.0;
end
t(1) = 2*addm00;

% INVERT S AND MULTIPLY BY t

T = inv(s)*t;

% FIRST ELEMENT OF T - 1 IS REFLECTION
% COEFFICIENT AND LAST VALUE OF T IS TRANSMISSION
% COEFFICIENT
% SINCE MEASURED TRANSMISSION IS REFERENCED TO
% INPUT PLANE IT IS MULTIPLIED BY PHASE CORRECTION

ref = T(1,1)-1;
s22c(ifr) = ref*exp(i*ak00*2*zn(n1+1));
s12c(ifr) = T(n1+1,1)*exp(i*ak00*zn(n1+1));

end

ss = load('FT_GRLK.60');

```

```

delf = (12.4-8.2)/201;
for j = 1:201
    fr(j) = 8.2 +(j-1)*delf;
end

for j = 1:201

    s11r(j) = ss(j,1);
    s11i(j) = ss(j,2);
    s21r(j) = ss(j+201,1);
    s21i(j) = ss(j+201,2);
    s12r(j) = ss(j+402,1);
    s12i(j) = ss(j+402,2);
    s22r(j) = ss(j+603,1);
    s22i(j) = ss(j+603,2);
end

plot(fr,s11r,fr,s11i,fr,real(s11c),fr,imag(s11c))
ok = input('Is plot ok =');
plot(fr,s21r,fr,s21i,fr,real(s21c),fr,imag(s21c))
ok = input('Is plot ok =');
plot(fr,s12r,fr,s12i,fr,real(s12c),fr,imag(s12c))
ok = input('Is plot ok =');
plot(fr,s22r,fr,s22i,fr,real(s22c),fr,imag(s22c))
ok = input('Is plot ok =');

```

%

Appendix B: MatLab code for estimation of dielectric parameters of multilayer substrate placed in a rectangular waveguide.

```
% This program estimates dielectric parameters of  
% multi-layer substrate . Measured data has to be loaded in  
% meas.dat file  
% X-band rectangular waveguide as a function  
% of frequency.
```

```
%Load measured data file
```

```
ss = load('meas.dat');
```

```
delf = (12.4-8.2)/201;  
for j = 1:201  
    fr(j) = 8.2 +(j-1)*delf;  
end
```

```
%Input number of layers  
n1 = input('n1 = ');
```

```
%Input dielectric constants  
for k = 1:2*n1  
    %First value is real part of dielectric constant  
    %second value is imaginary part of dielectri constant  
    x0(k) = input('x0(k) =');  
end
```

```
%Input location of interfaces
```

```
for k = 1:n1+1  
    zn(k) = input('zn(k) =');  
end
```

```
for j = 1:201
```

```
    s11r(j) = ss(j,1);  
    s11i(j) = ss(j,2);  
    s21r(j) = ss(j+201,1);  
    s21i(j) = ss(j+201,2);  
    s12r(j) = ss(j+402,1);  
    s12i(j) = ss(j+402,2);  
    s22r(j) = ss(j+603,1);
```

```

    s22i(j) = ss(j+603,2);
end
% CHECK WHETHER SYSTEM IS LOSSLESS OR LOSSY

for j = 1:201
s11 = s11r(j)+i*s11i(j);
s21 = s21r(j)+i*s21i(j);
s12 = s12r(j)+i*s12i(j);
s22 = s22r(j)+i*s22i(j);
freq = fr(j);

x0 = [1.0;0.0;1.0;0.0;1.0;0.0];
options = optimset('Display', 'iter', 'MaxFunEvals', 1000, 'TolFun', 1.e-08)
options = optimset('TolX', 1e-08)
[x,fval,flag] = fminsearch(@obj_ref,x0,options,s11,s21,s12,s22,freq,n1 ...
    ,zn);
er11(j) = x(1);
er12(j) = x(2);
er21(j) = x(3);
er22(j) = x(4);
er31(j) = x(5);
er32(j) = x(6);

fg1(j) = fval;
flag

end

outmat = [fr.',er11.',er12.'];
save('temp1','outmat','-ascii')

outmat = [fr.',er21.',er22.'];
save('temp2','outmat','-ascii')
outmat = [fr.',er31.',er32.'];
save('temp3','outmat','-ascii')
outmat = [fr.',fg1.'];
save('temp4','outmat','-ascii')

```

```

%
```

```

% This is object function

function f = obj_ref(x0,s11,s21,s12,s22,freq,n1,zn)

if n1 == 1
    er(1) = x0(1) - i*x0(2);

end

if n1 == 2
    er(1) = x0(1) -i*x0(2);
    er(2) = x0(3) -i*x0(4);
end
if n1 ==3
    er(1) = x0(1) -i*x0(2);
    er(2) = x0(3) -i*x0(4);
    er(3) = x0(5) -i*x0(6);
end

if n1 ==4
    er(1) = x0(1) -i*x0(2);
    er(2) = x0(3) -i*x0(4);
    er(3) = x0(5) -i*x0(6);
    er(4) = x0(7) -i*x0(8);
end

nlayer = n1; %number of layers redifined
n = nlayer+1; % Matrix order
t =zeros(n,1); % Right handside column matrix initilized
s =zeros(n); % Admittance matrix initilized
ak =zeros(n,1); % propagation column matrix

%Set dielectric constants redifined

%const is set to zeros (3 by 3)
const = zeros(n);
aa = 2.29;
bb = 1.02; %guide dimensions
% Free space dielectric constant
e0 = 1.e-9/(36*pi);
%f = 8.2e9 %frequency of microwave signal
c = 3.e10; % velocity of light in meters/second

u0 =4*pi*1.e-7; % Magnetic permeability of free space
n0 = (u0/e0)^.5; % Free space impedance

```

```

%for ifr = 1:201
%freq = fr(ifr); % Frequency in GHz
ak0 =2*pi*freq/30; % wave number in free space
% This loop calculates propagation
% constants in three layers
for k1 = 1:n1
const(k1) = ak0*ak0*(er(k1))-(pi/aa)^2;
if (real(const(k1)) < 0.0) ak(k1) = -i*(-const(k1))^0.5;
else ak(k1) = (const(k1))^0.5;
end
end

const0 = ak0^2-(pi/aa)^2;
if (real(const0) < 0.0) ak00 = -i*(-const0)^0.5;
else ak00 = (const0)^0.5;

end
ak11 = ak00;

% calculate admittance for layers:
for k1 = 1:n1
addm(k1) =ak(k1)/(ak0*n0);
end

addm00 = ak00/(ak0*n0);
addm11 = addm00;

% Initalize s matrix
for k1= 1:n1+1
for k2 = 1:n1+1
s(k1,k2) = 0.0;
end
end

% LOOP TO CALCULATE IMPEDANCE MATRIX
% EXCLUDING FIRST ROW AND LAST ROW
for k1 = 2:n1

k2 = k1-1;
arg1 = ak(k2)*(zn(k1)-zn(k1-1));
s(k1,k2) = -addm(k2)/(i*sin(arg1));
k2 = k1;
arg1 = ak(k2-1)*(zn(k1)-zn(k1-1));
arg2 = ak(k1)*(zn(k1+1)-zn(k1));
s(k1,k2) = addm(k2-1)*cos(arg1)/(i*sin(arg1)) + addm(k1)*cos(arg2)/(i*sin(arg2));
k2 = k1+1;

```

```

    arg2 = ak(k1)*(zn(k1+1)-zn(k1));
    s(k1,k2)= -addm(k1)/(i*sin(arg2));
end

% FIRST ROW OF IMPEDANCE MATRIX

s(1,1) = addm00 + cos(ak(1)*zn(2))/(i*sin(ak(1)*zn(2)))*addm(1);
    s(1,2) = -addm(1)/(i*sin(ak(1)*zn(2)));

% LAST ROW OF IMPEDANCE MATRIX

s(nlayer+1,nlayer) = -addm(nlayer)/(i*sin(ak(nlayer) ...
    *(zn(nlayer+1)-zn(nlayer))));
s(nlayer+1,nlayer+1) = addm(nlayer)*cos(ak(nlayer)* ...
    (zn(nlayer+1)-zn(nlayer)))/(i*sin(ak(nlayer)* ...
    (zn(nlayer+1)-zn(nlayer)))) + addm11;

% ONLY FIRST ELEMENT OF RIGHT HAND SIDE
% IS NON ZERO
for j = 1:nlayer+1
    t(j) = 0.0+i*0.0;
end
t(1) =2*addm00;

% INVERT S AND MULTIPLY BY t

T = inv(s)*t;

% FIRST ELEMENT OF T - 1 IS REFLECTION
% COEFFICIENT AND LAST VALUE OF T is TRANSMISSION
% COEFFICIENT
% SINCE MEASURED TRANSMISSION IS REFERENCED TO
% INPUT PLANE IT IS MULTIPLIED BY PHASE CORRECTION

s11c =T(1,1)-1;
s21c =T(n1+1,1)*exp(i*ak00*zn(n1+1));

f = real(s11c-s11)*real(s11c-s11) + imag(s11c-s11)*imag(s11c-s11) ...
    +real(s21c-s21)*real(s21c-s21) + imag(s21c-s21)*imag(s21c-s21);

% CALCULATING S22 AND S12

```

```
if n1 == 1
    zn1(1) = zn(2) - zn(2);
    zn1(2) = zn(2) - zn(1);
```

```
end
```

```
if n1 == 2
```

```
    zn1(1) = zn(3) - zn(3);
    zn1(2) = zn(3) - zn(2);
    zn1(3) = zn(3) - zn(1);
```

```
end
```

```
if n1 == 3
```

```
    zn1(1) = zn(4) - zn(4);
    zn1(2) = zn(4) - zn(3);
    zn1(3) = zn(4) - zn(2);
    zn1(4) = zn(4) - zn(1);
```

```
end
```

```
if n1 == 4
```

```
    zn1(1) = zn(5) - zn(5);
    zn1(2) = zn(5) - zn(4);
    zn1(3) = zn(5) - zn(3);
    zn1(4) = zn(5) - zn(2);
    zn1(5) = zn(5) - zn(1);
```

```
end
```

```
if n1 == 1
```

```
    er(1) = x0(1) - i*x0(2);
```

```
end
```

```
if n1 == 2
```

```
    er(2) = x0(1) - i*x0(2);
    er(1) = x0(3) - i*x0(4);
```

```
end
```

```
if n1 == 3
```

```
    er(1) = x0(5) - i*x0(6);
    er(2) = x0(3) - i*x0(4);
    er(3) = x0(1) - i*x0(2);
```

```
end
```

```
if n1 == 4
```

```
    er(1) = x0(7) - i*x0(8);
    er(2) = x0(5) - i*x0(6);
    er(3) = x0(3) - i*x0(4);
    er(4) = x0(1) - i*x0(2);
```

```
end
```

```

nlayer = n1; % number of layers redefined
n = nlayer+1; % Matrix order
t = zeros(n,1); % Right handside column matrix initilized
s = zeros(n); % Admittance matrix initilized
ak = zeros(n,1); % propagation column matrix
%Set dielectric constants redefined

%const is set to zeros (3 by 3)
const = zeros(n);
aa = 2.29;
bb = 1.02; %guide dimensions
% Free space dielectric constant
e0 = 1.e-9/(36*pi);
%f = 8.2e9 % frequency of microwave signal
c = 3.e10; % velocity of light in meters/second

u0 = 4*pi*1.e-7; % Magnetic permeability of free space
n0 = (u0/e0)^.5; % Free space impedance
%for ifr = 1:201
    %freq = fr(ifr); % Frequency in GHz
    ak0 = 2*pi*freq/30; % wave number in free space
    % This loop calculates propagation
    % constants in three layers
    for k1 = 1:n1
        const(k1) = ak0*ak0*(er(k1))-(pi/aa)^2;
        if (real(const(k1)) < 0.0) ak(k1) = -i*(-const(k1))^0.5;
        else ak(k1) = (const(k1))^0.5;
    end
end
%ak

%ok = input('ok = ');

% ak00 and ak11 are propagation constants
% for the free space regions of waveguide
const0 = ak0^2-(pi/aa)^2;
if (real(const0) < 0.0) ak00 = -i*(-const0)^0.5;
else ak00 = (const0)^0.5;

end
ak11 = ak00;
%ak0
%calculate addmitince for layers:
for k1 = 1:n1
    addm(k1) = ak(k1)/(ak0*n0);
end

```

```

%addm
%ok = input('ok =')
addm00 = ak00/(ak0*n0);
addm11 = addm00;
%addm00
%addm11
%ok = input('ok =')
% Initilize s matrix
for k1= 1:n1+1
    for k2 = 1:n1+1
        s(k1,k2) = 0.0;
    end
end
end

% LOOP TO CALCULATE IMPEDANCE MATRIX
% EXCLUDING FIRST ROW AND LAST ROW
for k1 = 2:n1

    k2 = k1-1;
    arg1 = ak(k2)*(zn1(k1)-zn1(k1-1));
    s(k1,k2) = -addm(k2)/(i*sin(arg1));
    k2 = k1;
    arg1 = ak(k2-1)*(zn1(k1)-zn1(k1-1));
    arg2 = ak(k1)*(zn1(k1+1)-zn1(k1));
    s(k1,k2) = addm(k2-1)*cos(arg1)/(i*sin(arg1)) + addm(k1)*cos(arg2)/(i*sin(arg2));
    k2 = k1+1;
    arg2 = ak(k1)*(zn1(k1+1)-zn1(k1));
    s(k1,k2)= -addm(k1)/(i*sin(arg2));
end

% FIRST ROW OF IMPEDANCE MATRIX

s(1,1) = addm00 + cos(ak(1)*zn1(2))/(i*sin(ak(1)*zn1(2)))*addm(1);
s(1,2) = -addm(1)/(i*sin(ak(1)*zn1(2)));

% LAST ROW OF IMPEDANCE MATRIX

s(nlayer+1,nlayer) = -addm(nlayer)/(i*sin(ak(nlayer) ...
*(zn1(nlayer+1)-zn1(nlayer))));
s(nlayer+1,nlayer+1) = addm(nlayer)*cos(ak(nlayer)* ...
(zn1(nlayer+1)-zn1(nlayer)))/(i*sin(ak(nlayer)* ...
(zn1(nlayer+1)-zn1(nlayer)))) + addm11;

% s
%ok = input('ok =')
% form cpmplex matrix

```

```

% ONLY FIRST ELEMENT OF RIGHT HAND SIDE
% IS NON ZERO
for j = 1:nlayer+1
    t(j) = 0.0 + i*0.0;
end
t(1) = 2*addm00;

% INVERT S AND MULTIPLY BY t

T = inv(s)*t;

% FIRST ELEMENT OF T - 1 IS REFLECTION
% COEFFICIENT AND LAST VALUE OF T is TRANSMISSION
% COEFFICIENT
% SINCE MEASURED TRANSMISSION IS REFERENCED TO
% INPUT PLANE IT IS MULTIPLIED BY PHASE CORRECTION

ref = T(1,1)-1;
s22c = ref*exp(i*ak00*2*zn1(n1+1));
s12c = T(n1+1,1)*exp(i*ak00*zn1(n1+1));

f = f+ real(s22c-s22)*real(s22c-s22) + imag(s22c-s22)*imag(s22c-s22) ...
    + real(s12c-s12)*real(s12c-s12) + imag(s12c-s12)*imag(s12c-s12);

```

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13. ABSTRACT (Maximum 200 words) A simple method is presented to estimate the complex dielectric constants of individual layers of a multilayer composite material. Using the MatLab Optimization Tools simple MatLab scripts are written to search for electric properties of individual layers so as to match the measured and calculated S-parameters. A single layer composite material formed by using materials such as Bakelite, Nomex Felt, Fiber Glass, Woven Composite B and G, Nano Material #0, Cork, Garlock, of different thicknesses are tested using the present approach. Assuming the thicknesses of samples unknown, the present approach is shown to work well in estimating the dielectric constants and the thicknesses. A number of two layer composite materials formed by various combinations of above individual materials are tested using the present approach. However, the present approach could not provide estimate values close to their true values when the thicknesses of individual layers were assumed to be unknown. This is attributed to the difficulty in modelling the presence of airgaps between the layers while doing the measurement of S-parameters. A few examples of three layer composites are also presented.				
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