MH4D Progress Report

Annual Report FY04

December 29, 2002 — June 6, 2003

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1st and 2nd Quarters FY04

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We report progress for the development of MH4D for the first and second quarters of FY2004, December 29, 2002 – June 6, 2003. The present version of MH4D can now solve the *full viscous and resistive MHD equations* using either an explicit or a semi-implicit time advancement algorithm. This fulfills with what we promised in the proposal.

In this report we describe progress in the following areas

Presentation of a Poster at the EGS-AGU-EUG Joint Assembly in Nice, France, April 6-11, 2003.

Presentation of a Poster at the 2003 International Sherwood Theory Conference in Corpus Christi, Texas, April 28-30 2003.

Code Development.

- Implementation of the MHD equations.
- Implementation of the semi-implicit algorithm.

Validation of the New Features Implemented in the Code

- The Sod's problem.
- Alfvén waves in a cylinder.

Details are given in the following sections.

1. PRESENTATION OF A POSTER AT EGS-AGU-EUG JOINT ASSEMBLY

We have attended the EGS-AGU-EUG Joint Assembly that was held in Nice, France between 6 and 11 April 2003. A poster illustrating the structure of the code and the most recent result was presented. (Dalton D. Schnack and Roberto Lionello `` Developing an MHD Algorithm on an Unstructured Mesh of Tetrahedra'', *Geophysical Research Abstracts, Vol. 5, 04691, 2003*). The text of the abstract follows:

> MH4D (Magnetohydrodynamics on a TETRAhedral Domain) is a massively-parallel, device-independent numerical code for the solution of the resistive and viscous MHD equations on an unstructured grid of tetrahedra. The unstructured grid allows the resolution to be increased in the regions of physical interest. Consequently, MH4D can model problems with a wide range of spatial scales (e.g., active regions in the large scale corona). A variational formulation of the differential operators ensures accuracy and the preservation of the analytical properties of the operators ($\nabla \cdot \mathbf{B} = 0$) and self-adjointness of the resistive and viscous operators. The combined semi-implicit treatment of the waves and implicit formulation of the diffusive operators can accommodate the wide range of time scales present in the

solar corona. The capability of mesh refinement and coarsening is also included. MH4D is currently capable of solving the resistive diffusion equation and the equations of hydrodynamics. We will present preliminary results.

2. PRESENTATION OF A POSTER AT THE SHERWOOD CONFERENCE

We have attended the 2003 International Sherwood Theory Conference that was held in Corpus Christi between 28 and 30 April 2003. A poster illustrating the structure of the code and the most recent result was presented. (R. Lionello and D. D. Schnack `` MH4D: an MHD Algorithm on an Unstructured Tetrahedral Grid'', *1E37*). The text of the abstract follows:

MH4D (Magnetohydrodynamics on a TETRAhedral Domain) is a massively-parallel, device-independent numerical code for the solution of the resistive and viscous MHD equations on an unstructured grid of tetrahedra. The unstructured grid allows the resolution to be increased in the regions of physical interest. Consequently, MH4D can model problems with a wide range of spatial scales (e.g., active regions in the large scale corona). A variational formulation of the differential operators ensures accuracy and the preservation of the analytical properties of the operators ($\nabla \cdot \mathbf{B} = 0$), and self-adjointness of the resistive and viscous operators. The combined semi-implicit treatment of the waves and implicit formulation of the diffusive operators can accommodate the wide range of time scales present in the solar corona. The capability of mesh refinement and coarsening is also included. MH4D is currently capable of solving the resistive diffusion equation and the equations of hydrodynamics. We are currently implementing the MHD operators and will present preliminary results.

3. CODE DEVELOPMENT

MH4D can now solve the full resistive and viscous MHD equations:

(1) $$\mathbf{B} = \nabla \times \mathbf{A}$$

(2) $$\mathbf{J} = \nabla \times \mathbf{B}$$

(3) $$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} - \eta \nabla \times \nabla \times \mathbf{A}$$

(4) $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

(5) $$\frac{\partial p}{\partial t} + \nabla \cdot (p \mathbf{v}) = -(\gamma - 1) p \nabla \cdot \mathbf{v}$$

(6) $$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \mathbf{J} \times \mathbf{B} + \nabla \cdot (\nu \rho \nabla \mathbf{v})$$

This set of equations can be advanced explicitly or semi-implicitly. We describe now how the magnetic operators and the semi-implicit solve have been implemented.

3.1 Implementation of the MHD Equations.

We have implemented subroutines to calculate the advective term in the induction equation for the vector potential and the Lorentz's force term in the momentum equation. The advection term, calculated on vertex *i*, has been written as

(7) $$(\mathbf{v}\times\overline{\mathbf{B}})_i = \mathbf{v}_i \times \frac{1}{N_i}\sum_{I(i)=1}^{N_i}\mathbf{B}_{I(i)},$$

where the value of the magnetic field, $\mathbf{B}_{I(i)}$, is the average of all the values of the magnetic field located on the centroids surrounding vertex *i*.

The expression for the Lorentz's force term is

(8) $$(\mathbf{J}\times\overline{\mathbf{B}}')_i = \mathbf{J}_i \times \frac{1}{4V_i}\sum_{I(i)=1}^{N_i}\mathbf{B}_{I(i)}V_{I(i)}.$$

$V_{I(i)}$ is the volume of the centroid *I* surrounding vertex *i* and V_i is the volume of the dual mesh element centered on vertex *i*. This volume average has been introduced to preserve the self-adjointness of the numerical representation of the MHD operator.

3.1 Implementation of the Semi-Implicit Algorithm

In the lower corona, the Alfvén and sound speed are large compared with the speed of the flow. An explicit algorithm would require a very small time-step to follow the propagation of short-wavelength, high frequency waves that are of minimal importance for the dynamics of the systems we want to model. For this reason we apply a semi-implicit treatment to the wave-like terms. A semi-implicit formulation avoids the time-step restriction imposed by high frequency compressional and torsional Alfvén waves and allows for an arbitrarily large time-step with respect to the normal modes of the system.. In order to implement the semi-implicit method, the momentum equation has been modified as follows:

(9) $$\rho\frac{\partial\mathbf{v}}{\partial t} = \mathbf{F} + \alpha\Delta t\mathsf{S}\cdot\frac{\partial\mathbf{v}}{\partial t},$$

where **F** represents the explicit forces appearing in Eq. (6), α is a numerical factor, and S is the semi-implicit operator. Our choice for S is the following

(10) $$\mathsf{S}\cdot\mathbf{v} = \mathbf{S} = \nabla\times\nabla\times(\mathbf{v}\times\mathbf{B})\times\mathbf{B}. + \nabla\gamma p\nabla\cdot\mathbf{v}.$$

S is the sum of the linearized MHD waves and sound waves operators. This choice of S is motivated by the fact that the Alfvén time scale determines the stiffness of the problem in the lower corona. However S is easier to invert than the full MHD operator. The expression for S minimizes the following functional

(11) $$I(\mathbf{v}) = \frac{1}{2}\int_V\left[|\nabla\times(\mathbf{v}\times\mathbf{B})|^2 + \gamma p(\nabla\cdot\mathbf{v})^2 + 2\mathbf{S}\cdot\mathbf{v}\right]dV.$$

We have made sure that the numerical discretization of S preserves the self-adjointess of the analytical operator. The operator is inverted using the same parallel, preconditioned

CG solver implemented to invert the resistive diffusion operator in the induction equation.

4. VALIDATION OF THE NEW FEATURES IMPLEMENTED IN THE CODE

4.1 The Sod's Problem

We have conducted another test of the implementation of the hydrodynamic equations in MH4D by reproducing the Sod's problem (Sod, G. A. 1978, J. Comp. Phys., 27, 1). This consists of the following: at t=0 a diaphgram separates a tube in two regions filled with a fluid with different densities and pressures.

$$\begin{cases} \rho_{\text{left}} = 1.0 \\ p_{\text{left}} = 1.0, \\ v_{z\text{left}} = 0.0 \end{cases} \begin{cases} \rho_{\text{right}} = 0.125 \\ p_{\text{right}} = 0.1 \\ v_{z\text{right}} = 0.0 \end{cases} . \tag{12}$$

The specific heat ratios γ is set to 1.4. At t>0 the diaphgram is broken. We have designed a cylindrically shaped grid of radius 0.5 and height 1 with 10300 vertices and 50525 cells. We have evolved the hydrodynamic equations in MH4D stopping before any wave has reached the left or right boundary. The results of the simulation are graphically presented in Fig. 1 (velocity), 2 (pressure), and 3 (density). A shock wave propagates to the right, a rarefaction wave to the left, and a contact discontinuity, visible in the density plot (Fig. 3), follows the shock.

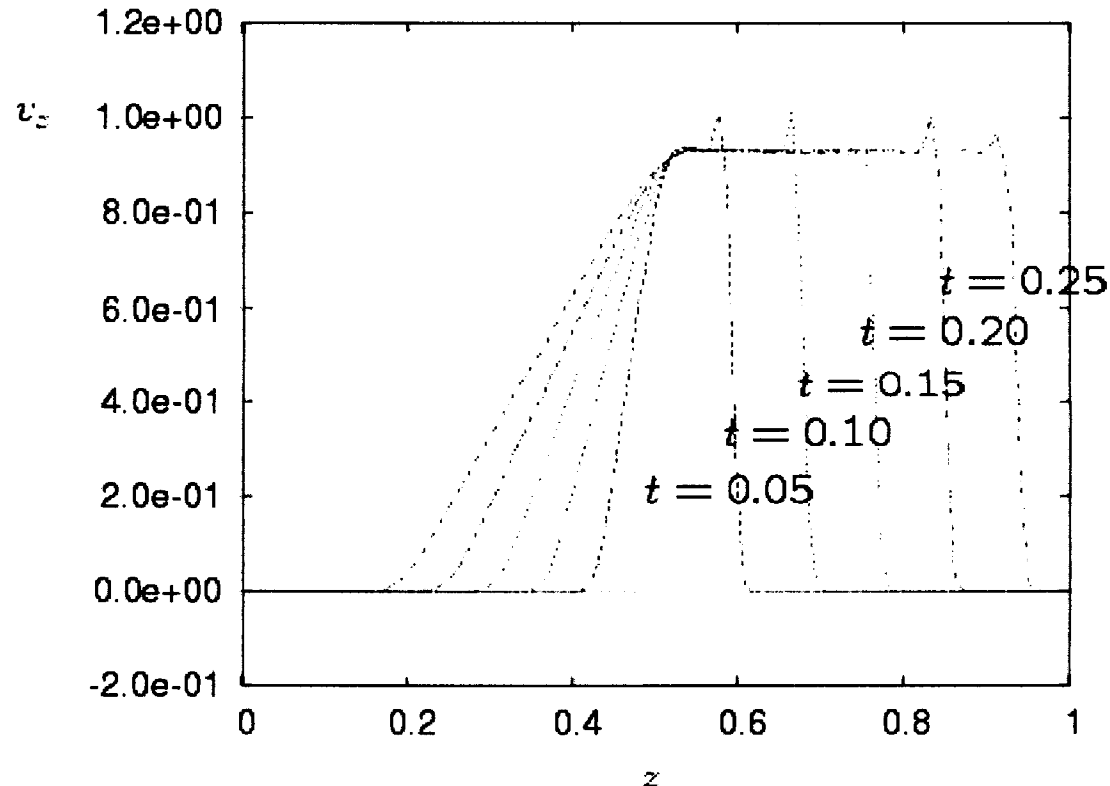

FIG. 1. The component of the velocity along the tube in the Sod's problem at different instants in time. After the diaphgram is broken a shockwave propagates from the left to the right.

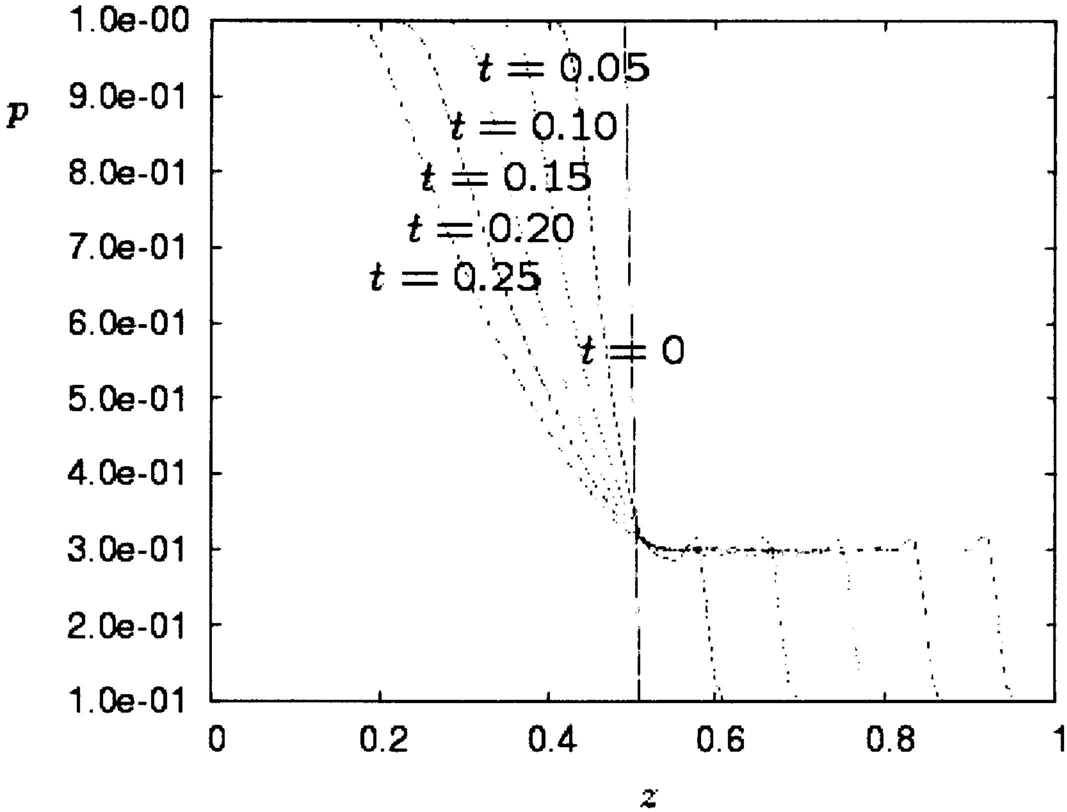

FIG. 2. Evolution of the pressure in the Sod's problem. A shock wave moves rightward and a rarefaction wave leftward.

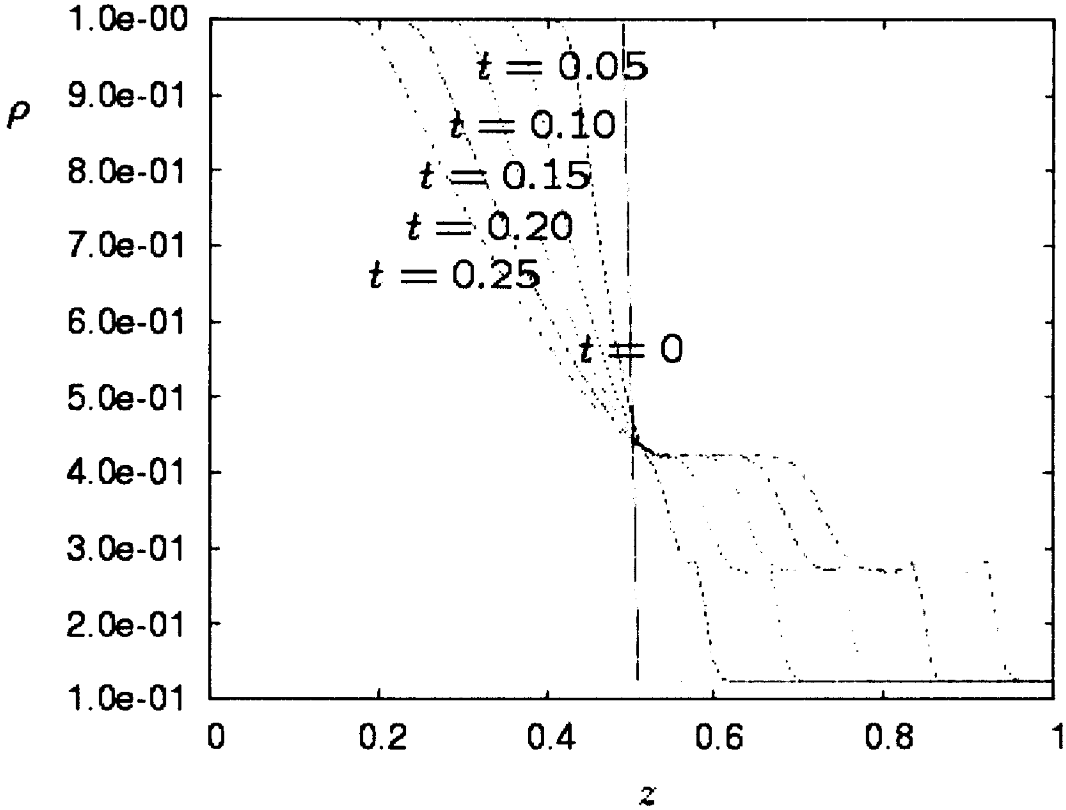

FIG. 3. Evolution of the density in the Sod's problem. We notice a shock wave propagating to the right followed by a contact discontinuity. A rarefaction wave moves to towards the left.

4.2 Alfvén Waves in a Cylinder

We have tested the implementation of the MHD equations by reproducing eigenfunctions of the linearized equations (i.e. Alfvén waves). The equations for linear Alfvén waves are:

$$\frac{\partial \delta \mathbf{A}}{\partial t} = \delta \mathbf{v} \times \mathbf{B}, \tag{13}$$

$$\rho \frac{\partial \delta \mathbf{v}}{\partial t} = \delta \mathbf{J} \times \mathbf{B}. \tag{14}$$

We have used δ to indicate the perturbed quantities. A solution (eigenvector) in a cylinder where $\mathbf{B} = B\hat{\mathbf{z}}$ is

$$\begin{cases} \delta A_r = \varepsilon f(r) \cos \omega t \sin kz \\ \delta B_\phi = \varepsilon k f(r) \cos \omega t \cos kz, \\ \delta v_\phi = \varepsilon k^2 f(r) \sin \omega t \sin kz \end{cases} \tag{15}$$

with angular speed

(16) $$\omega = k\sqrt{\frac{B^2}{\rho}}.$$

$f(r)$ is an arbitrary function of r. Equation (14) represents a torsional Alfvén wave.

For $k = \pi$, $\rho = 1$, $B = 1$, the theoretical period is $T = 2$. For a cylindrical computational domain of radius and height equal to one, we have designed a grid of 420 vertices and 1692 cells. We have applied the perturbation as in Eq. (15) and advanced Eqs. (1-6) with MH4D using both the explicit and the semi-implicit time advancement with no noticeable difference between the two. In Fig. 4 we show the time history of the kinetic energy, which shows a period of 1, as expected. In Figs. 5 and 6 we show respectively the poloidal component of the magnetic field at four instants in time.

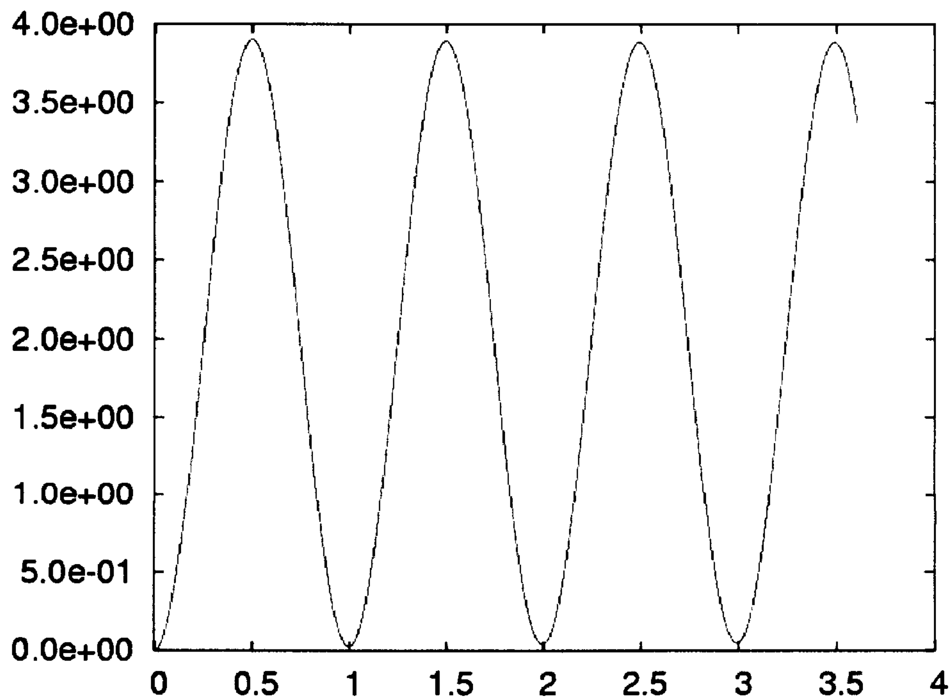

FIG. 4. Kinetic energy as a function of time for the case of the torsional Alfvén wave eigenmode. in a cylinder.

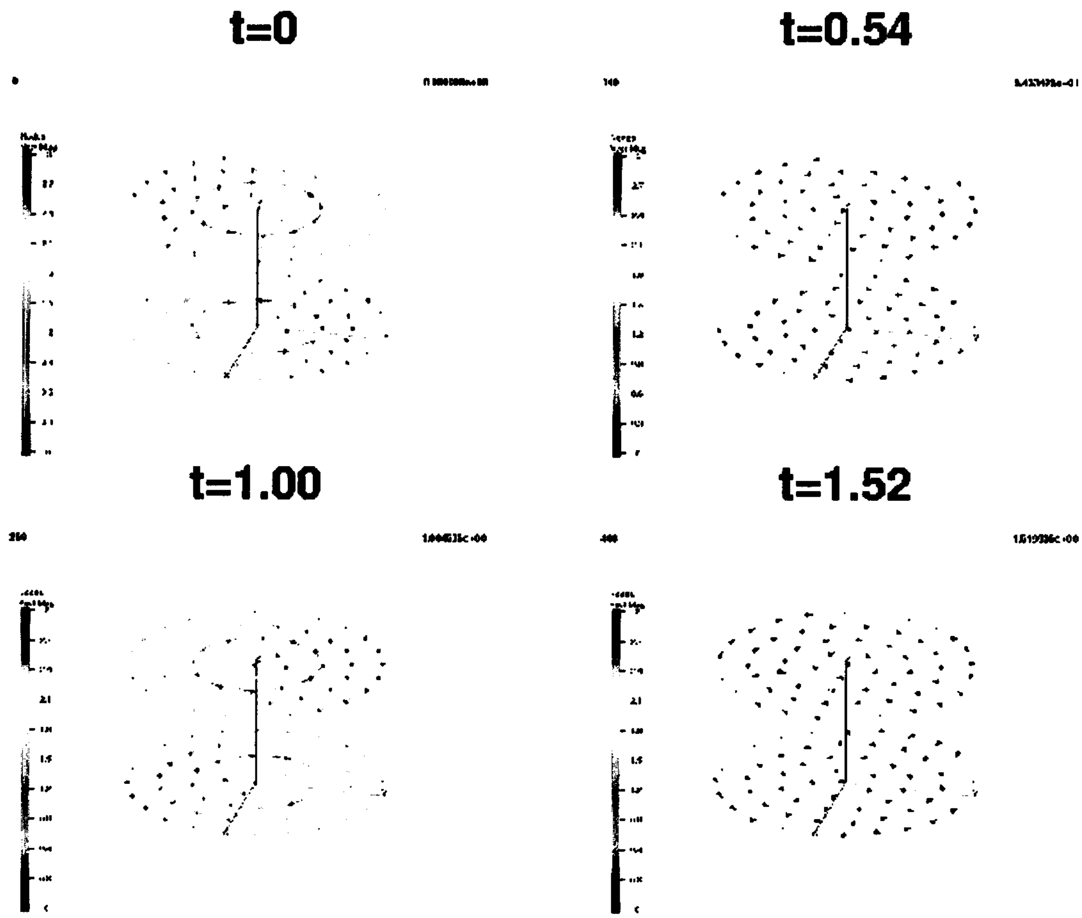

FIG. 5. The poloidal magnetic field is shown at four instants in time in the simulation of a torsional Alfvén wave in a cylinder obtained with MH4D. The vectors are colored according to their magnitude.

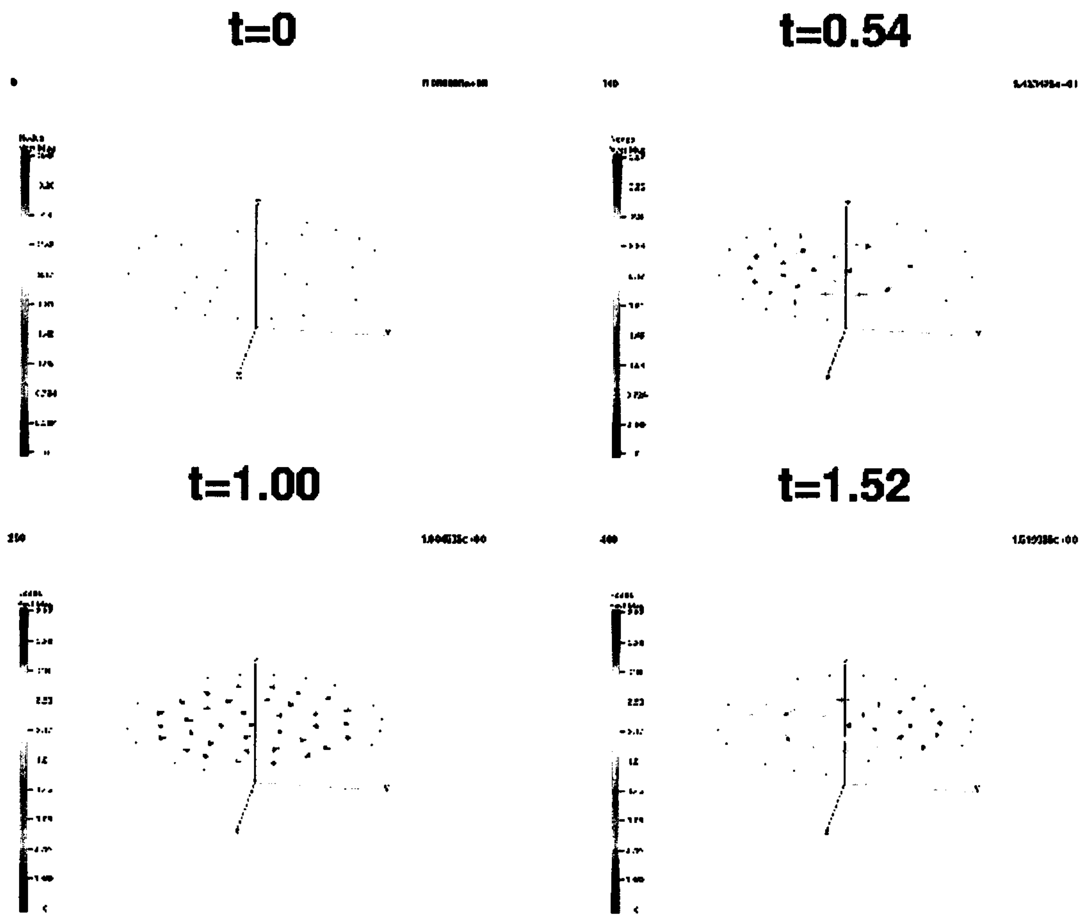

FIG. 6. The poloidal velocity field is shown at four instants in time in the simulation of a torsional Alfvén wave in a cylinder obtained with MH4D. The vectors are colored according to their magnitude.

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13. Abstract *(Maximum 200 Words)* *(abstract should contain no proprietary or confidential information)*

We report progress for the development of MH4D for the first and second quarters of FY2004, December 29, 2002 - June 6, 2003. The present version of MH4D can now solve the full viscous and resistive MHD equations using either an explicit or a semi-implicit time advancement algorithm. In this report we describe progress in the following areas. During the two last quarters we have presented poster at the EGS-AGU-EUG Joint Assembly in Nice, France, April 6-11, 2003, and a poster at the 2003 International Sherwood Theory Conference in Corpus Christi, Texas, April 28-30 2003. In the area of code development, we have implemented the MHD equations and the semi-implicit algorithm. The new features have been tested.

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